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# A general approach to defining a contractive like multi-valued mapping

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**Abstract**
The objective of this work is to illustrate a new concept to examine the existence of fixed points for multivalued *F*-contraction in the. In this generalize approach, we present the idea of multivalued *F*-contraction and also proved corresponding fixed point theorems in overall modular metric space with some specific assumption. We justify fixed point theorems for F-contractions of Hardy–Rogers type involving self-mappings characterized on metric spaces as well as ordered metric spaces. Then we implement our findings to create the existence of solutions for non-linear integral equations of certain type. An example as well as its application to multistage decision processes are illustrated to describe the usability of the derived theorems.

**Keywords:** fixed point, multi-valued mapping, metric space and hardy–rogers

## 1. Introduction

Many researchers in mathematics as well as in applied sciences are showing their interests in illustrating necessary as well as sufficient conditions, to stating the existence and uniqueness of fixed points for self-mappings. The key fundamental concept of these findings is the generic fixed point problem p = T p, where T: Ṣ → Ṣ is a self-mapping of a space Ṣ. It is well understood that a huge variety of mathematical and practical problems can be explained by minimizing them to an equivalent fixed point problem. In reality, by illustrating suitable operators, it is possible to explain an equilibrium problem by probing the fixed points of these operators. Furthermore, the solutions of differential equations can also be found in terms of fixed points of integral-differential operators. These solution sets can also be distinguished by a stability analysis of fixed point's sets.

These fundamental concepts are real encouragements to boost up the interest of mathematicians and researchers to stating extensions as well as generalizations of the Banach fixed point theorem [1], which is globally acknowledged as the basic findings of fixed point theory [2, 3]. In this proposed work, we carry forward these findings by illustrating existence and uniqueness fixed point theorems for a self-mapping. In this paper, our objective to analysis of Hardy–Rogers-type conditions, which demonstrate a generalized concept for Banach fixed point theorem. Here, the collaborative idea by Wardowski [4] for concept of F-contraction with extension of Hardy and Rogers [5].

The Banach contraction principle [1] is crucial analytical findings and treated as the origin of metric fixed point theory. It is prominent findings in many branches of mathematics for fixed point. These outcomes can be explained in various ways like Wordowski [4] illustrated the theory of F-contraction which generalized the Banach contraction principal in various directions. Later, Sgroi in 2013 [6] concluded a multivalued mapping of Wordowski's findings. Likewise, Chistyakov in 2010 [7] found the basics of modular metric spaces and illustrated few findings. The fixed point property in this space has been revealed by many researchers in current decade [8-13]. In this present work, we state a generalize approach to defining contractive like multivalued mapping. Our result is a partial extension of Hardy–Rogers-type conditions. We also give an application of our main findings to establish the existence of the solution of a non-linear integral equation.

**2. Basic Fundamental Concepts**

Many researchers have illustrated fixed point theory for self-mappings on partially ordered sets particularly in dealing with differential equations. Let's assume (Ş, d) be a metric space and (Ş, €) be a partially ordered non-empty set, therefore (Ş, d, €) is known as an ordered metric space. Furthermore, two elements p, q ∈ Ş are called comparable if x € y or y € x holds.

A self-mapping T on a partially ordered set (Ş, €) is called non-decreasing if Tp € Tq whenever p € q for all p, q ∈ Ş. Thus, an ordered metric space (Ş, d, €) is known as regular if for every non-decreasing sequence $\{p_n\}$ in Ş, convergent to some p ∈ Ş, we find $p_n$ € p for all n ∈ N ∪ {0}.

**Theorem 1**

Let's assume (Ş, d, €) be a complete ordered metric space, and assume T be a non-decreasing self-mapping on Ş. Let, there exist a continuous R ∈ R and Φ ∈ Ũ such that T is an ordered F-contraction of Hardy–Rogers type, that is,

$$\Phi(d(p,q))+R(d(T_p,T_q))\le R(\alpha d(p,q)+\beta d(p,T_p)+\gamma d(q,T_q)+\delta d(p,T_q)+\xi d(q,T_p) \qquad (1)$$

For all comparable p, q ∈ Ş with Tp≠Tq, where α, β, γ, δ, ζ ∈ [0, +∞[, α + β + γ + 2δ = 1, γ ≠ 1 and α + δ + ζ≤1. If the conditions given below are satisfied

a) There exists $p_0$ ∈ Ş such that $p_0$ € $Tp_0$
b) Ş is regular

Then T has a fixed point. Moreover, the set of fixed points of T is well-ordered if and only if T has a unique fixed point.

**Proof:** Let $p_0$ ∈ Ş be an arbitrary point such that (a) holds, and let's assume $\{p_n\}$ be the Picard sequence of initial point $p_0$, that is, $p_n = T^np_0 = Tp_{n-1}$. If $p_n = p_{n-1}$ for some n ∈ N, then $p_n$ is a fixed point of T and this defines the existence of a fixed point of T in Ş.

Let $d_n = d(p_n, p_{n+1}) > 0$ for all n ∈ N ∪ {0}.

As T is non-decreasing, we conclude that $p_0$ € $p_1$ € $p_2$...... € $p_n$ · · · (2)

That is, $p_{n-1}$ and $p_n$ are comparable and $Tp_{n-1} \neq Tp_n$ for all n ∈ N.

By means of theorem 1 proof, we get that $\{p_n\}$ is a Cauchy sequence. Since (Ş, d) is a complete metric space, there exists z ∈ Ş such that $\{p_n\} \to z$ as $n \to +\infty$. If z = T z, the proof is finished.

Let's assume that z ≠Tz. Since Ş is regular, from (2) we can say that $\{p_n\}$ and z are comparable and $Tp_n \neq Tz$ for all n ∈ N ∪ {0}. Now, by using (3), we obtain

$$d(T_p,T_q)<\alpha d(p,q)+\beta d(p,T_p)+\gamma d(q,T_q)+\delta d(p,T_q)+\xi d(q,T_p) \qquad (3)$$
$$d(z,T_z)\le d(z,p_{n+1})+d(Tp_n,T_z)$$
$$<d(z,p_{n+1})+\alpha d(p_n,z)+\beta d(p_n+p_{n+1})+\gamma d(z,T_z)+\delta d(p_n,T_z)+\xi d(z,p_{n+1})$$

By putting $n \to +\infty$ in the previous inequality, we get

$d(z,T_z)\le(\gamma+\delta)d(z,T_z)<d(z,T_z)$, which is a contradiction. Therefore z=Tz.

Further we assume, that the set of fixed points of T is well-ordered. We state that the fixed point of T is unique. Let's assume on the contrary that there exists another fixed point w in Ş such that z ≠w. Then, by using eq. (1) with p = z and q = w, we find

$$\Phi(d(z,w))+R(d(z,w))=\Phi(d(z,w))+R(d(T_z,T_w))$$
$$\le R(\alpha d(z,w)+\beta d(p,T_z)+\gamma d(w,T_w)+\delta d(z,T_w)+\xi d(w,T_z)$$
$$=R((\alpha+\delta+\xi)d(z,w))\le R(d(z,w))$$

Which is a contradiction, therefore z=w. Conversely, the entity is well ordered for set of fixed points of T when T has a unique fixed point.

**Theorem 2:** Let's assume (Ṣ, d) be a complete ordered metric space as well as assume T be a non-decreasing self-mapping over Ṣ. Let's assume that there exist a continuous R ∈ Ɍ and Φ ∈ Ũ such that T is an ordered F-contraction of Hardy–Rogers type. If the finding below are satisfied:

a) There exists $p_0$ ∈ Ṣ such that $p_0$€$Tp_0$
b) X is regular; Then, T has a fixed point. Furthermore, if $\alpha + 2\gamma + \delta + \zeta < 1$ and the following condition holds:
c) For all z, w ∈ Ṣ there exists v ∈ Ṣ such that z and v are comparable as well as w and v are comparable, then T has a unique fixed point.

**Proof:** The existence of a fixed point of T is an outcome of Theorem 1. Let's say z ∈ Ṣ be a fixed point of T. For all v ∈ Ṣ comparable with z such that Tz≠Tv, we hold

$$\begin{aligned}&\Phi(d(z,v))+R(d(Tz,Tv))\\&\le R(\alpha d(z,v)+\beta d(z,Tz)+\gamma d(v,Tv)+\delta d(z,Tv)+\xi d(v,Tz)\\&\le R(\alpha d(z,v)+\gamma d(v,z)+d(z,Tv)+\delta d(z,Tv)+\xi d(v,z)\\&=R((\alpha+\gamma+\xi)d(z,v))+(\gamma+\delta)d(z,Tv))\end{aligned}$$

Since F is non-decreasing, we figure out

$$d(z,Tv)<(\alpha+\gamma+\xi)d(z,v)+(\gamma+\delta)d(z,Tv)$$

and hence

$$(1-\gamma-\delta)d(z,Tv)<(\alpha+\gamma+\xi)d(z,v).$$

Since 1-γ-δ>0, we find

$$d(z,Tv)<\frac{\alpha+\gamma+\xi}{1-\gamma-\delta}d(z,v)$$

As T is non-decreasing, we find that z and $T^n v$ are comparable for all n ∈ N. If z≠ $T^n v$ for all n ∈ N, then d (z, $T^n v$) < $\lambda^n$ d (z, *v*) for all n ∈ N, where $\lambda = (\alpha + \gamma + \zeta)/ (1 - \gamma - \delta) < 1$.
From the previous inequality we find d (z, $T^n v$) → 0 as n → +∞.
Now, if z, w are two fixed points of T, by the condition (c) there exists *v* ∈ Ṣ such that z and *v* are comparable and w and *v* are comparable. If z= $T^n v$ or w= $T^n v$ for some n ∈ N, then z and w are comparable and the uniqueness of the fixed point follows since T is an F-contraction of Hardy–Rogers type.
Let's say that z ≠ $T^n v$ and w≠$T^n v$ for all n ∈ N.
Then d(z,w)≤ d(z,$T^n v$), d(w,$T^n v$) → 0 as n → +∞ and results d(z, w) = 0, that is, z = w.

### 3. Result for Multivalued F-Contrations

**Definition 1[4]**

Let's say F: $R^+$⟶R satisfying the conditions mentioned below:

a) F is strictly increasing on $R^+$,
b) for every sequence {$p_n$} in $R^+$, we have $\lim_{n\to\infty} p_n = 0$ if and only if $\lim_{n\to\infty} F(p_n) = -\infty$
c) there exists a number k∈(0,1) such that $\lim_{p\to 0^+} p^k F(p) = 0$

We denote by F the class of all function that satisfy the conditions (a-c).

**Example 1**

The following functions F: $R^+$⟶R belong to ꞙ:

1. F(p)=ln p, with p>0
2. $F(p) = \frac{-1}{\sqrt{p}}$, p>0

**Definition 2**

Let's say (Ş, w) be a modular metric space. Assume E be non-empty bounded subset of Ş. A multivalued mapping T: E⟶CB(E) is called F-contraction on Ş if F∈ƒ, and Φ∈ $R^+$, for all p,q ∈ E with q ∈ Tp there exists r ∈ Tq, such that w1(q, r)>0, the following inequality holds:

$$\Phi + F(w1(q,r)) \leq F(M(p,q)) \tag{4}$$

Where *M(p,q)=max{w1(p,q),w1(p,Tp),w1(q,Tq),w1(q,Tp)}.*

**Definition 3**

Let's say (Ş, w) be a modular metric space. Assume E be non-empty subset $Ş_w$. A multivalued mapping T:E⟶CB(E) is known to be F-contraction of Hardy-Rogers-type if F∈ƒ, and Φ∈ $R^+$, such that,

$$2\Phi + F(H_w(Tp,Tq)) \leq F(\alpha w_1(p,q) + \beta w_1(p,Tp) + \gamma w_1(q,Tq) + \xi w1(q,Tp)) \tag{5}$$

for all p,q ∈ E with $H_w(Tp,Tq)>0$, where α,β,γ,ζ≥0,α+β+γ=1 and γ≠1.

**Example 2**

Let's say F: R+⟶R be given by F(s) =ln p. For each multivalued mapping T: E⟶CB(E) satisfying equation 4 we have
w1 (q,r)≤$e^{-\Phi}$M(x,y), for all p,q ∈ E, q≠r
It is clear that for r, q ∈E such that q=r the previous inequality also holds.

**Example 3**

Let's say F: R+⟶R be given by F(s) =ln p. It is clear that F satisfies (a, b, c of definition 1) for any k ∈ (0, 1). Each mapping T: E⟶CB (E) satisfying Equation (5) is an F-contraction such that
$H_w$(Tp,Tq)≤ $e^{-\Phi}$w1(p,q), for all p, q ∈ E, Tp≠Tq
It is clear that for p, q ∈ E such that Tp=Tq the previous inequality also holds and hence T is a contraction.

**Theorem 1**

Let's assume (Ş, w) be a modular metric space. Let's say that w is a regular modular satisfying $\Delta_M$-condition and $\Delta_2$-condition. Let's say E be a nonempty w-bounded and w-complete subset of $Ş_w$. Let's say T: E⟶CB (E) be a continuous F-contraction. Then T has a fixed point.

**Proof:** Let $p_0$ ∈ E be an arbitrary point of E and choose $p_1$ ∈$Tp_0$. If $p_1=p_0$, then $p_1$ is a fixed point of T and the proof is completed. Suppose that $p_1 \neq p_0$. Since T is an F-contraction, then there exists $p_2$ ∈$Tp_1$ such that

$\Phi+F(w1(p_1, p_2)) \leq F(M(p_0, p_1))$ and $p_1 \neq p_2$

So, we can say that there exists $p_3$∈$Tp_2$ such that

$\Phi+F(w1(p_2,p_3)) \leq F(M(p_1,p_2))$ and $p_2 \neq p_3$

Continuing this process again and again, we get that there exists a sequence {$p_n$} with initial point $p_0$ such that $p_{n+1}$∈$Tp_n$, $p_{n+1} \neq p_n$ and

$\Phi+F(w1(p_n, p_{n+1})) \leq F(M(p_{n-1}, p_n))$ for all n ∈ N

It can also be written as $F(w1(p_n, p_{n+1})) < F(M(p_{n-1}, p_n))$ for all n ∈ N
Likewise,

$w1(p_n, p_{n+1}) < M(p_{n-1}, p_n)$ (Since F is strictly increasing)
$=\max\{w1(p_{n-1},p_n), w1(p_{n-1},Tp_{n-1}), w1(p_n, Tp_n), w1(p_n,Tp_{n-1})\}$
$=\max\{w1(p_{n-1}, p_n), w1(p_n, Tp_n)\}$
$\leq\max\{w1(p_{n-1}, p_n), w1(p_n, p_{n+1})\}$
Clearly, if $\max\{w1(p_{n-1}, p_n), w1(p_n, p_{n+1})\} = w1(p_n, p_{n+1})$, we have a contradiction and thus,
$\max\{w1(p_{n-1}, p_n), w1(p_n, p_{n+1})\} = w1(p_{n-1}, p_n)$.

Consequently, by ((a) of definition 1) we have

$$\Phi + F(w1(p_n, p_{n+1})) \leq F(w1(p_{n-1}, p_n)), \text{ for all n} \in N \tag{6}$$

With the help of eq. 6 we have

$$F(w1(p_n, p_{n+1})) \leq F(w1(p_{n-1}, p_n)) - \Phi \leq \ldots \leq F(w1(p_0, p_1)) - n\Phi, \text{ for all n} \in N \tag{7}$$

and hence, $\lim_{n\to\infty} F(w1(p_n, p_{n+1})) = -\infty$, by ((b) of definition 1) we can say that w1 ($p_n$, $p_{n+1}$)→0 as n→∞. Now, assume k ϵ (0, 1), such that, $\lim_{n\to\infty} (w1(p_n, p_{n+1}))^k F(w1(p_n, p_{n+1}))$. With eq. 7 the below shown holds for all n ∈ N.

$$(w1(p_{n+1}, p_n))^k (F(w1(p_{n+1}, p_n)) - F(w1(p_0, p_1))) \le -(w1(p_{n+1}, p_n))^k n\Phi \le 0 \quad (8)$$

By putting n→∞ in eq. 8, we find

$$\lim_{n\to\infty} n(w1(p_{n+1}, p_n))^k = 0$$

Then there exists $n_1$∈N such that n(w1($p_{n+1}$,$p_n$)) k≤1 for all n≥n1, that is,

$$w1(p_n, p_{n+1})) \le \frac{1}{n^{1/k}}$$ , for all n≥$n_1$

Now, for all m, n≥$n_1$ with m>n, we have

$$w_{m-n}(p_n, p_m) \le w_1(p_n, p_{n+1}) + w_1(p_{n+1}, p_{n+2}) + \ldots.. + w_1(p_{m-1}, p_m)$$

$$\le \frac{1}{n^{1/k}} + \frac{1}{(n+1)^{1/k}} + \ldots + \frac{1}{m^{1/k}}$$

$$< \sum_{i=n}^{\infty} \frac{1}{i^{1/k}}$$

Since the series $\sum_{i=n}^{\infty} \frac{1}{i^{1/k}}$ is convergent, this shows

$\lim_{m,n\to\infty} w_{m-n}(p_n, p_m) = 0$, since w satisfy ΔM-condition. Hence, we have

$$\lim_{m,n\to\infty} w_1(p_n, p_m) = 0$$

This shows that {$p_n$} is a w-Cauchy sequence. E is w-complete, there exists *v* ∈ E such that $p_n$→v as n→∞. Now, we proved that *v* is a fixed point of T.

**Theorem 2**

Let's assume (Ş, w) be a modular metric space. Let's say that w is a regular modular satisfying $\Delta_M$-condition and $\Delta_2$-condition. Let's say E be a nonempty w-bounded and w-complete subset of $Ş_w$. Let's say T: E⟶CB (E) be an F-contraction of Hardy-rogers type. Then T has a fixed point.

**Proof:** Let's say $P_0$ be an arbitrary point in E. As Tp is nonempty for all p ∈ Ş, we can choose $p_1$∈T$p_0$. If $p_1$∈T$p_1$, then $p_1$ is a fixed point of T and so the proof is complete.

Let's assume $p_1$∉T$p_1$. Then, since T$p_1$ is closed, w(p,T$p_1$)>0. On the other hand, from w($p_1$,T$p_1$)≤ $Ş_w$ (T$p_0$,T$p_1$) and ((a) of definition 1).

F(w($p_1$,T$p_1$))≤F($Ş_w$ (T$p_0$,T$p_1$))

With the help of eq. 5, we can say that

$$F(w(p_1,Tp_1)) \le F(S_w(Tp_0,Tp_1)) \le F(\alpha w_1(p_0, p_1) + \beta w_1(p_0,Tp_0) + \gamma w_1(p_1,Tp_1) + \xi w_1(p_1,Tp_0)) - 2\Phi$$

Since T$p_1$ is compact, there exists $p_2$∈T$p_1$ such that

w1 ($p_1$,$p_2$)=w1($p_1$,T$p_1$)

Then,

$$F(w_1(p_1, p_2)) = F(w(p_1,Tp_1)) \le F(S_w(Tp_0,Tp_1))$$

$$\le F(\alpha w_1(p_0, p_1) + \beta w_1(p_0,Tp_0) + \gamma w_1(p_1,Tp_1) + \xi w_1(p_1,Tp_0)) - 2\Phi$$

Thus,

$$\begin{aligned} F(w_1(p_1,p_2)) &\le F(S_w(Tp_0,Tp_1)) \\ &\le F(\alpha w_1(p_0,p_1)+\beta w_1(p_0,Tp_0)+\gamma w_1(p_1,Tp_1)+\xi w_1(p_1,Tp_0))-2\Phi \\ &\le F(\alpha w_1(p_0,p_1)+\beta w_1(p_0,p_1)+\gamma w_1(p_1,p_2))-2\Phi \\ &\le F((\alpha+\beta)w_1(p_0,p_1)+\gamma w_1(p_1,p_2)) \end{aligned}$$

So,

$$F(w_1(p_1,p_2))\le F((\alpha+\beta)w_1(p_0,p_1)+\gamma w_1(p_1,p_2))$$

Since F is strictly increasing, we illustrate that

$$w_1(p_1,p_2))\le(\alpha+\beta)w_1(p_0,p_1)+\gamma w_1(p_1,p_2)$$

and we can say

$$(1-\gamma)w_1(p_1,p_2))<(\alpha+\beta)w_1(p_0,p_1)$$

From $\alpha+\beta+\gamma=1$ and $\gamma\neq1$, we deduce that $1-\gamma>0$ and thus

$$w_1(p_1,p_2))<\frac{\alpha+\beta}{1-\gamma}w_1(p_0,p_1)=w_1(p_0,p_1)$$

In the continuation,

$\Phi+F(w_1(p_1,p_2))\le F(w_1(p_0,p_1))$

Consequently, we can express a sequence $\{p_n\}\subset E$ such that $p_n\notin Tp_n$, $p_{n+1}\in Tp_n$ and

$\Phi+F(w1(p_{n+1},p_{n+2}))\le F(w1(p_n,p_{n+1}))$ for all $n\in N\cup\{0\}$

At the ending as in the proof of Theorem 2, we find that $\{p_n\}$ is a w-Cauchy sequence. Since E is a w-complete modular metric space, there exists $v\in E$ such that $p_n\to v$ as $n\to\infty$. So, we prove that $v$ is a fixed point of T.

If there occurs an increasing sequence $\{n_k\}\subset N$ such that $p_{nk}\in Tv$ for all $k\in N$, since $Tv$ is w-closed and $p_{nk}\to v$, we have $v\in Tv$ and the proof is obtained.

Let's say that there exists $n_0\in N$ such that $p_n\notin Tv$ for all $n\ge n_0$. This concludes that $Tp_{n-1}\neq Tv$ for all $n\ge n_0$. Thus, using equation (5) with $p=p_n$ and $q=v$, we illustrate

$$2\Phi+F(S_w(Tp_n,Tv))\le F(\alpha w_1(p_n,v)+\beta w_1(p_n,Tp_n)+\gamma w_1(v,Tv)+\xi w_1(v,Tp_n))$$

Which says,

$$\begin{aligned} 2\Phi+F(w_1(p_{n+1},Tv)) &\le 2\Phi+F(S_w(Tp_n,Tv)) \\ &\le F(\alpha w_1(p_n,v)+\beta w_1(p_n,Tp_n)+\gamma w_1(v,Tv)+\xi w_1(v,Tp_n)) \\ &\le F(\alpha w_1(p_n,v)+\beta w_1(p_n,p_{n+1})+\gamma w_1(v,Tv)+\xi w_1(v,p_{n+1})) \end{aligned}$$

Though F is increasing strictly, so we obtain

$$w_1(p_{n+1},Tv)\le\alpha w_1(p_n,v)+\beta w_1(p_n,p_{n+1})+\gamma w_1(v,Tv)+\xi w_1(v,p_{n+1}))$$

Let's say $n\to\infty$ in the previous inequality, as $\gamma<1$

we have $w1(v, Tv)\le\gamma w1(v, Tv) < w1(v, Tv)$, which implies $w1(v, Tv)=0$.

Since $Tv$ is w-closed, we obtain that $v\in Tv$, that is, $v$ is a fixed point of T.

**References**


1. Banach S. Sur les opérations dans les ensembles abstraits et leur application aux équations integrals, Fundam. Math. 1922; 3:133-181.
2. Rus IA, Petru¸sel A, Petru¸sel G. Fixed Point Theory", Cluj Univ. Press, Cluj-Napoca, 2008.
3. Saadati R, Vaezpour SM, Vetro P, Rhoades BE. Fixed point theorems in generalized partially ordered G-metric spaces, Math. Comput. Modelling. 2010; 52:797-801.
4. Wardowski D. Fixed points of a new type of contractive mappings in complete metric spaces", Fixed Point Theory Appl, 2012.
5. Hardy GE, Rogers TD. A generalization of a fixed point theorem of Reich, Canada Mathematic Bulletin. 1973; 16:201-206.

6. Sgroi M, Vetro C. Multi-valued F-contractions and the solution of certain functional and integral equations, Filomat. 2013; 27:1259-1268.
7. Chistyakov VV. Modular metric spaces I basic concepts. Nonlinear Anal. 2010; 72:1-14.
8. Mongkolkeha C, Sintunavarat W, Kumam P. Fixed point theorems for contraction mappings in modular metric spaces, Fixed Point Theory Appl, 2011.
9. Chaipunya P, Mongkolkeha C, Sintunavarat W, Kumam P. Fixed-point theorems for multivalued mappings in modular metric spaces, Abstr. Appl. Anal, 2012.
10. Abdo AAN, Khamsi MA. Fixed point results of point-wise contractions in modular metric spaces, Fixed Point Theory Appl, 2013.
11. Abdou AAN, Khamsi MA. Fixed points of multivalued contraction mappings in modular metric spaces", Fixed Point Theory Appl, 2014.
12. Alfuraidan MR. Fixed points of multivalued contraction mappings in modular metric spaces, Fixed Point Theory Appl, 2015.
13. Kadelburg Z, Radenovic S. Remarks on some recent M. Borcut's results in partially ordered metric spaces, International Journal Nonlinear Anal. Appl. 2015; 6:96-104.
14. Saadati R, Vaezpour SM, Vetro P, Rhoades BE. Fixed point theorems in generalized partially ordered G-metric spaces, Mathematics Computing Modelling. 2010; 52:797-801.
15. Secelean NA. Iterated function systems consisting of F-contractions, Fixed Point Theory Appl, 2013, 277.
16. Sgroi M, Vetro C. Multi-valued F-contractions and the solution of certain functional and integral equations, Filomat. 2013 27(7):1259-1268.
17. Suzuki T. A new type of fixed point theorem in metric spaces", Nonlinear Anal., Theory Methods Appl. 2009; 71(11):5313-5317.
18. Udo-utun X. On inclusion of F-contractions in (δ, k)-weak contractions, Fixed Point Theory Appl, 2014, 65.
19. Wardowski D. Fixed points of a new type of contractive mappings in complete metric spaces", Fixed Point Theory Appl., 2012, 94.