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**Dr. Reena Garg**  
 Assistant, Professor  
 (Mathematics), Department of  
 Humanities and Sciences, YMCA  
 University of Science and  
 Technology, Faridabad,  
 Haryana, India

## Analysis of various factors of reliability using Boolean function technique in refinery unit

**Dr. Reena Garg**

### Abstract

For the system of multistation elements, the problems of developing Boolean reliability model has acknowledged on the basis of algebra of groups of incompatible events. The objective of this paper to find the various reliability factor of refinery unit which is a subunit of Vacuum Distillation Unit (VDU). The subunits used in the considered unit are Pump, Desalter, Furnace, atmospheric fractionator and gas separator and these subunits are connected through pipes. The reliability factor and mean time to failure (MTTF) is Exponentially and weibully distributed. We illustrate the use of technique by means of examples and introduce numerical results.

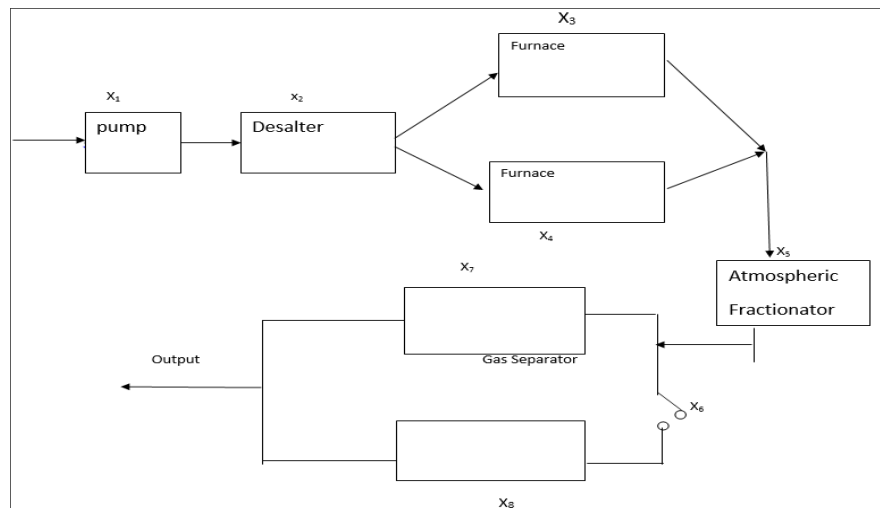
**Keywords:** Boolean function, reliability, vacuum distillation unit (VDU), MTTF, Desalter, refinery

### Introduction

About 80% of the refineries have a vacuum distillation unit. A secondary processing unit consisting of VDU. In the refining process, the atmospheric distillation unit is separate the lighter hydro carbon from the heavier oil based on the boiling point. The ADU is capable of boiling crude oil fraction to the temperature of 750°F. Above this temperature the oil will be thermally crack impedes the distillation process. The lighter products are boiled up and the heavier oil called bottoms remain at the bottom of ADU. These bottoms are run through a vacuum distillation column for further refinement.

These vacuum distillation columns are less than atmospheric pressure of 760 mm of mercury. Gas oils are slightly heavier than middle distillates such as jet fuel, Diesel, Kerosene. These gas oils are further refined to make such light cycle oil, gasoline.

The desalted crude enter the heat exchange network which use the vapour of column condenser, the pump around the circuits streams and the products are cooled and then enters the furnace where it is heated to about 340-372 °C.



**Fig 1**

**Correspondence**  
**Dr. Reena Garg**  
 Assistant, Professor  
 (Mathematics), Department of  
 Humanities and Sciences, YMCA  
 University of Science and  
 Technology, Faridabad,  
 Haryana, India

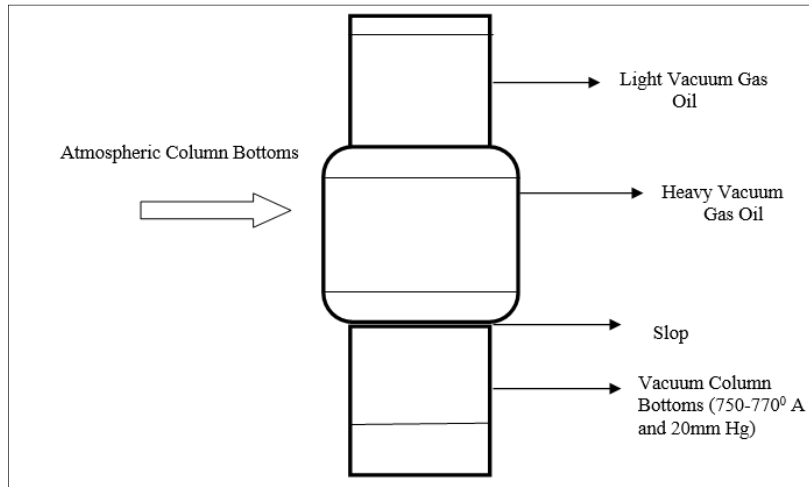


Fig 2

**Assumptions**

The following assumption have been associated with this model:

- a) Initially, the whole system is good and operable.
- b) Every component of the system remains either in good or bad state.
- c) There is no repair facility to repair a failed component.
- d) The whole system can fail due to failure of any one of its units.
- e) Failures are statistically-independent.

**Notations**

- $x_1$  : state of pump
- $x_2$  : state of desalter
- $x_3, x_4$  : states of furnace
- $x_5$  : state of atmospheric fractionator
- $x_6$  : state of switching device
- $x_7, x_8$  : states of gas separator
- $x_i (i=1,2,3,\dots,8)$  : 1 in good state; 0 is bad state.
- $x^i$  : negation of for  $x^i$  all  $i$
- $\wedge / \vee$  : conjunction/disjunction
- $||$  : this notation has used to represent logical matrix.
- $R_i$  : Reliability of  $i^{th}$  part of the system,  $\forall i=1,2,\dots,8$ .
- $Q_i$  :  $1-R_i$
- $R_s$  : Reliability of the whole system.
- $R_{sw}(t)/R_{SE}(t)$  : Reliability of the system as a whole when failures follow weibull/Exponential time distribution.

**Formulation of mathematical model**

By making use of Boolean function technique, the condition of capability of successful operation of the system in terms of logical matrix are expressed as shown below:

$$F(x_1, \dots, x_8) = \begin{vmatrix} x_1 & x_2 & x_3 & x_5 & x_7 \\ x_1 & x_2 & x_4 & x_5 & x_7 \\ x_1 & x_2 & x_3 & x_5 & x_6 & x_8 \\ x_1 & x_2 & x_4 & x_5 & x_6 & x_8 \end{vmatrix} \tag{1}$$

**Solution of the model**

By using algebra of logics, we may write eq<sup>n</sup> (1) again as,

$$F(x_1, x_2, \dots, x_8) = (x_1 x_2 x_5) \wedge f(x_1, x_2, \dots, x_8) \tag{2}$$

where

$$f(x_1, x_2, \dots, x_8) = (x_1 \ x_2 \ x_5) \wedge \begin{vmatrix} x_3 & x_7 \\ x_4 & x_7 \\ x_3 & x_6 & x_8 \\ x_4 & x_6 & x_8 \end{vmatrix} \quad (3)$$

Let

$$B_1 = \begin{vmatrix} x_3 & x_7 \end{vmatrix} \quad (4)$$

$$B_2 = \begin{vmatrix} x_4 & x_7 \end{vmatrix} \quad (5)$$

$$B_3 = \begin{vmatrix} x_3 & x_6 & x_8 \end{vmatrix} \quad (6)$$

$$B_4 = \begin{vmatrix} x_4 & x_6 & x_8 \end{vmatrix} \quad (7)$$

$$\Rightarrow \begin{vmatrix} x_1 & x_2 & x_5 \end{vmatrix} \wedge \begin{vmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{vmatrix} \quad [\text{from eq. (4),(5),(6),(7)}]$$

Using orthogonalization process

$$\begin{vmatrix} B_1 \\ B_1' & B_2 \\ B_1' & B_2' & B_3 \\ B_1' & B_2' & B_3' & B_4 \end{vmatrix} \quad (8)$$

Where

$$B_1' = \begin{vmatrix} x_3' \\ x_3' & x_7' \end{vmatrix} \quad (9)$$

$$B_1' B_2' = \begin{vmatrix} x_3' & x_4' & x_7' \end{vmatrix} \quad (10)$$

$$B_1' B_2' B_3' = \begin{vmatrix} x_3' & x_4' & x_6' & x_7' & x_8' \\ x_3' & x_4' & x_6' & x_7' & x_8' \end{vmatrix} \quad (11)$$

Using all these values in equation (8), we obtain

$$f(x_3, x_4, x_6, x_7, x_8) = \begin{vmatrix} x_3 & x_7 \\ x_3' & x_4' & x_7' \\ x_3 & x_4' & x_6' & x_7' & x_8 \\ x_3 & x_4 & x_6 & x_7 & x_8 \\ x_3' & x_4' & x_6' & x_7' & x_8' \end{vmatrix} \quad (13)$$

Using equation(13), equation (2) becomes

$$F(x_1, x_2, \dots, x_8) = \begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 \\ x & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 \end{vmatrix} \quad (14)$$

since equation (13) is the disjunction of disjoint conjunction, therefore, the reliability of the whole system is given by  $R_s = P_r\{F(x_1, x_2, \dots, x_8)\}$  Or

$$\begin{aligned} R_s &= R_1 R_2 R_5 [R_3 R_7 + R_3 R_4 R_7 + R_3 R_4 R_6 R_7 R_8 + R_3 R_4 R_6 R_7 R_8 + R_3 R_4 R_6 R_7 R_8] \\ &= R_1 R_2 R_5 [R_3 R_7 + (1 - R_3) R_4 R_7 + R_3 (1 - R_4) R_6 (1 - R_7) R_8 + R_3 R_4 R_6 (1 - R_7) R_8 + (1 - R_3) R_4 R_6 (1 - R_7) R_8] \\ &= R_1 R_2 R_5 [R_3 R_7 + R_4 R_7 - R_3 R_4 R_7 + R_3 R_6 R_8 (1 - R_4) (1 - R_7) + R_3 R_4 R_6 R_8 (1 - R_7) + R_4 R_6 R_8 (1 - R_3) (1 - R_7)] \\ &= R_1 R_2 R_5 [R_3 R_7 + R_4 R_7 - R_2 R_4 R_7 + R_3 R_6 R_8 - R_3 R_7 R_6 R_8 - R_4 R_6 R_3 R_8 + R_3 R_4 R_6 R_7 R_8 + R_3 R_4 R_6 R_8 - R_3 R_4 R_6 R_7 R_8 + R_4 R_6 R_8 - R_3 R_4 R_6 R_8 - R_4 R_6 R_7 R_8 + R_3 R_4 R_6 R_7 R_8] \\ &= R_1 R_2 R_3 R_5 R_7 + R_1 R_2 R_4 R_5 R_7 - R_1 R_2 R_3 R_4 R_5 R_7 + R_1 R_2 R_3 R_5 R_6 R_8 - R_1 R_2 R_3 R_7 R_5 R_6 R_8 - R_1 R_2 R_4 R_5 R_6 R_3 R_8 + R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 + R_1 R_2 R_3 R_4 R_5 R_6 R_8 - R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 + R_1 R_2 R_4 R_5 R_6 R_8 - R_1 R_2 R_3 R_4 R_5 R_6 R_8 - R_1 R_2 R_4 R_5 R_6 R_7 R_8 + R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 \end{aligned}$$

**Some Particular Cases**

**Case 1:** When reliability of each component is R.

$$2R^5 - 3R^7 + R^6 + R^8 \quad (15)$$

**Case 2:** When failure rates follow weibull distribution

$$R_{sw}(t) = \sum_{i=1}^7 e^{-\alpha_i t^b} - \sum_{j=1}^6 e^{-\beta_j t^b} \quad (16)$$

- $\alpha_1 = c + a_3 + a_7$
- $\alpha_2 = c + a_4 + a_7$
- $\alpha_3 = c + a_4 + a_6 + a_8$
- $\alpha_4 = c + a_3 + a_4 + a_6 + a_7 + a_8$
- $\alpha_5 = c + a_3 + a_4 + a_6 + a_8$
- $\alpha_6 = c + a_4 + a_6 + a_8$
- $\alpha_7 = c + a_3 + a_4 + a_6 + a_7 + a_8$
- $c = a_1 + a_2 + a_5$
- $\beta_1 = c + a_3 + a_4 + a_7$
- $\beta_2 = c + a_3 + a_4 + a_6 + a_8$
- $\beta_3 = c + a_4 + a_6 + a_7 + a_8$
- $\beta_4 = c + a_3 + a_4 + a_6 + a_7 + a_8$
- $\beta_5 = c + a_3 + a_4 + a_6 + a_8$
- $\beta_6 = c + a_4 + a_6 + a_7 + a_8$

**Case 3:** When failure rate follow exponential time distribution

Exponential distribution is nothing but a particular case of weibull distribution for  $b=1$  and is very useful for practical problems purpose. Therefore, the reliability of considered system as a whole at an instant 's' is expressed as:

$$R_{SE}(t) = \sum_{i=1}^7 e^{-\alpha_i t} - \sum_{j=1}^6 e^{-\beta_j t} \quad (17)$$

Where  $\alpha_i$  and  $\beta_j$ 's have mentioned earlier.

Also, an important reliability parameter, viz; M.T.T.F., in this case is given by

$$\begin{aligned} M.T.T.F &= \int_0^{\infty} R_{SE} dt \\ &= \sum_{i=1}^7 \frac{1}{\alpha_j} - \sum_{i=1}^6 \frac{1}{\beta_j} \end{aligned} \quad (18)$$

**Numerical Example**

For a numerical computation, setting:

(A)  $a_i (i=1,2,\dots,7) = 0.001$ ,  $\alpha=2$  and  $t=0,1,2,\dots$  In eq. (16)

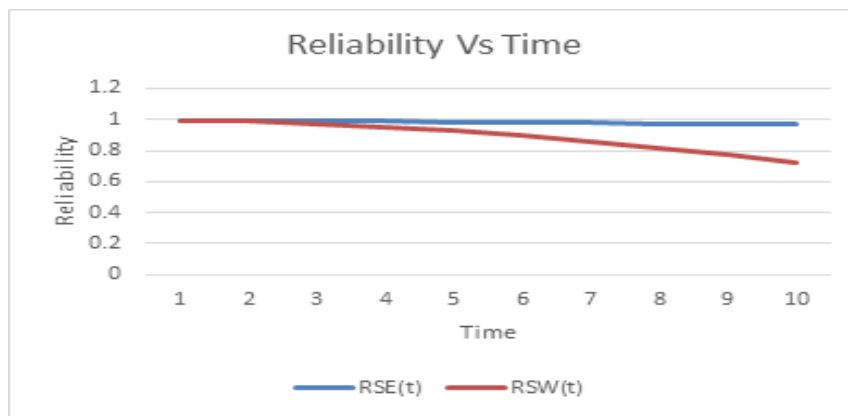
(B)  $a_i (i=1,2,\dots,7) = 0.001$ , and  $t=0,1,2,\dots$  in eq.(17)

(C)  $a_i (i=1,2,\dots,7) = 0,0.1,\dots,1.0$  in eq. (18)

one can compute the table 1 and 2. The corresponding graph have been shown through fig. 3 and 4. Respectively

**Table 1**

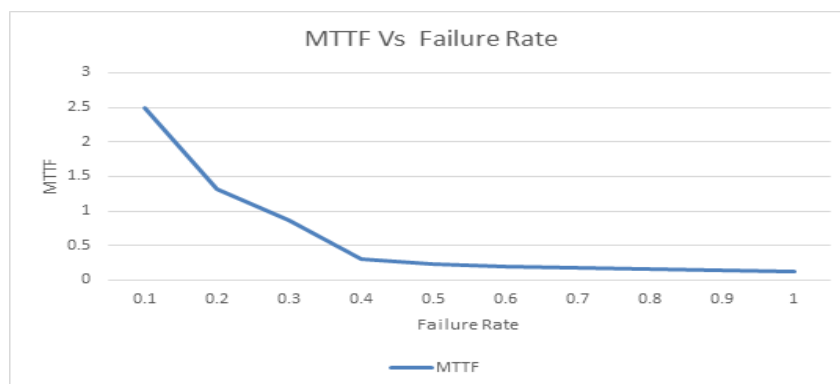
T	R <sub>sw</sub> (t)	R <sub>se</sub> (t)
0	1	1
1	0.9970015085	0.9970015085
2	0.9940060676	0.9880245381
3	0.9910137276	0.9731275449
4	0.9880245381	0.9524173155
5	0.985038548	0.9260614724
6	0.9820558059	0.8943031027
7	0.9790763598	0.8574750472
8	0.9761002572	0.8160112371
9	0.9731275449	0.7704526549
10	0.9701582693	0.7214460083



**Fig 3**

**Table 2**

A	MTTF
0	$\infty$
0.1	2.4920634921
0.2	1.3154761905
0.3	.86984127
0.4	.3005952381
0.5	0.2404761905
0.6	0.2003968254
0.7	0.1717687075
0.8	0.150297619
0.9	0.1335978836
1	0.1202380952



**Fig 4**

### Results and Discussion

In this paper the author has acknowledged a refinery unit which is a subunit of vacuum distillation unit for the analysis of various reliability parameter by utilizes the Boolean function technique and algebra of logics.

Table 1 computes the reliability of system with respect to time when failure rate is exponential and weibull distribution. An inspection of graph 'Reliability Vs time' fig. 3 compute that in case of exponential distribution, reliability of the complex system decreases approximately at uniform rate but in case of weibull distribution, failure rate decreases rapidly.

Table 2 and graph "M.T.T.F Vs failure rate" produce that M.T.T.F of the system decreases catastrophically in the beginning but after that it decreases approximately at a uniform rate.

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