International Journal of Statistics and Applied Mathematics 2017; 2(5): 131-135

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2017; 2(5): 131-135

www.mathsjournal.com
Received: 20-07-2017
Accepted: 21-08-2017

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$M^X/G/1$ vacation queueing system with server timeout

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Abstract
This paper is to analyze an $M^X/G/1$ vacation queueing system with server timeout. We consider single server vacation queueing system that operates in the following manner; when the system is empty, the server waits for fixed time C. At the expiration of this time, if no one arrives into the system then server takes vacation. If anyone arrived into the system during the time as well as in vacation the server commence service exhaustively and obtained an expression for expected system length of this queueing system. Numerical results are also illustrated.

Keywords: Vacation queueing system, server timeout, batch arrival and mean system length

1. Introduction

Queueing system with server vacations have been studied for the past few decades and applied in manufacturing, computer communication network systems. In vacation queuing model the server completely stops service when it is on vacation. Queueing system with server vacations idea was first discussed by Levy and Yechiali [6] they introduced the utilization of idle time in an M/G/1 queueing system. B.T. Doshi [2] did excellent surveys on the vacation models. Jau-chauank, Chian-Hangwu and Zhe George Zhang [4] also did research on various kinds of vacation models

There has been performed several researchers on the batch arrivals queueing system, which are used in many practical situation. Some of these previous works are briefly discussed here. Baba. Y [1] first studied an $M^X/G/1$ queueing system with multiple vacations. He derived the general queue length distribution at an arbitrary time. Also, he obtained the waiting time and busy period distribution under multiple vacation policy using supplementary variable technique. J.C.Ke [5] introduced the design of an $M^X/G/1$ queue with single and multiple vacation policy. For batch arrivals analytical results are derived with various batch size distributions by V. Vasantha Kumar, et.al. [9].

Vacation queueing systems with server timeout are studied by some of authors. Oliver C.Ibe [7] discussed an M/ G/1 vacation queueing system with server timeout. E.Ramesh Kumar and Y. Praby Loit [3] studied Vacation Bulk Queueing Model with setup time and server timeout.

The main objective of this paper is to analyze an $M^X/G/1$ vacation queueing system with server timeout and the expected system length for various batch size distributions is obtained. Further, numerical solution of this system is presented.

2. Model Description

Customers arrive in batches according to a compound Poisson with rate d. Let D(z) denotes the batch size distribution. The service time provided by a single server with a general distribution function. Whenever the system become empty the server waits for fixed time c is called server timeout, at this time if a customer arrive the server return to the system and do service. At the expiration of the time if no customer arrive the server takes vacation, here also if the server finds customer is waiting in the queue it return to the system during vacation and commence service to the customer exhaustively; otherwise it take another vacation, it is represented in diagrammatic form as given in fig 1 :

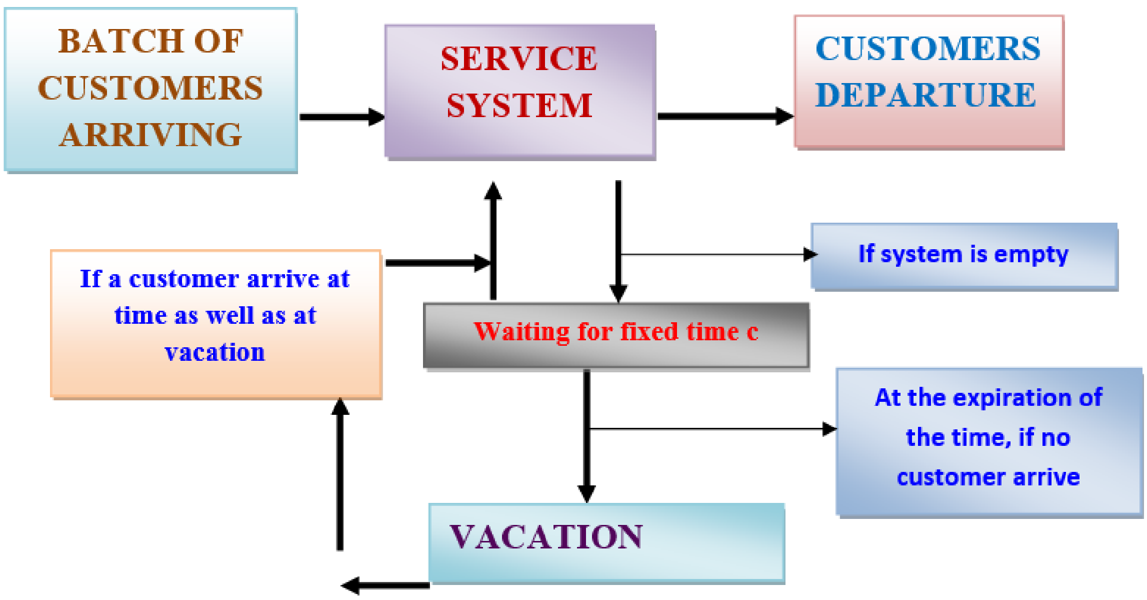

Fig 1: Batch arrivals in Queueing System with Vacation and Server timeout

3. Notation

The following notations are used to analyze the model.

λ	=	arrival rate.
μ	=	service rate.
d	=	batch arrival rate.
x	=	general distribution.
v	=	vacation.
A	=	number of customers in the system at the beginning of a busy period.
B	=	number of customers left behind by an arbitrary departing customer.
L	=	number of customers in the system at an arbitrary point in time.
C	=	fixed time.
D(z)	=	batch size distribution.
γ	=	mean of the vacation.
$F_x(x)$, $F_v(v)$	=	cumulative distribution function.
$f_x(x)$, $f_v(v)$	=	probability density function.
$G_A(z)$, $G_B(z)$& $G_L(z)$	=	Z-transforms of A,B&L.
$M_x(s)$, $M_v(s)$	=	S- transforms of X and V.
$E(W_q)$	=	Expected of mean waiting time in the system.
E(L)	=	Mean system length.

4. Analysis of the Model

The customer arrivals occur according to a compound Poisson process with arrival rate 'λ'. The batch of arriving customers joins in a single waiting line. An arriving batch has size "d". D (z) is the probability generating function of the number of customers in the arriving batch. D' and D" denote the first and second order derivatives of D (z) with respect to z. The service times follows general distribution with service rate μ. If however, the queue is empty, the server will wait for fixed time c. At this time if any customer arrive the server return to the system and commence service, by the expiration of the time, if no customer arrive then the server takes vacation. During the vacation customer arrives then the server returns to the system and starts service. It repeats as mentioned above manner. Here, the general service time, X is a random variable with cumulative distribution function, $F_X(x)$ with mean E(X) and its second moment E(X^2). Similarly it follows the S-transform of the probability density function of X; $F_X(x)$ is $M_X(s)$ is defined by

$$M_X(s) = E(e^{-SX}) = \int_0^\infty e^{-sx} f_X(x)dx \tag{1}$$

Here, arrivals are in batches, service is provided individually and its generating function is taken from the [ref 8] as given below

$$G_{L(M^X/G/1)}(Z) = \frac{(1-\rho)(1-z)M_X(\lambda-\lambda D(z))}{M_X(\lambda-\lambda D(z))-z} \tag{2}$$

Let the vacation time, V is also a random variable which is generally distributed with CDF $F_V(v)$ and s-transform $M_X(s)$. The mean of V is E (V) and its second moment E (V^2). Since X and V are mutually independent variables. Let the random variable A denotes if the batch of customers is arriving into the system at the beginning of busy period. The probability mass function of A is P_A (a) =P [A=a], whose z-transformation G_A (z) is given by $G_A(z) = E(z^A) = \sum_{a=1}^{\infty} z^a p_A(a)$ (3)

The mean of A is E (A) and its second moment is E (A^2). Let the random variable B, number of customers left behind by an arbitrary customer depart. The PMF of B is P_B (b), whose z-transform is given by $G_B(Z) = \dfrac{1-G_A(z)}{(1-z)E(A)}$, (4)

The mean B is E (B) and its second moment is E (B^2). Let L, denote the numbers of customers are in the system with the arbitrary point in time and it's PMF is $P_L(l)$ whose z-transform is given by $G_L(Z) = G_B(Z).G_{L(M^X/G/1)}(Z)$ (5)

The mean of L is E (L) and its second moment is E (L^2). Let the utilization factor be defined by $\rho = \dfrac{\lambda}{\mu}\{D'(1)\}$. (6)

Specified Batch Size Distributions

As the batch size'd' is a random variable it has a probability distribution. In particular Deterministic, Geometric and Positive Poisson are considered.

1.) If the batch size distribution is Deterministic, then the generating function equals to

D (z) = z^d (7)

This gives mean $\overline{D} = D'(1) = d$ and second moment $\overline{\overline{D}} = D''(1) = d^2 - d$, where d is the average batch size.

2.) The batch size distribution is Positive Poisson, then the generating function equals to

D (z)$= me^{-\alpha}(e^{\alpha z} - 1)/\alpha$, where m= $\alpha/(1-e^{-\alpha})$. (8)

This gives mean $\overline{D} = m$ and second moment $D''(1) = \dfrac{\alpha^2}{1-e^{-\alpha}}$, where m is the average batch size.

3.) If the batch size distribution is Geometric, then the generating function equals

D (z) = p [z^{-1}-(1-p)]$^{-1}$ (9)

This gives mean $D'(1) = \overline{a} = \dfrac{1}{p}$ and second moment $D''(1) = \overline{\overline{a}} = \dfrac{2(1-p)}{p^2}$, where $\dfrac{1}{p}$ is the average batch size. To determine Mean system length from the mean of the waiting time,
by applying little's law,

$E(W_q) = \dfrac{1}{\lambda}\dfrac{dG_L(z)}{dz}\Big|_{Z=1} - E(X)$ (10) (From Equation (5) and ref [8]) $\lambda E(W_q) + E(X) = \dfrac{dG_L(z)}{dz}\Big|_{Z=1}$

$\Longrightarrow \lambda E(W_q) + E(X) = \dfrac{d}{dz}G_L(z)\big|_{Z=1} = E(L)$ (10A)

If G_A (z) is known, we can obtain E (L).

Consider the following three mutually exclusive events associated with the servers experience after returning from a vacation:

1. The server returns from vacation, waits and commences another vacation without serving a customer; the probability of this event is $M_v(\lambda)\, e^{-\lambda c}$.
2. The server returns from vacation, waits and serves at least one customer before taking vacation; the probability of this event is $M_v(\lambda)\, \{1-e^{-\lambda c}\}$.
3. The server returns from vacation, and finds at least one waiting customer; the probability of this event is (1- $M_v(\lambda)$).

Under event 2, a busy period is initiated with exactly one customer in the system. Similarly under event 3, a busy period is initiated with at least one customer in the system. Therefore, if we define P_k as the probability of event k, given that busy period is initiated before the server commences another vacation, where k=2, 3 then we obtain the following results:

$$G_A(z) = zp_2 + \frac{M_v(\lambda - \lambda D(z)) - M_v(\lambda)}{1-M_v(\lambda)}\, p_3 . \quad (11)$$

Where $p_2 = \frac{M_V(\lambda)\{1-e^{-\lambda c}\}}{(1-e^{-\lambda c})M_V(\lambda)}$ (11a) and $p_3 = \frac{1-M_V(\lambda)}{(1-e^{-\lambda c})M_V(\lambda)}$ (11b)

From this we obtain the result $G_A^{'}(1) = E[A] = p_2 + \frac{\lambda D'(1)E(v)}{1-M_v(\lambda)} p_3$

$$= \frac{M_V(\lambda)D''(1) + \lambda D'(1)E(v)}{1-e^{-\lambda c}.M_v(\lambda)}. \quad (12)$$

and $G_A^{''}(1) = E[A^2] = \frac{\lambda D''(1)E(v) + \lambda^2 (D'(1))^2 E(v^2)}{1-M_v(\lambda)} p_3$

$$= \frac{\lambda D''(1)E(v) + \lambda^2 (D'(1))^2 E(v^2)}{1-e^{-\lambda c}.M_v(\lambda)}. \quad (13)$$

In particular, in the $M^X/G/1$ queueing model if X (Service time) is assumed to follow an exponential distribution with mean E[X] =1/μ, $E[X^2]$ =2/μ^2 and $\rho = \frac{\lambda}{\mu}\left\{D^{'}(1)\right\}$ (where D' (1)=1), also V(vacation times) assumed to follow an exponential distribution with E[V] =(1/ γ), $E[V^2]$ =2/ γ^2 and $M_V(s) = \frac{\gamma}{s+\gamma} \Rightarrow M_V(\lambda) = \frac{\gamma}{\lambda+\gamma}$

Then, by using the above results we obtain mean system length of $M^X/M/1$queueing system with server time out is

$$E(L) = \frac{\rho}{1-\rho} + \frac{[\lambda\gamma D''(1) + \lambda(2\lambda(D'(1))^2]}{2[\lambda D'(1)\gamma(\lambda+\gamma) - \gamma^3(1-e^{-\lambda c})]} + \frac{\rho D''(1)}{2(1-\rho)} \quad (14)$$

To the above equation, let us take limiting case for c

$$lt_{c\to 0} E(L) = \frac{\rho}{1-\rho} + \frac{[\lambda\gamma D''(1) + \lambda(2\lambda(D'(1))^2]}{2[\lambda D'(1)\gamma(\lambda+\gamma)]} + \frac{\rho D''(1)}{2(1-\rho)}. \quad (15)$$

and

$$lt_{c\to\infty} E(L) = \frac{\rho}{1-\rho} + \frac{[\lambda\gamma D''(1) + \lambda(2\lambda(D'(1))^2]}{2[\lambda D'(1)\gamma(\lambda+\gamma) - \gamma^3]} + \frac{\rho D^{'}(1)}{2(1-\rho)}. \quad (16)$$

Note: where E (L) monotonically increases as C increases

$$\frac{dE(L)}{dc} = -\frac{[\lambda\gamma D''(1) + 2\lambda^2(D'(1))^2]e^{-\lambda}\gamma^3}{2\{(\lambda D'(1)\gamma(\lambda+\gamma) - (1-e^{-\lambda})\gamma^3\}^2} < 0. \quad (17)$$

Which is consistent with the fact that multiple vacation scheme (c=∞) and has a smaller system length of the scheme (c=1). Also it shows that the s-transform of the PDF of the waiting time of system length is given by

$$M_w(s) = \frac{\lambda(1-\rho)\{1-G_A(1-s/\lambda)\}}{E[A]\{S-\lambda+\lambda M_X(S)\}}. \quad (18)$$

By using (11), (11a), (11b) & (12) in above equation we get

$$M_w(s) = \frac{(1-\rho)\{\lambda[1-M_V(s)] + s(1-e^{-\lambda c})M_V(\lambda)\}}{\{s(1-e^{-\lambda c})M_V(\lambda) + \lambda D^{'}(1)E(V)\}\{S-\lambda+\lambda M_X(S)\}}. \quad (19)$$

Which, on differentiation and evaluating at s=0, gives the same value of E (L) obtained in equation (14).

5. Numerical Illustrations

By fixing different values of the parameters in the equation (14) we get numerical results which are illustrated below:

Table 1: Effect of E(L) in different batch size distributions (Deterministic, Geometric and Positive Poisson) with fixed values d=2, λ=2, μ=50, c=1 and γ =0.25.

Parameters	Parameter Values	E(L) for Deterministic Distribution	E(L) for Geometric Distribution	E(L) for Positive Poisson Distribution
d	2	7.5515	7.8621	9.05123
	3	11.7012	8.8528	12.76686
	4	16.2671	10.294	17.20318
	5	21.466	11.8441	22.48156
	6	27.5421	24.8526	28.7996
λ	2	7.5515	7.86206	9.05123
	6	8.3980	8.79389	10.32097
	10	10.4412	11.74924	15.24963
	14	11.4641	13.27274	18.60797
	18	3.0614	15.6602	25.66514
μ	50	7.5515	7.86206	9.0513
	60	7.5345	7.83649	9.017707
	70	7.5205	7.81548	8.99023
	80	7.5088	7.79792	8.96731
	90	7.4989	7.78301	8.94790
c	1	7.5515	7.86206	9.05123
	2	7.5575	7.86828	9.05738
	3	7.55893	7.86912	9.05822
	4	7.55851	7.86924	9.05833
	5	7.55852	7.86925	9.05835
γ	0.25	7.55154	7.86206	9.05123
	0.75	2.39325	2.67044	3.04786
	1.5	1.22470	1.47757	1.66395
	2	0.97128	1.21771	1.3564
	2.5	0.8404	1.08655	1.19327

From the above table 1,

- As d, λ and c are increasing then mean system length E (L) increasing in all three batch size distributions.
- As μ and γ are increasing then mean system length E (L) is decreasing in all three batch size distribution.

6. Conclusion

In this model, we have derived an expression for the system length of $M^X/G/1$ vacation queueing system with server timeout, particular case of $M^X/M/1$ vacation Queueing System with server timeout expected system length is derived and numerical results are illustrated under a set of parameters by applying different distribution assumptions that shows the impact of the timeout period on the system length.

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