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Estimation of ratio of two population means using ratio type estimators in stratified sampling under non – Response

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Abstract
In present paper, our aim is to propose a class of ratio type estimators for the ratio of two population means using auxiliary information in stratified sampling under non-response based on proposed estimator by Tailor *et al* [24]. The biased and mean square error of the proposed estimator are expressed in ordered to compare with the existing conventional estimator for comparing the efficiency.

Keywords: Population mean, study character, auxiliary character, ratio estimator, relative bias, mean square error, stratified sampling, non-response

Introduction
The applications of ratio of two population means are in calculation of per capita income (socio-economics field), per hectare production of crops (agriculture field), per kilometre population density (population study), growth index rate, in field of education and medical etc.
Various research works have been done for estimation of ratio of two population means using auxiliary variable in simple random sampling technique by Singh [1], Tripathi [2, 3], Upadhyaya and Singh [4], Srivastava *et al* [5, 6], Singh *et al* [7, 8], and Singh and Singh [9]. But in some cases and situations, it has been seen that estimators in stratified sampling are more efficient than those of simple random sampling.
Tailor *et al* [20] have been suggested a ratio estimator of two population means using auxiliary information in stratified sampling. Due to the problem of non-response, we unable to find the information of all selected units in the sample. Thus, to handle this problem of non-response, Hansen and Hurwitz [10] developed a technique using sub sample from non- respondent. Using this technique, various researchers have been studied the estimation of ratio two population means using auxiliary variables under non-response Khare and Pandey [11], Khare and Sinha [12-15], Khare *et al* [16, 17], and N. kumar and Patel [18, 19].

Sampling strategy and estimation methodology
Let the heterogeneous population of size N is divided into k homogeneous strata of size N_i (i =1,2,3,……..,k) such that N(=N_1+N_2+N_3+……..+N_K) be the total population size. Then we can select a sample of size n_i(i=1,2,3,………,k)from each one of the strata using SRSWOR. Such that n(=n_1+n_2+n_3+……..+n_k)be total sample size.
In case of non-response, it is seen that n_{i1} units provide information but n_{i2} units be non-respondent of the sample size n_i(<N_i). By extensive efforts the data are later obtained from a random sample of u_{i2} of n_{i2} units such that $n_{i2} = K_i u_{i2}$ (K_i>1),
Where, 1/K_i be the sampling fraction among non-respondents in the ith stratum (i = 1,2,…..,k).
Then using Hansen-Hurwitz[10] technique, the sample mean of jth character,taken from the ith stratum,of study variable Y in case of non-response is given by
$\bar{y}^*_{ji} = \frac{n_{i1}\bar{y}_{jn_{i1}} + n_{i2}\bar{y}_{ju_{i2}}}{n_i}$; unbiased estimator of $\bar{Y}_{ji}$ in ith stratum for jth characteristics and
$\bar{y}_{jn_{i1}}$ and $\bar{y}_{ju_{i2}}$ are the means based on n_{i1} units for response group and u_{i2} units of sub sample of non-response group respectively for jth character in ith stratum.

Now,
$\bar{y}_{jst}^{*} = \sum_{i=1}^{k} p_i \bar{y}_{ji}^{*}$ such that - E($\bar{y}_{jst}^{*}$) = $\bar{Y}_j$; (j = 1,2)
And, V[$\bar{y}_{jst}^{*}$] = $\sum_{i=1}^{k}[\frac{Ni-ni}{Nini}]P_i^2\, S_{ji}^2 + \sum_{i=1}^{k}\left[\frac{ki-1}{ni}\right] W_{i2}\, P_i^2\, S_{ji(2)}^2$

Where, S_{ji}^2 and $S_{ji(2)}^2$ are population mean square errors of response group and non-response group for jth character in ith stratum respectively.

Such as - $S_{ji}^2 = \frac{1}{N_i-1}\sum_{h=1}^{N_i}(y_{jih} - \bar{Y}_{ji})^2$ and $S_{ji(2)}^2 = \frac{1}{N_{i2}-1}\sum_{h=1}^{N_{i2}}(y_{jih(2)} - \bar{Y}_{ji(2)})^2$

Similary, using Hansen-Hurwitz[10] technique, the sample mean of the ith stratum,of auxiliary variable X in case of non-response is given by

$\bar{x}_i^{*} = \frac{n_{i1}\bar{x}_{n_{i1}} + n_{i2}\bar{x}_{u_{i2}}}{n_i}$; unbiased estimator of $\bar{X}_i$ in ith stratum and

$\bar{x}_{n_{i1}}$ and $\bar{x}_{u_{i2}}$ are the means based on n_{i1} units for response group and u_{i2} units of sub sample of non-response group respectively in ith stratum of auxiliary variable X.

Now,
$\bar{x}_{st}^{*} = \sum_{i=1}^{k} p_i \bar{x}_i^{*}$ such that - E($\bar{x}_{st}^{*}$) = $\bar{X}$;
And, V[$\bar{x}_{st}^{*}$] = $\sum_{i=1}^{k}[\frac{Ni-ni}{Nini}]P_i^2\, S_{xi}^2 + \sum_{i=1}^{k}\left[\frac{ki-1}{ni}\right] W_{i2}\, P_i^2\, S_{xi(2)}^2$

Where, S_{xi}^2 and $S_{xi(2)}^2$ are population mean square errors of response group and non-response group in ith stratum respectively.
Such as - $S_{xi}^2 = \frac{1}{N_i-1}\sum_{h=1}^{N_i}(x_{ih} - \bar{X}_i)^2$ and $S_{xi(2)}^2 = \frac{1}{N_{i2}-1}\sum_{h=1}^{N_{i2}}(x_{ih(2)} - \bar{X}_{i(2)})^2$

Also, $W_{i2} = N_{i2}/N_i$; non-response rate in the ith stratum.
Here, we have proposed a class of estimators for ratio of two population means using stratified sampling technique based on non-response. In order to efficiency comparison, the relative bias and mean square error of proposed estimator are compared with existing estimators.

The estimators
Let us define $R_{st}^{*} = \frac{\bar{y}_{1st}^{*}}{\bar{y}_{2st}^{*}}$ be the unbiased estimator of R = $\frac{\bar{Y}_1}{\bar{Y}_2}$.
In order to derive bias relative and mean square error of above estimator,
Let us assume, $\bar{y}_{jst}^{*} = \bar{Y}_j(1 + \varepsilon_j)$; such that- E($\varepsilon_j$) = 0; j = 1,2
Also, E(ε_1^2) = $\sum_{i=1}^{k}[\frac{Ni-ni}{Nini}]P_i^2\, C_{1i}^2 + \sum_{i=1}^{k}\left[\frac{ki-1}{ni}\right] W_{i2}\, P_i^2\, C_{1i(2)}^2$
E(ε_2^2) = $\sum_{i=1}^{k}[\frac{Ni-ni}{Nini}]P_i^2\, C_{2i}^2 + \sum_{i=1}^{k}\left[\frac{ki-1}{ni}\right] W_{i2}\, P_i^2\, C_{2i(2)}^2$
E($\varepsilon_1\, \varepsilon_2$) = $\sum_{i=1}^{k}[\frac{Ni-ni}{Nini}]P_i^2\, C_{12i} + \sum_{i=1}^{k}\left[\frac{ki-1}{ni}\right] W_{i2}\, P_i^2\, C_{12i(2)}$

Then, expressing the estimator R_{st}^{*} in term of ε_j's, we get-
$R_{st}^{*} = R(1 + \varepsilon_1)(1 + \varepsilon_2)^{-1}$

Expanding the above expression and neglecting 3rd and higher degree of terms, we get-
$R_{st}^{*} = R(1 + \varepsilon_1 - \varepsilon_2 - \varepsilon_1\varepsilon_2 + \varepsilon_2^2)$
Thus, R.B.(R_{st}^{*}) = $E[R_{st}^{*} - R]/R$
= $\sum_{i=1}^{k}[\frac{Ni-ni}{Nini}]P_i^2\,\{C_{1i}^2 - C_{12i}\} + \sum_{i=1}^{k}\left[\frac{ki-1}{ni}\right] W_{i2}\, P_i^2\{ C_{1i(2)}^2 - C_{12i(2)}\}$ ………..1.1
And, M.S.E.(R_{st}^{*}) = $E[R_{st}^{*} - R]^2$ = R^2 E[$\varepsilon_1^2 + \varepsilon_2^2 - 2\,\varepsilon_1\varepsilon_2$]
=R^2[$\sum_{i=1}^{k}[\frac{Ni-ni}{Nini}]P_i^2\,\{C_{1i}^2 + C_{2i}^2 - C_{12i}\} + \sum_{i=1}^{k}\left[\frac{ki-1}{ni}\right] W_{i2}\, P_i^2\{ C_{1i(2)}^2 + C_{2i(2)}^2 - C_{12i(2)}\}$] ………..1.2
Where, $C_{ji}^2 = \frac{S_{ji}^2}{\bar{Y}_j}$; $C_{ji(2)}^2 = \frac{S_{ji(2)}^2}{\bar{Y}_j}$; $C_{12i} = \frac{S_{12i}}{\bar{Y}_1\bar{Y}_2}$ and $C_{12i(2)} = \frac{S_{12i(2)}}{\bar{Y}_1\bar{Y}_2}$ j = 1,2 and i = 1,2,3,……….,k
And, $S_{12i} = \frac{1}{N_i-1}\sum_{h=1}^{N_i}(y_{1ih} - \bar{Y}_{1i})(y_{2ih} - \bar{Y}_{2i})$;

$S_{12i(2)} = \frac{1}{N_i-1}\sum_{h=1}^{N_i}(y_{1ih(2)} - \bar{Y}_{1i(2)})(y_{2ih(2)} - \bar{Y}_{2i(2)})$

The ratio type estimator 0f population mean in stratified sampling, proposed by Tailor *et al*[20], is given as

$T_{1st} = \left[\frac{\bar{y}_{1st}}{\bar{y}_{2st}}\right]\left[\frac{\bar{X}}{\bar{x}_{st}}\right]$ ………….2

Adopting above idea of Tailor, we proposed the conventional (when complete information on X, but incomplete information on Y) and alternative estimator (when incomplete information on X and Y both) i.e. Z_1 and Z_2 respectively for ratio of two population means in stratified sampling in case of non-response, given by

$Z_1 = = \left[\frac{\bar{y}_{1st}^*}{\bar{y}_{2st}^*}\right]\left[\frac{\bar{x}_{st}}{\bar{X}}\right]$ and $Z_2 = \left[\frac{\bar{y}_{1st}^*}{\bar{y}_{2st}^*}\right]\left[\frac{\bar{x}_{st}^*}{\bar{X}}\right]$ …………..3

Where, $\bar{x}_{st} = \sum_{i=1}^{k} p_i\bar{x}_i$ such that - $E(\bar{x}_{st}) = \bar{X}$;

And, $V[\bar{x}_{st}] = \sum_{i=1}^{k}[\frac{Ni-ni}{Nini}]P_i^2\, S_{xi}^2$

The proposed class of estimators

After combining and generalizing, the above proposed estimators we suggest a class of estimators of ratio of two population means in stratified sampling in case of non-response, given by

$G_{st} = \left[\frac{\bar{y}_{1st}^*}{\bar{y}_{2st}^*}\right]\left[\frac{a\bar{x}_{st}+b}{a\bar{X}+b}\right]^{\alpha}\left[\frac{a\bar{x}_{st}^*+b}{a\bar{X}+b}\right]^{\beta} = R_{st}^*\left[\frac{a\bar{x}_{st}+b}{a\bar{X}+b}\right]^{\alpha}\left[\frac{a\bar{x}_{st}^*+b}{a\bar{X}+b}\right]^{\beta}$ ………..4

Where, a, b, α, and β are arbitrary chosen constants.

consist family of estimators –

S. No.	Family of estimators	a	B	α	B
1	$G1_{st} = R_{st}^*$	a	B	0	0
2	$G2_{st} = R_{st}^*\frac{\bar{x}_{st}^*}{\bar{X}} = Z_2$	1	0	0	1
3	$G3_{st} = R_{st}^*\frac{\bar{X}}{\bar{x}_{st}^*}$ Tailor type estimator	1	0	0	-1
4	$G4_{st} = R_{st}^*\frac{\bar{x}_{st}}{\bar{X}} = Z_1$	1	0	1	0
5	$G5_{st} = R_{st}^*\frac{\bar{X}}{\bar{x}_{st}}$ Tailor type estimator	1	0	-1	0

Relative Bias and Mean Square Error

In order to derive relative bias and mean square error of above class of estimators,

Let us assume, $\bar{x}_{st} = \bar{X}(1 + \varepsilon_3)$ and $\bar{x}_{st}^* = \bar{X}(1 + \varepsilon_4)$; such that- $E(\varepsilon_m) = 0$; m = 3,4

Also, $E(\varepsilon_3^2) = \sum_{i=1}^{k}[\frac{Ni-ni}{Nini}]P_i^2\, C_{xi}^2$

$E(\varepsilon_4^2) = \sum_{i=1}^{k}[\frac{Ni-ni}{Nini}]P_i^2\, C_{xi}^2 + \sum_{i=1}^{k}\left[\frac{ki-1}{ni}\right] W_{i2}\, P_i^2\, C_{xi(2)}^2$

$E(\varepsilon_1\, \varepsilon_3) = \sum_{i=1}^{k}[\frac{Ni-ni}{Nini}]P_i^2\, C_{1xi}$

$E(\varepsilon_2\, \varepsilon_3) = \sum_{i=1}^{k}[\frac{Ni-ni}{Nini}]P_i^2\, C_{2xi}$

$E(\varepsilon_1\, \varepsilon_4) = \sum_{i=1}^{k}[\frac{Ni-ni}{Nini}]P_i^2\, C_{1xi} + \sum_{i=1}^{k}\left[\frac{ki-1}{ni}\right] W_{i2}\, P_i^2\, C_{1xi(2)}$

$E(\varepsilon_2\, \varepsilon_4) = \sum_{i=1}^{k}[\frac{Ni-ni}{Nini}]P_i^2\, C_{2xi} + \sum_{i=1}^{k}\left[\frac{ki-1}{ni}\right] W_{i2}\, P_i^2\, C_{2xi(2)}$

$E(\varepsilon_3\, \varepsilon_4) = \sum_{i=1}^{k}[\frac{Ni-ni}{Nini}]P_i^2\, C_{xi}^2$

Then, expressing the estimator R_{st}^* in term of ε_j's and ε_m's we get-

$G_{st} = \left[\frac{\bar{Y}_1(1 + \varepsilon 1)}{\bar{Y}_2(1 + \varepsilon 2)}\right]\left[\frac{a\bar{X}(1 + \varepsilon 3)+b}{a\bar{X}+b}\right]^{\alpha}\left[\frac{a\bar{X}(1 + \varepsilon 4)+b}{a\bar{X}+b}\right]^{\beta}$

$= R(1 + \varepsilon_1)(1 + \varepsilon_2)^{-1}(1 + \varphi\, \varepsilon_3)^{\alpha}(1 + \varphi\varepsilon_3)^{\beta}$ where, $\varphi = \frac{a\bar{X}}{a\bar{X}+b}$

Expanding the above expression and neglecting 3rd and higher degree of terms, we get-

$G_{st} = R[\, 1 + \varepsilon_1 - \varepsilon_2 + \varphi\,(\alpha\varepsilon_3 + \beta\varepsilon_4) + \varepsilon_2^2 + (\varphi^2/2)\,\{\alpha(\alpha-1)\varepsilon_3^2 + \beta(\beta-1)\varepsilon_4^2\} - \varepsilon_1\,\varepsilon_2 + \varphi^2(\alpha\beta\, \varepsilon_3\, \varepsilon_4) + \varphi\,(\alpha\, \varepsilon_1\varepsilon_3 + \beta\, \varepsilon_1\varepsilon_4 - \alpha\, \varepsilon_2\varepsilon_3 - \beta\, \varepsilon_2\varepsilon_4)]$

Now, R.B.(G_{st}) = E$[G_{st} - R]$

= R.B.(R^*_{st}) +($\varphi^2/2$) [α(α-1) $\sum_{i=1}^{k}[\frac{Ni-ni}{Nini}]P_i^2\, C_{xi}^2$ + β(β-1){ $\sum_{i=1}^{k}[\frac{Ni-ni}{Nini}]P_i^2\, C_{xi}^2$ + $\sum_{i=1}^{k}\left[\frac{ki-1}{ni}\right] W_{i2}\, P_i^2\, C_{xi(2)}^2$}] + φ^2αβ $\sum_{i=1}^{k}[\frac{Ni-ni}{Nini}]P_i^2\, C_{xi}^2$ + φα $\sum_{i=1}^{k}[\frac{Ni-ni}{Nini}]P_i^2\{C_{1xi} - C_{2xi}\}$

+ φβ[$\sum_{i=1}^{k}[\frac{Ni-ni}{Nini}]P_i^2\, \{C_{1xi} - C_{2xi}\}$+$\sum_{i=1}^{k}\left[\frac{ki-1}{ni}\right] W_{i2}\, P_i^2\, \{C_{1xi(2)} - C_{2xi(2)}\}$]

R.B.(G_{st}) = R.B.(R^*_{st}) +(φ/2) (α + β) $\sum_{i=1}^{k}[\frac{Ni-ni}{Nini}]P_i^2$[φ{ (α + β) – 1}$C_{xi}^2$ + 2$\{C_{1xi} - C_{2xi}\}$]

+ (φ/2) β $\sum_{i=1}^{k}\left[\frac{ki-1}{ni}\right] W_{i2}\, P_i^2$[φ(β – 1) $C_{xi(2)}^2$ +2 $\{C_{1xi(2)} - C_{2xi(2)}\}$] ……..4.1

M.S.E.(G_{st}) = E$[G_{st} - R]^2$

=R^2E[$\varepsilon_1^2 + \varepsilon_2^2 + \varphi^2 (\alpha^2 \varepsilon_3^2 + \beta^2 \varepsilon_4^2) - 2\varepsilon_1 \varepsilon_2 + 2\varphi^2(\alpha\beta\, \varepsilon_3 \varepsilon_4) + 2\varphi (\alpha\, \varepsilon_1\varepsilon_3 + \beta\, \varepsilon_1\varepsilon_4 - \alpha\, \varepsilon_2\varepsilon_3 - \beta\, \varepsilon_2\varepsilon_4)$]

= R.B.(R^*_{st}) +R^2[{φ (α + β)}^{2} $\sum_{i=1}^{k}[\frac{Ni-ni}{Nini}]P_i^2\, C_{xi}^2$ + $\varphi^2\beta^2$ $\sum_{i=1}^{k}\left[\frac{ki-1}{ni}\right] W_{i2}\, P_i^2\, C_{xi(2)}^2$

+ φ (α + β)$\sum_{i=1}^{k}[\frac{Ni-ni}{Nini}]P_i^2\, \{C_{1xi} - C_{2xi}\}$+ 2φβ $\sum_{i=1}^{k}\left[\frac{ki-1}{ni}\right] W_{i2}\, P_i^2\, \{C_{1xi(2)} - C_{2xi(2)}\}$] …….4.2

M.S.E.(G_{st}) = M.S.E.(R^*_{st}) + R^2[φ (α + β) $\sum_{i=1}^{k}[\frac{Ni-ni}{Nini}]P_i^2$ {φ(α + β) C_{xi}^2 + 2$(C_{1xi} - C_{2xi})$}

+ φ β $\sum_{i=1}^{k}\left[\frac{ki-1}{ni}\right] W_{i2}\, P_i^2$ {φβ $C_{xi(2)}^2$ +2 $(C_{1xi(2)} - C_{2xi(2)})$}] ....….4.3

To obtain optimum α ($\widehat{\alpha}$) and optimum β ($\widehat{\beta}$), we have to minimize M.S.E.(G_{st}) with respect to α and β respectively. In this order, differentiating it w.r.t. α and β respectively and equating derivatives to zero.

The M.S.E.(G_{st}) is minimum when –

$\frac{\partial}{\partial\alpha}$ M.S.E.(G_{st}) = 0 and $\frac{\partial}{\partial\beta}$ M.S.E.(G_{st}) = 0

then, we get- $\widehat{\alpha} = \frac{-B^*_{st}}{\varphi A^*_{st}}$ and $\widehat{\beta} = \left[\frac{B^*_{st}}{\varphi A^*_{st}} - \frac{B_{st}}{\varphi A_{st}}\right]$ ……..(*)

Where, $A_{st} = \sum_{i=1}^{k}[\frac{Ni-ni}{Nini}]P_i^2\, C_{xi}^2$; $B_{st} = \sum_{i=1}^{k}[\frac{Ni-ni}{Nini}]P_i^2\, \{C_{1xi} - C_{2xi}\}$

$A^*_{st} = \sum_{i=1}^{k}\left[\frac{ki-1}{ni}\right] W_{i2}\, P_i^2\, C_{xi(2)}^2$ and, $B^*_{st} = \sum_{i=1}^{k}\left[\frac{ki-1}{ni}\right] W_{i2}\, P_i^2\, \{C_{1xi(2)} - C_{2xi(2)}\}$

And, $C_{xi}^2 = \frac{S_{xi}^2}{\bar{X}}$; $C_{xi(2)}^2 = \frac{S_{xi(2)}^2}{\bar{X}}$; $C_{jxi} = \frac{S_{1xi}}{\bar{Y}_j\bar{X}}$ and $C_{jxi(2)} = \frac{S_{jxi(2)}}{\bar{Y}_j\bar{X}}$ j = 1,2 and i = 1,2,3,……….,k

And, $S_{jxi} = \frac{1}{N_i-1}\sum_{h=1}^{N_i}(y_{jih} - \bar{Y}_{ji})(x_{ih} - \bar{X}_i)$;

$S_{jxi(2)} = \frac{1}{N_i-1}\sum_{h=1}^{N_i}\left(y_{jih(2)} - \bar{Y}_{ji(2)}\right)\left(x_{ih(2)} - \bar{X}_{i(2)}\right)$

Using (*) in 4(b), we get-

M.S.E.(G_{st})$_{opt}$ = M.S.E.(R^*_{st}) - R^2 $\left[\frac{B^{*2}_{st}}{A^*_{st}} + \frac{B^2_{st}}{A_{st}}\right]$ …………..4.4

in order to obtain the expression of R.B. and M.S.E. of family of estimators, we put the different values of a, b, α, and β in 4(a) and 4(c) respectively. Then, we have

I] $G2_{st}$ ⇒ (a b α β) ⇒ (1 0 0 1)

R.B.($G2_{st}$) = R.B.(R^*_{st}) + $\sum_{i=1}^{k}[\frac{Ni-ni}{Nini}]P_i^2\{C_{1xi} - C_{2xi}\}$ + $\sum_{i=1}^{k}\left[\frac{ki-1}{ni}\right] W_{i2}\, P_i^2\, \{C_{1xi(2)} - C_{2xi(2)}\}$ ....…….5.1

M.S.E.($G2_{st}$) = M.S.E.(R^*_{st}) + R^2[$\sum_{i=1}^{k}[\frac{Ni-ni}{Nini}]P_i^2\{C_{xi}^2$ + 2$(C_{1xi} - C_{2xi})$}

+ $\sum_{i=1}^{k}\left[\frac{ki-1}{ni}\right] W_{i2}\, P_i^2\{C_{xi(2)}^2$ +2 $(C_{1xi(2)} - C_{2xi(2)})$}] ……….5.2

II] $G3_{st}$ ⇒ (a b α β) ⇒ (1 0 0 -1)

R.B.($G3_{st}$) = R.B.(R^*_{st}) + $\sum_{i=1}^{k}[\frac{Ni-ni}{Nini}]P_i^2$ [C_{xi}^2 - $\{C_{1xi} - C_{2xi}\}$]

+ $\sum_{i=1}^{k}\left[\frac{ki-1}{ni}\right] W_{i2}\, P_i^2$ [$C_{xi(2)}^2$ - $\{C_{1xi(2)} - C_{2xi(2)}\}$] ……….6.1

M.S.E.($G3_{st}$) = M.S.E.(R^*_{st}) + R^2[$\sum_{i=1}^{k}[\frac{Ni-ni}{Nini}]P_i^2\{C_{xi}^2$ - 2$(C_{1xi} - C_{2xi})$}

+ $\sum_{i=1}^{k}\left[\frac{ki-1}{ni}\right] W_{i2}\, P_i^2\{C_{xi(2)}^2$- 2 $(C_{1xi(2)} - C_{2xi(2)})$}] ……….6.2

III] $G4_{st}$ ⇒ (a b α β) ⇒ (1 0 1 0)

R.B.($G4_{st}$) = R.B.(R^*_{st}) + $\sum_{i=1}^{k}[\frac{Ni-ni}{Nini}]P_i^2\{C_{1xi} - C_{2xi}\}$ ……….7.1

M.S.E.($G4_{st}$) = M.S.E.(R^*_{st}) + R^2[$\sum_{i=1}^{k}[\frac{Ni-ni}{Nini}]P_i^2\{C_{xi}^2$ + 2$(C_{1xi} - C_{2xi})$}] ……….7.2

IV] $G5_{st}$ ⇒ (a b α β) ⇒ (1 0 -1 0)

R.B.($G5_{st}$) = R.B.(R^*_{st}) + $\sum_{i=1}^{k}[\frac{Ni-ni}{Nini}]P_i^2$ [C_{xi}^2 - $\{C_{1xi} - C_{2xi}\}$] ……….8.1

M.S.E.($G5_{st}$) = M.S.E.(R^*_{st}) + R^2[$\sum_{i=1}^{k}[\frac{Ni-ni}{Nini}]P_i^2\{C_{xi}^2$ - $2(C_{1xi}-C_{2xi})\}$] ……….8.2

Efficiency comparison

Theoretical comparison

[1] On Comparing section 4.4 with 1.2; the proposed estimator G_{st} will be more efficient than R^*_{st} iff

M.S.E. $[G_{st}]_{opt}$ - M.S.E. $[R^*_{st}]$ < 0

$\Rightarrow R^2\left[\frac{B_{st}^{*2}}{A_{st}^*}+\frac{B_{st}^2}{A_{st}}\right] > 0$ clearly, $R^2 > 0$

Then, either, $\frac{B_{st}^{*2}}{A_{st}^*} > \frac{-B_{st}^2}{A_{st}}$

Or, $\frac{B_{st}^{*2}}{A_{st}^*} > 0 < \frac{B_{st}^2}{A_{st}}$

It is obvious, in both cases [(i)(B_{st}) or (B^*_{st}) < 0 (ii) (B_{st}) or (B^*_{st}) > 0], $B_{st}^2 > 0 < B_{st}^{*2}$

Thus, in condition $A_{st} > 0 < A^*_{st;}$ the proposed estimator G_{st} is always efficient than the existing estimator R^*_{st}.

[2] On Comparing section 4.4 with 5.2; the proposed estimator G_{st} will be more efficient than $G2_{st}$ iff

M.S.E. $[G_{st}]_{opt}$ - M.S.E. $[G2_{st}]$ < 0

$\Rightarrow R^2\left\{\left[\frac{B_{st}^{*2}}{A_{st}^*}+\frac{B_{st}^2}{A_{st}}\right] + (A_{st} + 2B_{st} + A^*_{st} + 2B^*_{st})\right\} > 0$

$\Rightarrow R^2\left[\frac{(A_{st}+B_{st})^2}{A_{st}} + \frac{(A_{st}^*+B_{st}^*)^2}{A_{st}^*}\right] > 0$

Obviously, terms R^2, $(A_{st}+B_{st})^2$, and $(A_{st}^*+B_{st}^*)^2$ be always positive.

Thus, in condition $A_{st} > 0 < A^*_{st}$; the proposed estimator G_{st} is always efficient than the estimator $G2_{st}$.

[3] On Comparing section 4.2 with 6.2; the proposed estimator G_{st} will be more efficient than $G3_{st}$ iff

M.S.E. $[G_{st}]_{opt}$ - M.S.E. $[G3_{st}]$ < 0

$\Rightarrow R^2\left\{\left[\frac{B_{st}^{*2}}{A_{st}^*}+\frac{B_{st}^2}{A_{st}}\right] + (A_{st} - 2B_{st} + A^*_{st} - 2B^*_{st})\right\} > 0$

$\Rightarrow R^2\left[\frac{(A_{st}-B_{st})^2}{A_{st}} + \frac{(A_{st}^*-B_{st}^*)^2}{A_{st}^*}\right] > 0$

Obviously, terms R^2, $(A_{st}-B_{st})^2$, and $(A_{st}^*-B_{st}^*)^2$ be always positive.

Thus, in condition $A_{st} > 0 < A^*_{st}$; the proposed estimator G_{st} is always efficient than the estimator $G3_{st}$.

[4] On Comparing section 4.4 with 7.2; the proposed estimator G_{st} will be more efficient than $G4_{st}$ iff

M.S.E. $[G_{st}]_{opt}$ - M.S.E. $[G4_{st}]$ < 0

$\Rightarrow R^2\left\{\left[\frac{B_{st}^{*2}}{A_{st}^*}+\frac{B_{st}^2}{A_{st}}\right] + (A_{st} + 2B_{st})\right\} > 0$

$\Rightarrow R^2\left[\frac{(A_{st}+B_{st})^2}{A_{st}} + \frac{(B_{st}^*)^2}{A_{st}^*}\right] > 0$

Obviously, terms R^2, $(A_{st}+B_{st})^2$, and $(B_{st}^*)^2$ be always positive.

Thus, in condition $A_{st} > 0 < A^*_{st}$; the proposed estimator G_{st} is always efficient than the estimator $G4_{st}$.

[5] On Comparing section 4.4 with 8.2; the proposed estimator G_{st} will be more efficient than $G5_{st}$ iff

M.S.E. $[G_{st}]_{opt}$ - M.S.E. $[G5_{st}]$ < 0

$\Rightarrow R^2\left\{\left[\frac{B_{st}^{*2}}{A_{st}^*}+\frac{B_{st}^2}{A_{st}}\right] + (A_{st} - 2B_{st})\right\} > 0$

$\Rightarrow R^2\left[\frac{(A_{st}-B_{st})^2}{A_{st}} + \frac{(B_{st}^*)^2}{A_{st}^*}\right] > 0$

Obviously, terms R^2, $(A_{st}-B_{st})^2$, and $(B_{st}^*)^2$ be always positive.

Thus, in condition $A_{st} > 0 < A^*_{st}$; the proposed estimator G_{st} is always efficient than the estimator $G5_{st}$.

In each efficiency situation, it has been seen that if $A_{st} > 0 < A^*_{st}$ then, the proposed estimator G_{st} is always efficient than each family of estimators i.e. Gq_{st} (q = 1,2,…..,5).

Where, $A_{st} = \sum_{i=1}^{k}[\frac{Ni-ni}{Nini}]P_i^2\, C_{xi}^2$; and $A_{st}^* = \sum_{i=1}^{k}\left[\frac{ki-1}{ni}\right]\, W_{i2}\, P_i^2\, C_{xi(2)}^2$

Since, $\frac{Ni-ni}{Nini} > 0$, $\frac{ki-1}{ni} > 0$, $P_i^2 > 0$, $W_{i2} > 0$, $C_{xi}^2 > 0$ and $C_{xi(2)}^2 > 0$

So, A_{st} and A_{st}^* will be always positive. Thus, now it is clear that the proposed estimator G_{st} will be always efficient than each family of estimators i.e. Gq_{st} (q = 1, 2,…..,5).

Due to unavailability of data based on ratio of two population means in stratified sampling under non-response, we are unable to compare to the proposed estimator with existing estimators empirically.

Conclusion

In present paper, we have proposed a class of ratio type estimators for ratio of two population means in stratified sampling under non-response and derive and discuss their properties. After comparing their mean square errors, we have conclude that the proposed estimator in optimum case provides better estimate than usual conventional and alternative estimators in any condition.

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