International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2017; 2(5): 104-109
© 2017 Stats & Maths
www.mathsjournal.com
Received: 16-07-2017
Accepted: 17-08-2017

RS Wali
Department of Mathematics
Bhandari and Rathi College,
Guledagudd, Karnataka, India

Vijayalaxmi R Patil
Department of Mathematics
Rani Channamma University
Belagavi, Karnataka, India

Correspondence
Vijayalaxmi R Patil
Department of Mathematics
Rani Channamma University
Belagavi, Karnataka, India

On RGWαLC-Continuous and RGWαLC-Irresolute maps in topological spaces

RS Wali and Vijayalaxmi R Patil

Abstract
In this paper we introduced RGWαLC-Continuous, RGWαLC*-Continuous, RGWαLC**-Continuous, sub-RGWαLC*-Continuous and RGWαLC-Irresolute Maps which are weaker than LC-Continuous and stronger than GβLC- Continuous and study some of their properties and their relationship with w-lc continuous, θ-lc-continuous, lδc-continuous and π-lc continuous etc.

Keywords: RGWαLC-Continuous, RGWαLC*-Continuous, RGWαLC**-Continuous, sub-RGWαLC*-Continuous and RGWαLC-Irresolute Maps

1. Introduction

Bourbaki [7] defined a subset of a topological space as locally closed if it is the intersection of an open set and a closed set. Stone [6] used the term FG for locally closed subset. Ganster and Reilly [5] used locally closed sets to define LC-continuity and LC-irresoluteness. Sundaram [9] introduced the concepts of generalized locally closed sets, GLC-continuous maps and GLC-irresolute maps and investigated some of their properties. Also various authors have contributed to the development of generalizations of locally closed sets and locally continuous maps in topological spaces.

In this paper, three new classes of continuous maps are introduced namely RGWαLC-Continuous, sub-RGWαLC*-Continuous and RGWαLC-Irresolute Maps and studied some of their properties.

2. Preliminaries

Throughout this paper (X, τ) (or simple X) represents topological space on which no separation axioms are assumed unless otherwise mentioned. For a subset A of X, cl(A), int(A) and Ac denote the closure of A, the interior of A and the complement of A respectively. Let us recall the following definitions, which are useful in the sequel.

Definition 2.1: A subset A of topological space (X, τ) is called a

1. generalized closed set(briefly g-closed) [8] if cl(A) ⊆ U whenever A ⊆ U and U is open in X.
2. regular generalized weakly α-closed set [10] (briefly rgwα-closed set) if rαcl(A)⊆U whenever A⊆U and U is wα-open in (X, τ).
3. regular generalized weakly α-open set if A^c is a [13] rgwα-closed.
4. g-continuous [15] if $f^{-1}(V)$ is g-closed in X for every closed subset V of Y.
5. regular generalized weakly α- continuous [11] (briefly rgwα- continuous) if $f^{-1}(V)$ is rgwα-closed set in X for every closed set V in Y.
6. irresolute [14] if $f^{-1}(V)$ is semi- closed in X for every semi-closed subset V of Y
7. regular generalized weakly α- irresolute [11] (briefly rgwα- irresolute) if $f^{-1}(V)$ is rgwα-closed set in X for every rgwα-closed set V in Y.
8. Locally closed (briefly LC) set [5] if A=U⋂F, where U is open and F is closed in X.
9. θg-lc set [2] if A=U⋂F, where U is θg-open and F is θg-closed in X.
10. θg-lc* set [2] if A=U⋂F, where U is θg-open and F closed in X.
11. θg-lc set** [2] if A=U⋂F, where U is open and F θg-closed in X

12. g-lc set [3] if A=U⋂F, where U is g-open and F is g-closed in X.
13. g-lc* set [3] if A=U⋂F, where U is g-open and F closed in X.
14. g-lc set** [3] if A=U⋂F, where U is open and F g-closed in X.
15. w-lc set if A=U⋂F [16] where U is w-open and F is w-closed in X.
16. w-lc* set if A=U⋂F [16] where U is w-open and F closed in X.
17. w-lc** set if A=U⋂F [16] where U is open and F is w-closed in X.
18. rg-lc set if A=U⋂F [1] where U is g-open and F rg-closed in X.
19. rg-lc* set if A=U⋂F [1] where U is g-open and F closed in X.
20. rg-lc** set if A=U⋂F [1] where U is open and F is rg-closed in X.
21. regular generalized weakly α-locally closed [12] (briefly rgwα-locally closed) if A=U∩F where U is rgwα-open in (X, τ) and F is rgwα-closed in (X, τ).
22. regular generalized weakly α-locally open (briefly rgwα-locally open) if A^c is rgwα-locally closed.
23. LC-continuous [5] if $f^{-1}(V)$ is a lc set of (X, τ) for every open set V of (Y, σ).

Definition 2.2: A subset A of a topological space (X, τ) is called
1. door space [2] if every subset of X is either open or closed in (X, τ).
2. submaximal space [2] if every dense subset of (X, τ) is open in(X, τ).
3. RGWα-submaximal if every dense set in it is RGWα-open in(X, τ).

3. RGWαLC-Continuous maps.

In this section, we define RGWαLC-continuous, RGWαLC*-continuous, RGWαLC**-continuous functions which are weaker than LC-continuous function and stronger than GβLC-continuous function and some of their properties are studied.

Definition 3.1: A function f: (X, τ) →(Y, σ) from a topological space X in to a topological space Y is called RGWαLC-continuous if for each closed set V in Y, $f^{-1}(V)$ is RGWα-LC set in X.

Definition 3.2: A function f: (X, τ) →(Y, σ) from a topological space X in to a topological space Y is called RGWαLC*-continuous if for each closed set V in Y, $f^{-1}(V)$ is RGWα-LC* set in X.

Definition 3.3: A function f: (X, τ) →(Y, σ) from a topological space X in to a topological space Y is called RGWαLC**-continuous if for each closed set V in Y, $f^{-1}(V)$ is RGWα-LC** set in X.

Theorem 3.4: Let f: (X, τ) →(Y, σ) be a function. Then we have the following.
1. If f is locally-continuous then it is RGWαLC-continuous.
2. If f is w-lc-continuous (resp. θ-lc-continuous, lδc-continuous, π-lc continuous) then it is RGWαLC-continuous.
3. If f is RGWαLC-continuous then it is GβLC-continuous.
4. If f is RGWαLC*-continuous then it is RGWαLC-continuous.
5. If f is RGWαLC**- continuous then it is RGWαLC-continuous.

Proof: (1) Assume that f is locally-continuous. Let V be a closed set in Y. Then $f^{-1}(V)$ is locally closed in X. But, every locally closed set is RGWαLC set. Thus $f^{-1}(V)$ is RGWαLC in X. Therefore f is RGWαLC -continuous.
Similarly (2) to (5) can be proved.
However the converse of the above theorem need not be true as seen from the following examples.

Example 3.5: Let X={a,b,c,d,e}, Y={a,b,c,d} τ ={X, ∅,{a},{d},{e},{a,d},{a,e},{d,e},{a,d,e}} and σ= {Y, ∅,{a},{b},{a,b},{a,b,c}}. Let f: (X, τ) →(Y, σ) be the function defined f(a)=a, f(b)=b, f(c)=c, f(d)=d, f(e)=a. Then f is RGWαLC-continuous but not locally continuous, since for the closed set {c,d} in (Y, σ), f^{-1}({c,d })= {c,d} is not locally closed in (X, τ).

Example 3.6: Let X=Y = {a,b,c}, τ= {X, ϕ, {a}, {b}, {a,b}} and σ= {Y, ϕ, {a}}. Let f: (X, τ) →(Y, σ) be the identity map function. Then f is RGWαLC-continuous but not θ-lc-continuous and lδc-continuous, since for the closed set {b,c} in (Y, σ), f^{-1}({b,c})= {b,c} is not θ-locally closed and δ-locally closed set in (X, τ).

Example 3.7: Let X={a,b,c,d} and Y={a,b,c}, τ= {X, ϕ, {a}, {b}, {a,b}, {a,b,c}} and σ={Y, ϕ, {a}}. Let f: (X, τ) →(Y, σ) be the function defined f(a)=a, f(b)=b, f(c)=c, f(d)=a,. Then f is RGWαLC-continuous but not πlc-continuous, since for the closed set {b,c} in (Y, σ), f^{-1}({b,c})= {b,c} is not π-locally closed in (X, τ).

Example 3.8: Let X={a,b,c,d,e}, Y={a,b,c,d} τ ={X, ∅,{a},{d},{e},{a,d},{a,e},{d,e},{a,d,e}} and σ= {Y, ∅,{a},{b},{a,b},{a,b,c}}. Let f: (X, τ) →(Y, σ) be the function defined f(a)=a, f(b)=b, f(c)=c, f(d)=d, f(e)=a. Then f is RGWαLC-continuous but not w-lc-continuous, since for the closed set {a,c,d} in (Y, σ), f^{-1}({a,c,d })= {a,c,d} is not w-locally closed in (X, τ).

Example 3.9: Let X=Y= {a,b,c}, τ={X, ϕ,{a},{b,c}} and σ ={Y, ϕ, {a}}. Let map f: X→Y defined by f(a)=b, f(b)= a, f(c)=c, then f is GβLC-continuous but not RGWαLC-continuous as a closed set F={b,c} in Y, $f^{-1}(F)$= f^{-1}{b,c}={a,c} which is not rgwα-locally closed set.

Example 3.10: Let X={a,b,c,d} and Y={a,b,c}, τ= {X, ϕ, {a}, {b}, {a,b}, {a,b,c}} and σ={Y, ϕ, {a}}. Let f: (X, τ) →(Y, σ) be the function defined f(a)=a, f(b)=c, f(c)=b, f(d)=a,. Then f is RGWαLC-continuous but not RGWαLC*-continuous and RGWαLC**-continuous, since for the closed set {b} in (Y, σ), f^{-1}({b})= {c} is not rgwα-lc* set in (X, τ) and closed set {a,c} in (Y, σ), f^{-1}({a,c})= {a,b,d} is not rgwα-lc** set in (X, τ).

Remark 3.11: From the above discussion and known results we have the following implications
In the following diagram, A→B means A implies B but not conversely.

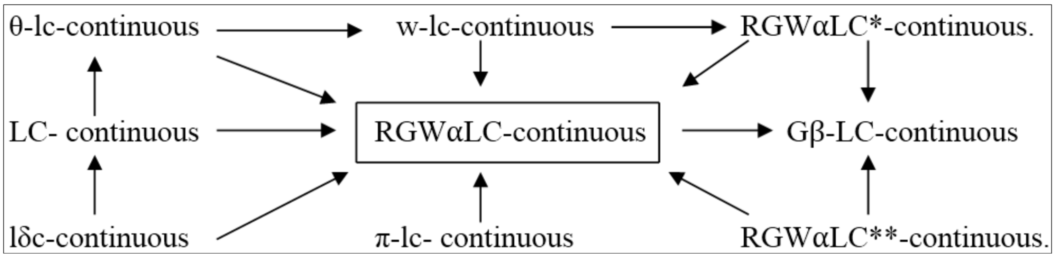

Fig 1

Theorem 3.12: Any map defined on a door space is RGWαLC-continuous (resp. RGWαLC*-continuous, RGWαLC**-continuous).
Proof: Let f: (X, τ) →(Y, σ) be a function and (X, τ) be a door space and (Y, σ) be any space. Let A∈σ. Then by assumption on (X, τ), f^{-1}(A) is either open or closed. In both the cases f^{-1}(A)
∈RGWαLC(X, τ) (resp. RGWαLC*(X, τ), RGWαLC**(X, τ)) and therefore f is RGWαLC-continuous (resp. RGWαLC*-continuous, RGWαLC**-continuous).

Theorem 3.13: A topological space (X, τ) is rgwα-submaximal if and only if every function having (X, τ) as domain is RGWαLC*-continuous.
Proof: Let f: (X, τ) →(Y, σ) be a function and (X, τ) be rgwα-submaximal. Then by theorem 3.50 of [12] P(X) = RGWαLC*(X, τ). If U is any open set of (Y, σ), f^{-1}(U) ∈P(X)= RGWαLC*(X,τ) and so f is RGWαLC*-continuous.
Conversely, assume that every function having (X, τ) as domain be RGWαLC*-continuous. Consider the sierpinski space Y= {0, 1} with σ ={Y, ϕ, {0}}. Let U be a sub set of (X, τ) and define f: (X, τ) →(Y, σ) by f(x) =0 for every x∈U and f(x)=1 for every x∉U. By assumption f^{-1}({0}) =U ∈ RGWαLC*(X,). Therefore we have P(X) = RGWαLC*(X, τ) and so (X, τ) is rgwα-submaximal, by theorem 3.50 of [12].

Theorem 3.14: If f: (X, τ) →(Y, σ) is RGWαLC*-continuous and a sub set B is both open and closed in (X, τ), then restriction f_B: (B, τ_B) →(Y, σ) is RGWαLC*-continuous.
Proof: Let G be open set of (Y, σ). By hypothesis f^{-1}(G) is rgwα-lc* set in (X, τ). Then f^{-1}(G) = U⋂F, for some rgwα-open set U and closed set F of (X, τ) Then f_B^{-1} (G) = B ⋂ f^{-1}(G) = B⋂U⋂F = (B⋂U) ⋂ (B⋂F). Since (B⋂F) is closed in (X, τ) and (B⋂F) ⊂B, B⋂F is closed in (B, τ_B). Since B⋂U is rgwα-open in (X, τ), B⋂U⊂B and B is regular open in (X, τ), B⋂U is rgwα-open in (B, τ_B). This shows that f_B^{-1} (G) ∈RGWαLC*(B, τ_B) and hence f_B is RGWαLC*-continuous.

Theorem 3.15: Let f: (X, τ) →(Y, σ) be RGWαLC-continuous and B an open set in Y containing f(X). Then f_B: (X, τ) → (B, τ_B) is RGWαLC-continuous.
Proof: Let V be an open set in B. Then V is open in (Y, σ), since B is open in (Y, σ). Therefore by hypothesis, f^{-1} (V) is rgwα-lc set in (X, τ). That is f: (X, τ) →(Y, σ) be RGWαLC-continuous.

Remark 3.16: Composition of two RGWαLC-continuous maps need not be RGWαLC-continuous as seen from the following example.

Example 3.17: Let X=Y=Z= {a,b,c} and τ={X, ϕ, {a}}, σ= {Y, ϕ, {a},{b,c}} and η= {Z, ϕ, {b}}. Let f: (X, τ) →(Y, σ) be the function defined f(a)=b, f(b)=a, f(c)=c, and g:(Y, σ) →(Z, η) be a function defined as g(a)=b, g(b)=a, g(c)=c. Then both f and g are RGWαLC-continuous but their composition gof: (X, τ) →(Z, η) is not RGWαLC-continuous, since for a closed set {a,c} in (Z, η), $(gof)^{-1}$({a,c})=f^{-1}(g^{-1}({a,c})= f^{-1}({b,c}) = {a,c}which is not rgwα-lc set in (X, τ).

Theorem 3.18: Let f: (X, τ) →(Y, σ) be RGWαLC-continuous (resp. RGWαLC*-continuous, RGWαLC**-continuous) and g: (Y, σ) → (Z, η) is continuous, then gof: (X, τ) → (Z, η) is
RGWαLC-continuous (resp. RGWαLC*-continuous, RGWαLC**-continuous).
Proof: Let U be an open set in (Z, η). Since g is continuous g^{-1}(U) is open in (Y, σ). Since f is RGWαLC-continuous, f^{-1}(g^{-1}(U)) = $(gof)^{-1}$(U) is rgwα-lc set in (X, τ)and hence gof: (X, τ) → (Z, η) is RGWαLC-continuous.

Definition 3.19: A function f: (X, τ) →(Y, σ) is called sub-RGWαLC*-continuous if there is a basis β for (Y, σ) such that f^{-1}(U) ∈ RGWαLC*(X, τ) for each U∈β.

Theorem 3.20: Let f: (X, τ) →(Y, σ) be a function. Then we have the following.

1. If f is RGWαLC*-continuous, then f is sub-RGWαLC*-continuous.
2. f is sub-RGWαLC*-continuous if and only if there is a subbasis S for (Y, σ) such that f^{-1}(V)∈ RGWαLC*(X, τ) for each V∈S.
3. (iii)If f is sub-LC-continuous, then f is sub-RGWαLC*-continuous.

Proof: (i) Let f be RGWαLC*-continuous, β be basis of (Y, σ) and V∈β. By hypothesis f^{-1}(V)∈ RGWαLC*(X, τ) and so f is sub-RGWαLC*-continuous.
(ii) Let f be sub-RGWαLC*-continuous. It follows from assumption that there is a basis β for (Y, σ) such that f^{-1}(U) ∈ RGWαLC*(X, τ) for each U∈β. Since βis also a subbasis for(Y, σ) the proof is obvious.
Conversely, let S be a subbasis of (Y, σ) and β= {A⊂Y: A is an intersection of finitely many sets belonging to S}. Then β is a basis for (Y, σ). For each U∈β, U=⋂{F_i: F∈S, i∈∧} where ∧ is a finite set. Now f^{-1}(U)= ⋂{f^{-1}(F_i): F_i ∈S, i∈∧}. By assumption, f^{-1}(F_i) ∈ RGWαLC*(X, τ) for each F_i ∈S, i∈∧. Also ⋂{f^{-1}(F_i): F_i ∈S, i∈∧}= f^{-1}(U)∈ RGWαLC*(X, τ), by Theorem (i) and hence f is sub-RGWαLC*-continuous.
(iii) Follows from LC(X, τ)⊂ RGWαLC(X, τ).

Remark 3.21: The following examples show that the converses of (i) and (ii) of Theorem are not always true.

Example 3.22: Let X= {a,b,c,d}, Y={a,b,c}, τ={X, ϕ, {a}, {a,b}} and σ={Y, ϕ, {a}, {c}, {a,c}}. Then RGWαLC*(X, τ) = P(X)-{a,b,d}, and β= {{a},{b},{c}} is a basis for (Y, σ). Define f: (X, τ) →(Y, σ) by f(a)=f(d)=a, f(b)=b, f(c)=c. Then f is sub-RGWαLC*-continuous but not RGWαLC*-continuous, since for the closed set {a,b}of (Y, σ), f^{-1}({a,b})={a,b,d}which is not rgwα-lc* set in (X, τ).

Example 3.23: Let X= Y={a,b,c}, τ={X, ϕ, {a}, {b}, {a,b}} and σ={Y, ϕ, {a}, {a,b}}. Then LC(X,τ) = {X, ϕ, {a}, {b}, {c}, {d}, {a,b}, {a,c}, {a,d}, {b,c}, {a,b,c}, {a,c,d}, {b,c,d}}, RGWαLC*(X, τ) = P(X) and β= {Y,{a}, {a,b}}is a base for (Y, σ). Define f: (X, τ) →(Y, σ) by f(a)=c, f(b)=d, f(c)=a, f(d)=b. Then f is sub-RGWαLC*-continuous but not sub-LC-continuous, since for the set {a,b}∈β, f^{-1}({a,b})={c,d} which is not lc-set in (X, τ).

Remark 3.24: The composition of a sub-RGWαLC*-continuous function and a continuous function need not be a sub-RGWαLC*-continuous.

Solution: Take a sub-RGWαLC*-continuous function f: (X, τ) →(Y, σ) which is not RGWαLC*-continuous (for example the function in example). Hence there is a set V∈σ such that f^{-1}(V) ∉ RGWαLC*(X, τ). Let η= {Y, ϕ, V}. Then η is a topology on Y and the identity function g: (Y, σ) → (Y, η) is continuous. But composition gof: (X, τ) →(Y, η) is not sub-RGWαLC*-continuous.

4. RGWαLC -Irresolute maps
In this section, we define RGWαLC- Irresolute, RGWαLC*- Irresolute, RGWαLC**- Irresolute functions which are weaker than w-LC-irresolute functions and stronger than GβLC-Irresolute functions.

Definition 4.1: A function f: (X, τ) →(Y, σ) from a topological space X in to a topological space Y is called RGWαLC-irresolute if for each RGWα-LC set V in Y, f^{-1}(V) is RGWα-LC set in X.

Definition 4.2: A function f: (X, τ) →(Y, σ) from a topological space X in to a topological space Y is called RGWαLC*-Irresolute if for each RGWα-LC* set V in Y, f^{-1}(V) is RGWα-LC* set in X.

Definition 4.3: A function f: (X, τ) →(Y, σ) from a topological space X in to a topological space Y is called RGWαLC**-Irresolute if for each RGWα-LC** set V in Y, f^{-1}(V) is RGWα-LC** set in X.

Theorem 4.4: Let f: (X, τ) →(Y, σ) be a function. Then we have the following
(i)If f is rgwα-irresolute, then it is RGWαLC-irresolute.
(ii)If f is RGWαLC-irresolute (resp. RGWαLC*-irresolute, RGWαLC**-irresolute), then it is RGWαLC-continuous (resp. RGWαLC*-continuous, RGWαLC**-continuous).

Proof: (i) Let f is rgwα-irresolute, and V∈ RGWαLC(Y, σ). Then V=U⋂F for some rgwα-open set U and some rgwα-closed set F in (Y, σ). We have f^{-1}(V) = f^{-1}(U⋂F) =f^{-1}(U)⋂f^{-1}(F) where f^{-1}(U) is a rgwα-open set in(X, τ), since is rgwα-irresolute. This shows that f^{-1}(V) ∈ RGWαLC(X, τ) and hence f is RGWαLC-irresolute.
(ii)Follows from the fact every open set is rgwα-lc, rgwα-lc* and rgwα-lc**.

Remark 4.5: The converse of the Theorem 4.4 need not be true in general as seen from the following example.

Example 4.6: Let X= Y= {a,b,c,d}, τ={X, ϕ, {a}, {b}, {a,b,c}} and σ={Y, ϕ, {a}, {a,b}}. Define a function f:(X, τ) →(Y, σ) by f(a)= b, f(b)=c, f(c)=d, f(d)=a. Then f is RGWαLC-irresolute but not rgwα-irresolute, since for the rgwα-closed set {b, c} of (Y, σ), f^{-1}{b, c}= {a,b} which is not rgwα-closed set in (X, τ).

Example 4.7: Let X= Y= {a,b,c}, τ={X, ϕ, {a}} and σ={Y, ϕ, {a}, {a,b}, {a,c}}. Let f: (X, τ) →(Y, σ) be an identity map. Then f is RGWαLC-continuous but not RGWαLC-irresolute, since for the rgwα-lc set {a,b} in (Y, σ), f^{-1}({a,b})={a,b} which is not rgwα-lc set in (X, τ)

Theorem 4.8: Let f: (X, τ) →(Y, σ) be a function. Then we have the following.
(i)If f is w-LC-irresolute (resp. w-LC*-irresolute, w-LC**-irresolute), then it is RGWαLC-irresolute (resp. RGWαLC*-irresolute, RGWαLC**-irresolute).
(ii)If f is RGWαLC-irresolute (resp. RGWαLC*-irresolute, RGWαLC**-irresolute), then it is Gβ-LC-irresolute (resp. Gβ-LC*-irresolute, Gβ-LC**-irresolute).

Proof: (i) Follows from the fact that every w-lc set (resp. w-lc* set, w-lc** set) is a rgwα-lc set (resp. rgwα-lc* set, rgwα-lc** set).
(ii)Follows from the fact that every rgwα-lc set(resp. rgwα-lc* set, rgwα-lc** set) is a gβ-lc set (resp. gβ-lc* set, gβ-lc** set).

Remark 4.9: the converse of the Theorem need not be true in general as seen from the following example.

Example 4.10: Let X= Y= {a,b,c,d}, τ={X, ϕ, {a}, {a,b } and σ={Y, ϕ, {a}, {b}, {a,b,c}}. Let f: (X, τ) →(Y, σ) be an identity map. Now w-LC(X, τ)={ X, ϕ, {a}, {b}, {a,b},{c,d}, {b,c,d}, RGWαLC(X, τ)=P(X), w-LC(Y, σ)= {X, ϕ, {a}, {b},{c}, {d}, {a,b}, {a,c}, {b,c}, {c,d}, {a,b,c}, {a,c,d}, {b,c,d} and RGWαLC(Y, σ)= P(Y). Then f is RGWαLC-irresolute but not w-LC-irresolute, since for the w-LC set {c} of (Y, σ), f^{-1}{c} ={c} which is not w-LC set in (X, τ).

Example 4.11: Let X= Y= {a,b,c}, τ={X, ϕ, {a}} and σ={Y, ϕ, {a}, {b},{a,b}} Let f: (X, τ) →(Y, σ) be an identity map. Then f is GβLC -irresolute but not RGWαLC-irresolute, since for the rgwα-lc set {a,c}of (Y, σ), f^{-1} ({a,c})={a,c} which is not rgwα-lc set in (X, τ).

Remark 4.12: The following example shows that
(i)RGWαLC-irresolute need not be RGWαLC*-irresolute.
(ii)RGWαLC-irresolute need not be RGWαLC**-irresolute.

Example 4.13: Let X= Y= {a,b,c,d}, τ={X, ϕ, {a}, {b}, {a,b,c}} and σ={Y, ϕ, {a}, {a,b}}. Let f: (X, τ) →(Y, σ) be an identity map. Then f is RGWαLC -irresolute but not RGWαLC*-irresolute, since for the rgwα-lc* set {c} of (Y, σ), f^{-1} ({c})={c} which is not rgwα-lc* set in (X, τ).

Example 4.14: Let X= Y= {a,b,c,d}, τ={X, ϕ, {a}, {a,b}} and σ={Y, ϕ, {a}, {b}, {a,b,c}}. Let f: (X, τ) →(Y, σ) be an identity map. Then f is RGWαLC -irresolute but not RGWαLC**-irresolute, since for the rgwα-lc** set {a,d} of (Y, σ), f^{-1} ({a,d})={a,d} which is not rgwα-lc** set in (X, τ).

Theorem 4.15: Any map defined on a door space is RGWαLC-irresolute.
Proof: Proof is similar to that of Theorem

Theorem 4.16: Let f: (X, τ) →(Y, σ) and g: (Y, σ) → (Z, η) be two maps. Then

1. gof: (X, τ) → (Z, η) is RGWαLC-irresolute (resp. RGWαLC*-irresolute, RGWαLC**-irresolute) and f is RGWαLC-irresolute (resp. RGWαLC*-irresolute, RGWαLC**-irresolute) and g is RGWαLC-irresolute (resp. RGWαLC*-irresolute, RGWαLC**-irresolute).
2. gof: (X, τ) → (Z, η) is RGWαLC-continuous if f is RGWαLC-irresolute and g is RGWαLC-irresolute.

Proof: (i) Let V∈ RGWαLC (Z, η). Since g is RGWαLC-irresolute, g^{-1}(V) ∈ RGWαLC(Y, σ). Since f is RGWαLC-irresolute,f^{-1}(g^{-1}(V))= $(gof)^{-1}$(V) is rgwα-lc set in (X, τ). Therefore gof is RGWαLC- irresolute.
(ii)Let V∈η. Since g is RGWαLC-continuous, g^{-1}(V) ∈ RGWαLC(Y, σ). Since f is RGWαLC-irresolute, f^{-1}(g^{1}(V)= $(gof)^{-1}$(V) ∈ RGWαLC(X, τ). Therefore gof is RGWαLC-continuous.

5. Conclusion

In this paper we have introduced and studied the properties of RGWαLC-continuous and RGWαLC-irresolute maps in topological spaces. Our future extension is to study rgwα-locally separation axioms in Topological Spaces.

6. Acknowledgement

The Authors would like to thank the referees for useful comments and suggestions.

7. Reference

1. Arockiarani I, Balachandran K, Ganster M. Regular generalized locally closed sets and rgl-continuous functions, Indian J Pure Appl Math. 1997; 28:661-669.
2. Arockiarani I, Balachandran K. on θ-g locally closed set (pre print).
3. Balachandran K, sundaram P, Maki H. g-lc and glc-continuous function, indian j pure appp Math. 1966; 27:235-244
4. Balachandran K, Sundaram P, Maki H. Generalized locally closed sets and GLC continuous functions, Indian J Pure. Appl. Math. 1996; 27:235-244.

5. Ganster M, Reilly IL. Locally closed sets and LC-continuous functions, Internal J Math. and Math. Sci. 1989; 12:417-424.
6. Stone M. Application of the theory of Boolean rings to general topology, Trans. Amer. Math. Soc. 1937; 41:374-481.
7. Bourbaki N. General topology, Addison-Wesley, Reading, Mass, 1966.
8. Levine N. Generalized closed sets in topology, Rend. Circ Mat. Palermo. 1970; 19(2):89-96.
9. Sundaram P. Studied on generalizations of continuous maps in topological spaces, Ph.D. Thesis, Bharathiar University, Coimbatore, 1991.
10. wali RS, vijayalaxmi R. Patil On rgwα-closed set in Topological spaces Jl of comp. and math. sci. 2017; 8(3):62-70.
11. wali RS, vijayalaxmi R. Patil On rgwα-continuous and rgwα-irresolute maps in Topological spaces Int Jl of Math. Trends and Technology. 2017, 45(1).
12. wali RS, vijayalaxmi R. Patil On rgwα- locally closed set in Topological spaces (processing).
13. wali RS, vijayalaxmi R. Patil On rgwα-open set in Topological spaces Int Jl of Mathematical Archeive. 2017; 8(4):1-11
14. Crossley SG, Hildebrand SK. Semi-topological properties, Fund. Math. 1972; 74:233-254.
15. Benchalli SS, Patil PG, Rayanagaudar TD. ωα-Closed sets in Topological Spaces, The Global J Appl. Math. and Math. Sci. 2009; 2:53-63.
16. Sheik john M. a study on generalization of closed sets on continuous maps in topological & bitopological spaces, Ph. D thesis, Bharathiar university, Coimbatore, 2002.