International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2017; 2(5): 136-143

www.mathsjournal.com
Received: 13-07-2017
Accepted: 16-08-2017

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Non-Linear Alfven wave with finite amplitude in viscous plasma

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Abstract
In this paper, we investigate the propagation of nonlinear Alfven waves with small and finite amplitude in a viscous plasma. This study is an extension to viscous plasma of the work of Borcia and Ignat (1998), to see the effect of viscosity on the propagation of Alfven waves. The organization of the paper is as follows. The problem is formulated in terms of basic equations in section 2. In section 3, first, second and third order perturbation equations are obtained by employing asymptotic perturbation technique. A nonlinear Schrondinger equation is also obtained to describe the nonlinear behavior of Alfven wave. In section 4, the nonlinear Schrodinger (NLS) equation is discussed and conclusions are drawn.

Keywords: Non-Linear Alfven wave, viscous plasma, finite amplitude

1. Introduction
Alfven waves play an important role in the dynamics of space plasma, which are often permeated by a large scale magnetic field (Belcher and Davis 1971; Cross 1988; Tu and Marsch 1995). The evolution of nonlinear Alfven waves which have been observed in solar wind, planetary bow shocks, and interplanetary shocks, comet atmosphere (Scarf *et al*. 1986 ; Buti 1990; Kennel *et al*. 1988) are described by Vector derivative Schrondinger (VDNLS) and DNLS (for circularly polarized parallel propagation of waves) equations. The derivative nonlinear Schrodinger (DNLS) equation appropriate to degenerate Alfven waves was first derived by Rogister (1971) who used kinetic theory. Subsequently, Mjolhus (1976) and Mio *et al*. (1976) derived the fluid theory version of the DNLS equation. Circularly polarized Alfven wave of arbitrary amplitude are an exact solution of the full set of ideal MHD explanations in the absence of gravity and other external forces. These waves remain compressible for any arbitrary amplitude. But, in contrast to the circularly polarized Alfven wave, behavior of linearly polarized Alfven wave depends upon the wave amplitude. In the linear limit, these waves make the magnetic field and plasma oscillate in the transversal direction with respect to the background magnetic field, and perturb the absolute value of the magnetic field.

For linearly polarized Alfven wave, Scheurwater (1990) investigated the non-linear behavior of finite amplitude Alfven wave beam with finite width propagating in an ideal and uniform magneto plasma. He shows the nonlinearities lead to a transversal defocusing of the beam and is describe by nonlinear Schrodinger wave equation. He pointed out that the longitudinal ponderomotive force due to the beam defocusing accelerates upto the velocity $V \equiv c_{A0}\varepsilon^2\alpha/(1-\beta)$, where c_{A0} is Alfven speed and ε is the ratio of magnetic field in the beam and background. Borcia and Ignat (1998) studied the propagation of nonlinear quasimonochromatic Alfven waves with small and finite amplitude into an ideal MHD plasma. They considered the similar problem as Scheurwater (1990) except the nonlinearities which were assumed to be more accentuated. The width of the beam was considered negligible. They found that the shock wave develops in an astrophysical plasma due to the nonlinear Alfven wave under certain conditions.

2. Formulation of the problem

We consider a viscous compressible plasma and the plasma motion is governed by the MHD equations mass conservation equation

$$\frac{d\rho}{dt} + \rho\nabla.v = 0 , \tag{1}$$

Momentum equation

$$\rho\frac{dv}{dt} = -\nabla p + \frac{1}{4\pi}(\nabla\times B)\times B + \mu(\nabla^2 v + \frac{1}{3}\nabla(\nabla.v)) , \tag{2}$$

Induction equation

$$\frac{\partial B}{\partial t} = \nabla\times(v\times B) , \tag{3}$$

Equation of state

$$\frac{dp}{dt} + pc_{s0}^2\nabla.v = 0 . \tag{4}$$

Here ρ is the plasma density, v is the velocity, p is the plasma pressure and B is the induction of the magnetic field.

We consider the equilibrium state in which the ambient magnetic field is taken to be uniform and directed along the z-axis. The other parameters of the plasma in the unperturbed state are given by,

$$\vec{v} = 0, \qquad \rho = \rho_0, \qquad p = p_0, \qquad \vec{B} = \vec{B}_0 = B_0\hat{z},\ (B_0 = \text{const}) ,$$

3. The perturbation procedure

We assume plasma perturbations restricted along z-axis. Since the perturbation is caused by a low frequency, small amplitude Alfven wave propagating along z-axis, so the dependence of the perturbation is considered only on z and t. Using derivative expansion method (Jeffrey and Kawahara 1982; Swanson) the medium perturbations are expanded into asymptotic series

$$W(z,t,\varepsilon) = \sum \varepsilon^n W_n(z_0, z_1, ...z_N, t_0, t_1, ...t_N) + O(\varepsilon^{N+1}) , \tag{5}$$

where

$$z_n = \varepsilon^n z_0, \qquad t_n = \varepsilon^n t_0 ,$$

and W_0, W_n/W_{n-1} are bounded for each z_n and t_n. The weakness of the nonlinearity is described by the nondimensional parameter $r(r < 1)$. As the dependent variables $(v_y)_m$, $(v_z)_n$, p_m, p_n and $(B_y)_m$ denoted by W, are represented by the expansion given below

$$\frac{\partial W}{\partial t} = \sum \varepsilon^n \frac{\partial W}{\partial t_n} , \tag{6}$$

$$\frac{\partial W}{\partial z} = \sum \varepsilon^n \frac{\partial W}{\partial z_n} , \tag{7}$$

For simplicity it is assumed that the perturbations z-axis we negligible and the magnetic field along z is given by $B_z = B_0$ = constant. Substituting equations (5), (6) and (7) into equations (1)-(4) and equating the coefficients of each power of ε on both the sides of the corresponding equations, we obtain first order, second order and third order perturbation equations.

The first order, O(ε) equations, are

$$\frac{\partial v_{y1}}{\partial t_0} = \frac{B_0}{4\pi\rho_0}\frac{\partial B_{y1}}{\partial z_0} + v\frac{\partial v_{y1}}{\partial t_0^2} , \tag{8}$$

$$\frac{\partial v_{z1}}{\partial t_0} = \frac{1}{\rho_0}\frac{\partial p_1}{\partial z_0} + \frac{4}{3}v\frac{\partial^2 v_{z1}}{\partial z_0^2} , \tag{9}$$

$$\frac{\partial B_{y1}}{\partial t_0} = B_0\frac{\partial v_{y1}}{\partial z_0} , \tag{10}$$

$$\frac{\partial \rho_1}{\partial t_0} = \rho_0 \frac{\partial v_{y1}}{\partial z_0}, \tag{11}$$

$$\frac{\partial \rho_1}{\partial t_0} = -\rho_0 c_{s0}^2 \frac{\partial v_{z1}}{\partial z_0}, \tag{12}$$

The solution for the Alfven mode obtained from the equations (8)-(12) is

$$B_{y1} = A_y(t_1, t_2, t_3, ..., z_1, z_2, z_3, ...)e^{i\theta} + c.c., \tag{13}$$

$$v_{y1} = \frac{B_0 \omega}{4\pi \rho_0 k v_A^2}(A_y e^{i\theta} + c.c.), \tag{14}$$

$$v_{z1} = \rho_1 = p_1 = B_{z1} = 0, \tag{15}$$

Here A_y represented the complex amplitude of B_{y1}, c.c. denotes the complex conjugate of the preceding term,

$$\theta = kz_0 - \omega t_0,$$

$$v_A = \sqrt{\frac{B_0^2}{4\pi\rho_0}},$$

Frequency ω and wavenumber k are related by

$$\omega^2 - k^2 v_A^2 + i v k^2 \omega^2 = 0, \tag{16}$$

Which is the dispersion relation for linear Alfven waves propagating in viscons plasma. The second order, $O(\varepsilon^2)$ perturbations, equations are

$$\rho_0\left(\frac{\partial v_{y2}}{\partial t_0} + \frac{\partial v_{y1}}{\partial t_1}\right) + \rho_1 \frac{\partial v_{y1}}{\partial t_0} + \rho_0 v_{z1}\frac{\partial v_{y1}}{\partial z_0} = \frac{B_0}{4\pi}\left(\frac{\partial B_{y2}}{\partial z_0} + \frac{\partial B_{y1}}{\partial z_1}\right) + \rho_0 v \frac{\partial^2 v_{y2}}{\partial z_0^2}, +2\rho_0 v \frac{\partial^2 v_{y1}}{\partial z_0 \partial z_1} + \rho_1 v \frac{\partial^2 v_{y1}}{\partial z_0^2}, \tag{17}$$

$$\rho_0\left(\frac{\partial v_{z2}}{\partial t_0} + \frac{\partial v_{z1}}{\partial t_0}\right) + \rho_1 \frac{\partial v_{z1}}{\partial t_0} + \rho_0 v_{z1}\frac{\partial v_{z1}}{\partial z_0} = -\frac{\partial p_2}{\partial z_0} - \frac{\partial p_1}{\partial z_1} - \frac{1}{4\pi} B_{y1}\frac{\partial B_{y1}}{\partial z_0},$$

$$+\frac{4}{3}\rho_0 v \frac{\partial^2 v_{z2}}{\partial z_0^2} + \frac{2}{3}\rho_0 v \frac{\partial^2 v_{z1}}{\partial z_0 \partial z_1} + \frac{4}{3}\rho_1 v \frac{\partial^2 v_{z1}}{\partial z_0^2}, \tag{18}$$

$$\frac{\partial B_{y2}}{\partial t_0} + \frac{\partial B_{y1}}{\partial t_1} + v_{z1}\frac{\partial B_{y1}}{\partial z_0} = -B_{y1}\frac{\partial v_{z1}}{\partial z_0} + B_0\left(\frac{\partial v_{y2}}{\partial z_0} + \frac{\partial v_{y1}}{\partial z_1}\right), \tag{19}$$

$$\frac{\partial \rho_2}{\partial t_0} + \frac{\partial \rho_1}{\partial t_1} + v_{z1}\frac{\partial \rho_1}{\partial z_0} = -\rho_0\left(\frac{\partial v_{z2}}{\partial z_0} + \frac{\partial v_{z1}}{\partial z_1}\right) - \rho_1 \frac{\partial v_{z1}}{\partial z_0}, \tag{20}$$

$$\frac{\partial p_2}{\partial t_0} + \frac{\partial p_1}{\partial t_1} + v_{z1}\frac{\partial p_1}{\partial z_0} = -\rho_0 c_s^2\left(\frac{\partial v_{z2}}{\partial z_0} + \frac{\partial v_{z1}}{\partial z_1}\right) - \rho_1 c_s^2 \frac{\partial v_{z1}}{\partial z_0}, \tag{21}$$

By making use of linear solutions (13)-(15) in equations (16)-(18), we get

$$\rho \frac{\partial v_{y2}}{\partial t_0} - \frac{B_0}{4\pi}\frac{\partial B_{y2}}{\partial z_0} - \rho_0 v \frac{\partial^2 v_{y2}}{\partial z_0^2} = \frac{B_0 \omega}{4\pi k v_A^2}\left(\frac{\partial A_y}{\partial t_1} e^{i\theta} + c.c.\right)$$

$$+\frac{B_0}{4\pi}\left(1 - \frac{2iv\omega}{v_A^2}\right)\left(\frac{\partial A_y}{\partial z_1} e^{i\theta} + c.c.\right), \tag{22}$$

$$\frac{\partial B_{y2}}{\partial t_0} - B_0 \frac{\partial v_{y2}}{\partial z_0} = -\left(\frac{\partial A_y}{\partial t_1} e^{i\theta} + c.c\right) - \frac{B_0^2 \omega}{4\pi\rho_0 k v_A^2}\left(\frac{\partial A_y}{\partial z_1} e^{i\theta} + c.c.\right), \tag{23}$$

From equations (22) and (23), we obtain

$$\frac{\partial^2 B_{y2}}{\partial t_0^2} - v_A^2 \frac{\partial^2 B_{y2}}{\partial z_0^2} - v\frac{\partial^3 B_{y2}}{\partial z_0^2 \partial t_0} = (2i\omega - k^2 v)\left(\frac{\partial A_y}{\partial t_1} e^{i\theta} + c.c.\right)$$
$$+\frac{i}{k}\left(\omega^2 + k^2 v_A^2 - ivk^2 v_A^2\right)\left(\frac{\partial A_y}{\partial t_1} e^{i\theta} + c.c.\right), \quad (24)$$

Using dispersion relation (16), equation (24) reduces to

$$\frac{\partial^2 B_{y2}}{\partial t_0^2} - v_A^2 \frac{\partial^2 B_{y2}}{\partial z_0^2} - v\frac{\partial^3 B_{y2}}{\partial z_0^2 \partial t_0} = \frac{i(\omega^2 + k^2 v_A^2)}{\omega}\left(\frac{\partial A_y}{\partial t_1} e^{i\theta} + c.c.\right) + \frac{2i\omega^2}{k}\left(\frac{\partial A_y}{\partial t_1} e^{i\theta} + c.c.\right), \quad (25)$$

Since B_{y2}/B_{y1} must be bounded for all z_0 and t_0 in (25), so the condition of nonsecularity gives,

$$\frac{\partial A_y}{\partial t_1} + \alpha \frac{\partial A_y}{\partial z_1} = 0, \quad (26)$$

where

$$\alpha = \frac{2\omega^3}{k(\omega^2 + k^2 v_A^2)}$$

The equation (26) implies

$$A_y = A_y(\xi, z_1, z_2, z_3, ..., t_2, t_3, ...),$$

where

$$\xi = z_1 - \alpha t_1.$$

From equations (16) and (25), we get the solution for B_{y2} and v_{y2} in the second order perturbation as

$$B_{y2} = A_y^{(2)}(z_1, z_2, ..., t_1, t_2 ...)e^{2i\theta} + c.c., \quad (27)$$

$$v_{y2} = -\frac{B_0 \omega}{4\pi k v_A^2}(A_y^{(2)} e^{2i\theta} + c.c.)), \quad (28)$$

where $A_y^{(2)}$ will be determined in the next order approximation. Now in order to get the solutions for remaining variables v_{z2}, ρ_2 and p_2 we eliminate ρ_2 and p_2 from the equations (18), (20) and (21) of the set of second order perturbations and obtain a differential equation for the variable v_{z2}

$$\frac{\partial^2 vz_{z2}}{\partial t_0^2} - c_s^2 \frac{\partial^2 v_{z2}}{\partial z_0^2} - \frac{4}{3} v \frac{\partial^3 v_{z2}}{\partial z_0^2 \partial t_0} = -\frac{k\omega}{2\pi\rho_0}(A_y^2 e^{2i\theta} + c.c.)). \quad (29)$$

On solving equation (29), we get

$$v_{z2} = \frac{\omega}{8\pi\rho_0 k(v_A^2 - c_s^2 - \frac{2}{3} iv\omega)}(A_y^2 e^{2i\theta} + c.c.) + \overline{v}_{s2}(z_1, z_2, ..., t_1, t_2, ...), \quad (30)$$

Using solution (30) in equations (20)-(21), we get the solutions for ρ_2 and p_2.

$$\rho_2 = \frac{1}{8\pi(v_A^2 - c_s^2 - \frac{2}{3} iv\omega)}(A_y^2 e^{2i\theta} + c.c.) + \overline{\rho}_2(z_1, z_2, ..., t_1, t_2, ...); \overline{\rho}_2 = \rho_0 \frac{\overline{v}_{z2}}{v_A}, \quad (31)$$

$$p_2 = \frac{c_s^2}{8\pi(v_A^2 - c_s^2 - \frac{2}{3} iv\omega)}(A_y^2 e^{2i\theta} + c.c.) + \overline{p}_2(z_1, z_2, ..., t_1, t_2, ...); \overline{p}_2 = c_s^2 \overline{\rho}_2, \quad (32)$$

where $\overline{v}_{z2}, \overline{\rho}_2, \overline{p}_2$ are slow functions of higher-order slow variables.

On observing the second order approximation equations (30) to (32) we see that the longitudinal components of Alfven mode appear due to the Lorentz force component and viscous term in equation (18). A coupling between the Alfven and the acoustic modes also occur in this order. The results obtained till here are in agreement with that of Borcia and Ignat (1998) except for the fact that there occurs resonant interactions between the two types of modes when sound velocity coincides with the Alfven velocity in equations (26)-(28). On the contrary our equations do not show resonance when Alfven velocity matches with the sound velocity. This is because of the extra viscosity term which we have added in the equation of momentum. The perturbation method which we have adopted is valid in our study when

$$\left|\frac{v_{z2}}{v_A}\right|,\ \left|\frac{\rho_2}{\rho_0}\right|,\left|\frac{p_2}{p_0}\right|<1,$$

i.e., if

$$\frac{B_{y'}}{B_0}<\sqrt{2\left(1-\beta-\frac{2i\nu\omega}{3v_A^2}\right)} \tag{33}$$

Here β is the ratio of sound velocity to that of Alfven velocity(i.e. c_s^2/v_A^2) and $B_y^{'}=\varepsilon A_y$ which represents the linear perturbation amplitude of the magnetic field induction after 0y axis.

The third order, $O(\varepsilon^3)$, problem is

$$\begin{aligned}
&\rho_0\left(\frac{\partial v_{y3}}{\partial t_0}+\frac{\partial v_{y2}}{\partial t_1}+\frac{\partial v_{y1}}{\partial t_2}\right)+\rho_1\left(\frac{\partial v_{y2}}{\partial t_0}+\frac{\partial v_{y1}}{\partial t_1}\right)\\
&+\rho_2\frac{\partial v_{y1}}{\partial t_0}+\rho_0 v_{z1}\left(\frac{\partial v_{y2}}{\partial z_0}+\frac{\partial v_{y1}}{\partial z_1}\right)+\rho_0 v_{z2}\frac{\partial v_{y1}}{\partial z_0}+\rho_1 v_{z1}\frac{\partial v_{y1}}{\partial z_0}\\
&=\frac{B_0}{4\pi}\left(\frac{\partial B_{y3}}{\partial z_0}+\frac{\partial B_{y2}}{\partial z_1}+\frac{\partial B_{y1}}{\partial z_2}\right)+\rho_0\nu\frac{\partial^2 v_{y3}}{\partial z_0^2}\\
&+2\rho_0\nu\frac{\partial^2 v_{y2}}{\partial z_0\partial z_1}+\rho_0\nu\frac{\partial^2 v_{y1}}{\partial^2 z_1}+\rho_1\nu\frac{\partial^2 v_{y2}}{\partial z_0^2}+2\rho_1\nu\frac{\partial^2 v_{y2}}{\partial z_0^2}+\frac{\partial^2 v_{y2}}{\partial z_0^2}+\rho_2\nu\frac{\partial^2 v_{y1}}{\partial z_0^2},
\end{aligned} \tag{34}$$

$$\begin{aligned}
&\rho_0\left(\frac{\partial v_{z3}}{\partial t_0}+\frac{\partial v_{z2}}{\partial t_1}+\frac{\partial v_{z1}}{\partial t_2}\right)+\rho_1\left(\frac{\partial v_{z2}}{\partial t_0}+\frac{\partial v_{z1}}{\partial t_1}\right)\\
&+\rho_2\frac{\partial v_{z1}}{\partial t_0}+\rho_0 v_{z1}\left(\frac{\partial v_{z2}}{\partial z_0}+\frac{\partial v_{y1}}{\partial z_1}\right)+\rho_0 v_{z2}\frac{\partial v_{z1}}{\partial z_0}+\rho_1 v_{z1}\frac{\partial v_{z1}}{\partial z_0}\\
&=-\frac{\partial p_3}{\partial z_0}-\frac{\partial p_2}{\partial z_1}-\frac{\partial p_1}{\partial z_2}-\frac{B_{y1}}{4\pi}\left(\frac{\partial B_{y2}}{\partial z_0}+\frac{\partial B_{y1}}{\partial z_1}\right)-\frac{B_{y2}}{4\pi}\frac{\partial B_{y1}}{\partial z_0}+\frac{4}{3}\rho_0\nu\frac{\partial^2 v_{z3}}{\partial z_0^2}\\
&+\frac{8}{3}\rho_0\nu\frac{\partial^2 v_{z2}}{\partial z_0\partial z_1}+\frac{4}{3}\rho_0\nu\frac{\partial^2 v_{z1}}{\partial z_1^2}+\frac{4}{3}\rho_1\nu\frac{\partial^2 v_{z2}}{\partial z_0^2}+\frac{8}{3}\rho_1\nu\frac{\partial^2 v_{z1}}{\partial z_0\partial z_1}
\end{aligned} \tag{35}$$

$$\begin{aligned}
&\frac{\partial B_{y3}}{\partial t_0}+\frac{\partial B_{y2}}{\partial t_1}+\frac{\partial B_{y1}}{\partial t_2}+v_{z1}\left(\frac{\partial B_{y2}}{\partial z_0}+\frac{\partial B_{y1}}{\partial z_1}\right)+v_{z2}\frac{\partial B_{y1}}{\partial z_0}\\
&=-B_{y1}\left(\frac{\partial v_{z2}}{\partial z_0}+\frac{\partial v_{z1}}{\partial z_1}\right)-B_{y2}\frac{\partial v_{z1}}{\partial z_0}+B_0\left(\frac{\partial vy_3}{\partial z_0}+\frac{\partial v_{y2}}{\partial z_1}+\frac{\partial v_{y1}}{\partial z_2}\right),
\end{aligned} \tag{36}$$

$$\begin{aligned}
&\frac{\partial \rho_3}{\partial t_0}+\frac{\partial \rho_2}{\partial t_1}+\frac{\partial \rho_1}{\partial t_2}+v_{z1}\left(\frac{\partial \rho_2}{\partial z_0}+\frac{\partial \rho_1}{\partial z_1}\right)\\
&-\rho_0\left(\frac{\partial v_{z3}}{\partial z_0}+\frac{\partial v_{z2}}{\partial z_1}+\frac{\partial v_{11}}{\partial z_2}\right)-\rho_1\left(\frac{\partial v_{z3}}{\partial z_0}+\frac{\partial v_{z1}}{\partial z_1}\right),
\end{aligned} \tag{37}$$

$$\frac{\partial p_3}{\partial t_0}+\frac{\partial p_2}{\partial t_1}-\frac{\partial p_1}{\partial t_2}+v_{z1}\left(\frac{\partial p_2}{\partial z_0}+\frac{\partial p_1}{\partial z_1}\right)$$

$$= -\rho_0 c_s^2 \left(\frac{\partial v_{z3}}{\partial z_0} + \frac{\partial v_{z2}}{\partial z_1} - \frac{\partial v_{z1}}{\partial z_2} \right) - \rho_1 c_{so}^2 \left(\frac{v_{z2}}{\partial z_0} + \frac{\partial v_{z1}}{\partial z_1} \right), \tag{38}$$

Now we obtain the equations for p_3 and B_{y3} with the help of first and second order solution as

$$\frac{\partial^2 p_3}{\partial t_0^2} - c_s^2 \frac{\partial^2 p_3}{\partial z_0^2} - \frac{4}{3}\nu \frac{\partial^2 p_3}{\partial z_0 \partial t_0} = \frac{i(\omega + \frac{4}{3}\nu k)c_s^2}{8\pi(v_A^2 - c_s^2 - \frac{2}{3} i\nu\omega)} \left(\frac{\partial A_y^2}{\partial t_1} + \frac{\omega}{k} \frac{\partial A_y^2}{\partial z_1} \right) e^{i\theta} + c.c$$

$$- \frac{2ikc_s^2}{8\pi(v_A^2 - c_s^2 - \frac{2}{3} i\nu\omega)} \left[\left(\frac{\omega}{k} \frac{\partial A_y^2}{\partial t_1} + c_s^2 \frac{\partial A_y^2}{\partial z_1} - i\frac{16}{3}\omega\nu \frac{\partial A_y^2}{\partial z_1} \right) e^{2i\theta} + c.c \right]$$

$$- \frac{k^2 c_s^2}{4\pi} [9 A_y A_y^{(2)} e^{3i\theta} + A_y^* A_y^{(2)} e^{i\theta} + c.c] + \frac{c_s^2}{4\pi} \left[2ikA_y \frac{\partial A_y}{\partial z_1} e^{2i\theta} + c.c \right] \tag{39}$$

$$\frac{\partial^2 B_{y3}}{\partial t_0^2} - v_A^2 \frac{\partial^2 B_{y3}}{\partial z_0^2} - \nu \frac{\partial^3 B_{y3}}{\partial z^2 \partial t_0} = (i\omega - k^2\nu) \left(\frac{\partial A_y}{\partial t_2} + \frac{\omega}{k} \frac{\partial A_y}{\partial z_2} \right) e^{i\theta} + c.c.$$

$$- \left(\frac{9\omega(\omega + 3ik^2\nu)}{8\pi\rho_0(v_A^2 - c_s^2 - \frac{2}{3} i\nu\omega)} A_y^3 e^{3i\theta} + \frac{\omega(\omega + ik^2\nu)}{8\pi\rho_0(v_A^2 - c_s^2 - \frac{2}{3} i\nu\omega)} |A_y|^2 A_y e^{i\theta} + c.c \right)$$

$$+ iv_A^2 \left[\left(\frac{\omega}{v_A^2} \frac{\partial A_y}{\partial t_2} + k \frac{\partial A_y}{\partial z_2} \right) e^{i\theta} + c.c \right] + \frac{i\omega\nu k^3}{8\pi\rho_0 k(v_A^2 - c_s^2 - \frac{2}{3} i\nu\omega)}$$

$$[3A_y^3 e^{3i\theta} + |A_y|^2 A_y e^{i\theta} + c.c] + \left[\left(\frac{\omega^2}{v_A} - k\omega + \frac{i\omega\nu k^2}{v_A} \right) A_y \bar{v}_{z2} e^{i\theta} + c.c \right]$$

$$- i\omega\nu \left(\frac{\partial^2 A_y}{\partial z_1^2} e^{i\theta} + c.c \right) + 2k\nu\omega \frac{\partial A_y}{\partial z_2} e^{i\theta} + c.c. \tag{40}$$

In equation (39) the condition of nonsecularity for the $e^{i\theta}$ term leads to

$$A_y^{(2)} = 0 . \tag{41}$$

This leads to the vanishing of the hydrodynamic components in the third approximation equations completely and annulling of the typical transversal components of the Alfven mode v_{y2}, B_{y2}.

$$i\left(\frac{\partial A_y}{\partial t_2} + \alpha \frac{\partial A_y}{\partial z_2} \right) - \frac{i\omega^2\nu}{\omega^2 + k^2 v_A^2} \frac{\partial^2 A_y}{\partial z_1^2} + \frac{\omega}{\omega^2 + k^2 v_A^2} \left(\frac{\omega^2}{v_A} - \omega k + \frac{i\omega\nu k^2}{v_A} \right) A_y \bar{v}_{z2}$$

$$- \frac{\omega^3}{8\pi\rho_0(\omega^2 + k^2 v_A^2)(v_A^2 - c_s^2 - \frac{2}{3} i\nu\omega)} |A_y|^2 A_y = 0 \tag{42}$$

4. Discussion and conclusions

In the absence of viscosity, equation (38) of Borcia and Ignat (1998) coincides with our equation (42). By using relation (16), the equation (42) takes the form

$$i\left(\frac{\partial A_y}{\partial t_2} + \alpha \frac{\partial A_y}{\partial t_2} \right) + D \frac{\partial^2 A_y}{\partial z_1^2} + qA_y \bar{v}_{z2} - N|A_y|^2 A_y = 0 \tag{43}$$

where the dispersive coefficient D is given by

$$D = -\frac{i\omega^2\nu}{\omega^2 + k^2 v_A^2}$$

and the nonlinear coefficient N is given by

$$N = \frac{\omega^3}{8\pi\rho_0(\omega^2 + k^2 v_A^2)(v_A^2 - c_s^2 - \frac{2}{3}i\nu\omega)},$$

and

$$q = \frac{\omega}{\omega^2 + k^2 v_A^2}\left(\frac{\omega^2}{v_A^2} + k\omega + \frac{i\omega\nu k^2}{v_A}\right),$$

which has mathematical similarity with the NLS obtained by Nakariakov *et al*. (1997).
Introducing the transformations

$$\xi = z_2 - \alpha t_2,$$

$$\eta = z_1,$$

$$\tau = t_2,$$

We get the equation (43) in the following form

$$i\frac{\partial A_y}{\partial \tau} + D\frac{\partial^2 A_y}{\partial \eta^2} = N\,|A_y|^2\,A_y - q v_{z2} A_y. \tag{44}$$

This equation looks like the nonlinear Schrodinger equation (NLS) and has similarity with (41) the nonlinear Schrodinger equation obtained by Scheurwater (1990). The equation describes the evolution of solution and the solution has been discussed by several researchers (Kakutani and Sujimoto 1974; Mjolhus 1976; Mio *et al*. 1976; and Rauf and Tataronis 1995). An inverse scattering method has been developed to solve the NLS by Kaup and Newell (1978) and Kawata *et al*. (1980). Using a pseudopotential formulation. Hada *et al*. (1989) discussed and classified the stationary solutions of DNLS equation. They showed that the solutions consist of rich family of non-linear Alfven waves and solitons with parallel and oblique propagation directions. Many of the properties were reviewed by Mjolhus and Wyller (1986, 1988). A detailed study of he nature of the solutions can be found in the aove mentioned papers.
In the presence of viscosity, dispersive term appears in the non-linear Schrodinger Wave equation which do not appear when the viscous term is absent. Thus in the absence of viscosity order dispersive effects are required to contain the non-linear growth of the wave. It implies that the viscosity provides the dispersion term to the non-linear growth of the Alfven wave. Considering a plane wave solution.

$$A_y = |A_y|\exp(i[l\eta - \Omega\tau]),$$

and substituting into the non-linear Schrodinger Wave equation (44), we obtain the following nonlinear dispersion relation

$$\Omega = Dl^2 + N\,|A_y|^2 - q\bar{v}_{z2}$$

Following Kumar and Srivastava (1990), the amplitude dependent relative frequency shift is given by

$$\frac{\Delta\omega}{\omega} = \frac{N\,|A_y|^2 - q\bar{v}_{z2}}{\omega},$$

And amplitude dependent wave number shift induced by nonlinearities is given by

$$\frac{\Delta k}{k} = -\frac{N\,|A_y|^2 - q\bar{v}_{z2}}{k}$$

It is useful to calculate the frequency and wave number shifts induced by nonlinearities from an experimental point of view.
Our results would be useful in solar atmosphere and interstellar space, for example, Alfven waves propagating in solar corona and stellar winds derive by Alfven waves.

5. References

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