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Regular mildly generalized closed and regular mildly generalized open sets in bitopological spaces

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Abstract
The aim of this paper is to introduced the concept of (τ_i, τ_j)-Regular Mildly Generalized (briefly (τ_i, τ_j)-RMG) closed and (τ_i, τ_j)-Regular Mildly Generalized (briefly (τ_i, τ_j)-RMG) open sets and study their basic properties in bitopological spaces. Further we define and study new neighborhood namely (τ_i, τ_j)-Regular Mildly Generalized (briefly (τ_i, τ_j)-RMG-nhd), (τ_i, τ_j)-Regular Mildly Generalized (briefly (τ_i, τ_j)-RMG-closure) closure and (τ_i, τ_j)-Regular Mildly Generalized (briefly (τ_i, τ_j)-RMG-interior) interior and discuss some of their properties in bitopological spaces. Also give some characterizations and applications of it.

Mathematics Subject Classification 2010: 54C08, 54C010, 54C20

Keywords: RMG-closed set, RMG-open, (τ_i, τ_j)-RMG-closed, (τ_i, τ_j)-RMG-open, (τ_i, τ_j)-gr-open, (τ_i, τ_j)-RMG-nhd, (τ_i, τ_j)-RMG-closure, (τ_i, τ_j)-RMG-interior

1. Introduction

A triple(X, τ_1, τ_2) where X is a non-empty set and τ_1, τ_2 are topologies on X is called bitopological space. J. C. Kelly [5] initiated the systematic study of such spaces in 1963. In 1986, T. Fukutake [3] introduced the concept of generalized closed sets in bitopological spaces and after that several authors turned their attention towards generalizations of various concepts of topology by considering bitopological spaces. M. A. Jalic [4], N. Nagaveni [8], I. Arockiarani [1] and M. Sheik John [11] introduced (i, j)-pre-open, (i, j)-wg-closed, (i, j)-rg-closed and (i, j)-g^*-closed sets in bitopological spaces. Also R. S. Wali and Nirani Laxmi[12, 13] introduce and studied the properties of RMG-closed and RMG-open sets in topological space. The purpose of this paper to introduced and investigate the concept of (τ_i, τ_j)-RMG-closed and (τ_i, τ_j)-RMG-open sets which are introduced in a bitopological space by analogy with RMG-closed and RMG-open sets in bitopological space.

2. Preliminaries

Throughout this paper the space X always means(X, τ_1, τ_2) bitopological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a space X. by τ_i-int(A), τ_i-cl(A) we shall denote the interior and closure of $A \subset X$ with respect to the topology τ_i for i=1, 2. By (i, j) means pair of topologies (τ_i, τ_j) and $X-A$ or A^c denotes the complement of A in X.

Now we recall the following known definitions and results that are used in our work;

Definition 2.1: A subset A of a topological space X is called
(i) Pre-open [7], if $A \subseteq$int(cl(A)) and pre-closed if cl(int(A))$\subseteq A$.

Definition 2.2: A subset A of a topological space X is called
(i) Generalized closed (briefly g-closed) [6] if cl(A) $\subseteq U$ whenever $A \subseteq U$ and U is open in X.
(ii) Weakly generalized closed (briefly wg-closed) [8] if cl(int(A)) $\subseteq U$ whenever $A \subseteq U$ and U is open in X.

(iii) Strongly generalized closed (briefly g*-closed) [11] if cl(A) ⊆U whenever A⊆U and U is g-open in X.
(iv) Mildly generalized closed (briefly mildly g-closed) [10] if cl(int(A)) ⊆U whenever A⊆U and U is g-open in X.
(v) Regular weakly closed (briefly rw-closed) [2] if cl(A)⊆U whenever A⊆U and U is regular semi open in X.
(vi) Regular generalized closed (briefly rg-closed) [9] if cl(A)⊆U whenever A⊆U and U is regular open set in X.
(vii) Regular Mildly Generalized closed(briefly RMG-closed) [12] if cl(int(A)) ⊆U whenever A⊆U and U is rg-open in X.
The complements of above all closed sets are their respective open sets in the same topological space X.

Definition 2.3: Let i, j∈{1, 2} be fixed integers. A subset A of a bitopological space (X, τ_i, τ_j) is said to
(i) (τ_i, τ_j)-pre-open [4] if A⊆ τ_i-int[τ_j-cl(A)]
The compliment of (τ_i, τ_j)-pre-open set is called (τ_i, τ_j)-pre-closed set.

Definition 2.4: Let i, j∈{1, 2} be fixed integers. A subset A of a bitopological space (X, τ_i, τ_j) is called
(i) (i, j)-g-closed [3] if τ_j-cl (A) ⊆U whenever A⊆U and U is open in τ_i.
(ii) (i, j)-wg-closed [8] if τ_j-cl(τ_i-int(A)) ⊆U whenever A⊆U and U is open in τ_i.
(iii) (i, j)-g*-closed [11] if τ_j-cl(A) ⊆U whenever A⊆U and U∈GO(X, τ_i).
(iv) (i, j)-rw-closed [2] if τ_j-cl(A)⊆U whenever A⊆U and U∈RSO(X, τ_i).
(v) (i, j)-rg-closed [1] if τ_j-cl(A)⊆U whenever A⊆U and U is regular open set in τ_i.
The complements of above all closed sets are their respective open sets in the same bitopological space X.

3.(τ_i, τ_j)-Rmg-Closed Sets and Some of Their Properties.

Definition 3.1: Let (i, j) ∈{1, 2} be fixed integers. In a bitopological space(X, τ_1, τ_2), a subset A⊆X is said to be (i, j)-Regular Mildly Generalized closed (briefly, (i, j)-RMG-closed) set if τ_j-cl(τ_i-int(A))⊆G, whenever A ⊆ G and G∈RGO(X, τ_i).
We denote the family of all (i, j)-RMG-closed sets in a bitopological space (X, τ_1, τ_2)by $D_{RMG}(\tau_i, \tau_j)$ or D_{RMG}(i, j).

Remark 3.2: By setting $\tau_1 = \tau_2$ in Definition 3.1, an (i, j)-RMG closed set reduces to an RMG-closed set in X.
First we prove that the class of (i, j)-RMG-closed sets properly lies between the class of (i, j)-pre-closed sets and the class of (i, j) mildly-g-closed sets.

Theorem 3.3: If A is a (i, j)-pre-closed subset of (X, τ_1, τ_2), then A is (i, j)-RMG-closed, but converse is not true.

Proof: Let A be a (i, j)-pre-closed subset of (X, τ_1, τ_2). Let U is rg-open in (X, τ_i) such that A⊆G. Since A be a(i, j)- pre-closed subset of (X, τ_1, τ_2)that is τ_j-cl(τ_i-int(A))⊆A, we have
τ_j-cl(τ_i-int(A))⊆A⊆G i e τ_j-cl(τ_i-int(A))⊆G. Therefore A is (i, j)-RMG-closed.

Example 3.4: Let X={a, b, c}, τ_1 ={X, ∅, {b}, {c}, {b, c}} and τ_2 ={X, ∅, {a}, {c}, {a, c}, {b, c}}. Then the subset {a, c} is (1, 2)-RMG closed set, but not (1, 2)-pre-closed set in the bitopological space (X, τ_1, τ_2).

Theorem 3.5: If A be a (i, j)-RMG-closed subset of (X, τ_1, τ_2), then A is (i, j)-mildly-g-closed, but converse is not true.

Proof: Let A be a (i, j)-RMG-closed subset of (X, τ_1, τ_2). Let G∈GO(X, τ_i) be such that A⊆G. Since GO(X,τ_i)⊆RGO(X, τ_i), we have G∈RGO(X, τ_i). Then by hypothesis, τ_j-cl(τ_i-int(A)) ⊆G. Therefore A is (i, j)-mildly-g-closed.

Example 3.6: Let X={a, b, c, d}, τ_1 ={X, ∅, {a}, {b}, {a, b}, {a, b, c}} and τ_2 ={X, ∅, {a}, {b, c}, {a, b, c}}. Then the subsets {b, d} and {a, b, d} are (1, 2)-mildly-g-closed sets, but not (1, 2) RMG-closed sets in the bitopological space(X, τ_1, τ_2).

Theorem 3.7: If A is τ_j-closed subset of bitopological space (X, τ_1, τ_2), then the set A is (i, j)-RMG-closed, but converse is not true.

Proof: Let G∈RGO(X, τ_i) be such that A⊆G. Then by hypothesis, τ_j-cl(τ_i-int(A))⊆ τ_j-cl(A)=A, which implies τ_j-cl(τ_i-int(A))⊆G. Therefore A is RMG-closed.

Example 3.8: Let X={a, b, c}, τ_1 ={X, ∅, {a}} and τ_2 ={X, ∅, {a}, {b}, {a, b}}. Then the subset {b} is (1, 2)-RMG-closed set, but not τ_2-closed set in the bitopological space (X, τ_1, τ_2).

Remark 3.9: τ_j-pre-closed sets and (i, j)-RMG-closed sets are independent as seen from the following example.

Example 3.10: Let X={a, b, c}, τ_1 ={X, ∅, {b}, {c}, {b, c}} and τ_2 ={X, ∅, {a}, {c}, {a, c}, {b, c}}. Then the subset {a, c) is (1, 2)-RMG-closed set, but not τ_2-pre-closed sets in the bitopological space (X, τ_1, τ_2).

Example 3.11: Let X={a, b, c}, τ_1 ={X, ∅, {a}, {b}, {a, b}} and τ_2 ={X, ∅, {a}, {a, b}}. Then the subset {b} is τ_2-pre-closed set, but not (1, 2)-RMG-closed sets in the bitopological space (X, τ_1, τ_2).

Remark 3.12: τ_j-mildly-g-closed sets and (i, j)-RMG-closed sets are independent as seen from the following example.

Example 3.13: Let X={a, b, c, d}, τ_1 ={X, ∅, {a}, {b}, {a, b}, {a, b, c}} and τ_2 ={X, ∅, {a}, {b}, {a, b}, {b, c}, {a, b, c}}. Then the subsets {b, d) and {a, b, d} are τ_2-mildly-g-closed set, but not (1, 2)-RMG-closed sets in the bitopological space(X, τ_1, τ_2).

Example 3.14: Let X={a, b, c, d}, τ_1 ={X, ∅, {a}, {b, c}, {a, b, c}} and τ_2 ={X, ∅, {a}, {b}, {a, b}, {b, c}, {a, b, c}}. Then the subset {b) is (1, 2)-RMG-closed set, but not τ_2-mildly-g-closed set in the bitopological space (X, τ_1, τ_2).

Remark 3.15: (i, j)-g-closed sets and (i, j)-RMG-closed sets are independent as seen from the following example.

Example 3.16: Let X={a, b, c, d}, τ_1 ={X, ∅, {a}, {b}, {a, b}, {a, b, c}} and τ_2 ={X, ∅, {a}, {b, c}, {a, b, c}}. Then the subset {c} is (1, 2)-RMG-closed set, but not (1, 2)-g-closed set. Also the subsets {b, d} and (a, b, d} are (1, 2)-g-closed sets, but not (1, 2)-RMG-closed set in the bitopological space(X, τ_1, τ_2).

Remark 3.17: (i, j)-g^*-closed sets and (i, j)-RMG-closed sets are independent as seen from the following example.

Example 3.18: Let X={a, b, c, d}, τ_1 ={X, ∅, {a}, {b}, {a, b}, {a, b, c}} and τ_2 ={X, ∅, {a}, {b, c}, {a, b, c}}. Then the subset {c} is (1, 2)-RMG-closed set, but not (1, 2)-g^*-closed set. Also the subsets {b, d} and (a, b, d} are (1, 2)-g^*-closed sets, but not (1, 2)-RMG-closed set in the bitopological space(X, τ_1, τ_2).

Theorem 3.19: If A be a (i, j)-RMG-closed subset of (X, τ_1, τ_2), then A is (i, j)-wg-closed, but converse is not true.

Proof: Let A be a (i, j)-RMG-closed subset of (X, τ_1, τ_2). Let G∈O(X, τ_i) be such that A⊆G. Since O(X, τ_i)⊆RGO(X, τ_i), we have G∈RGO(X, τ_i). Then by hypothesis, τ_j-cl(τ_i-int(A)⊆G. Therefore A is (i, j)-mildly-closed.

Example 3.20: Let X={a, b, c, d}, τ_1 ={X, ∅, {a}, {b}, {a, b}, {a, b, c}} and τ_2 ={X, ∅, {a}, {b, c}, {a, b, c}}. Then the subsets {b, d} and {a, b, d} are (1, 2)-wg-closed set, but not (1, 2) RMG-closed set in the bitopological space (X, τ_1, τ_2).

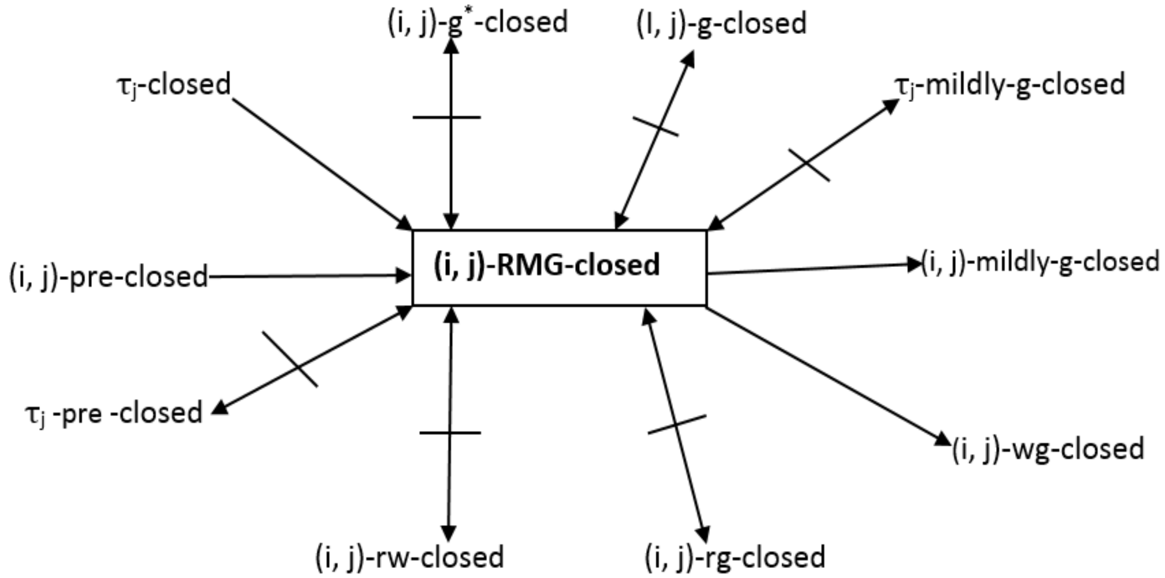
Remark 3.21: (i, j)-RMG-closed sets and (i, j)-rw-closed sets are independent as seen from the following example.

Example 3.22: Let X={a, b, c, d}, τ_1 ={X, ∅, {a}, {b}, {a, b}, {b, c}, {a, b, c}} and τ_2 ={X, ∅, {a}, {b, c}, {a, b, c}}. Then the subsets {a, b}, {a, c}, {b, d}, {a, b, c} and {a, b, d} are (1, 2)-rw-closed sets, but not (1, 2)-RMG-closed sets. Also the subset {c} is (1, 2)-RMG-closed set, but not (1, 2)-rw-closed set in the bitopological space (X, τ_1, τ_2).

Remark 3.23: (i, j)-RMG-closed sets and (i, j)-rg-closed sets are independent as seen from the following example.

Example 3.24: Let X={a, b, c, d}, τ_1 ={X, ∅, {a}, {b}, {a, b}, {b, c}, {a, b, c}} and τ_2 ={X, ∅, {a}, {b, c}, {a, b, c}}. Then the subsets {a, b}, {a, c}, {b, d}, {a, b, c} and {a, b, d} are (1, 2)-rg-closed sets, but not (1, 2)-RMG-closed sets. Also the subset {c} is (1, 2)-RMG-closed set, but not (1, 2)-rg-closed set in the bitopological space (X, τ_1, τ_2).

Remark 3.25: From the above discussion and know results we have the following implications.



Remark 3.26: The intersection of two (i, j)-RMG-closed sets is generally not a (i, j)-RMG-closed set as seen from the following example.

Example 3.27: Let X={a, b, c}, τ_1 ={X, ∅, {b}, {c}, {b, c}} and τ_2 ={X, ∅, {a}, {c}, {a, c}}. Then the subsets {a, c} and {b, c} are (1, 2)-RMG-closed sets, but {a, c}∩{b, c}={c} is not a (1, 2) RMG-closed set in bitopological space (X, τ_1, τ_2).

Remark 3.28: The union of two (i, j)-RMG-closed sets is generally not a (i, j)-RMG-closed set as seen from the following example.

Example 3.29: Let X={a, b, c, d}, τ_1 ={X, ∅, {a}, {b, c}, {a, b, c}} and τ_2 ={X, ∅, {a}, {b}, {a, b}, {b, c}, {a, b, c}}. Then the subsets {b} and {c} are (1, 2)-RMG-closed sets, but {b}∪{c}={b, c} is not a (1, 2)-RMG-closed set in bitopological space (X, τ_1, τ_2).

Remarks 3.30: The family D_{RMG}(X, τ_1, τ_2) is generally not equal to the family D_{RMG}(X, τ_2, τ_1) as seen from the following example.

Example 3.31: Let X={a, b, c d},τ_1={X, ∅,{a},{b},{a, b}, {a, b, c}}, τ_2={X, ∅,{a},{b, c}, {a, b, c}}. Then D_{RMG}(X, τ_1, τ_2)= {X, ∅, {c}, {d}, {a, d}, {c, d}, {a, c, d}, {b, c, d}} and D_{RMG}(X,τ_2,τ_1)= {X, ∅, {b}, {c}, {d}, {a, d}, {b, d},{c, d}, {a, b, d} {a, c, d}, {b, c, d}}. Therefore D_{RMG}(X, τ_1, τ_2)≠ D_{RMG}(X, τ_2, τ_1).

Theorem 3.32: If $\tau_1 \subseteq \tau_2$ and RGO(X, τ_1) ⊆ RGO(X, τ_2) in(X, τ_1, τ_2), then D_{RMG}(X, τ_1, τ_2) ⊒D_{RMG}(X, τ_2, τ_1).

Proof: Let A be a (τ_2, τ_1)-RMG-closed set and G be an τ_1-rg-open set containing A. By the assumption $\tau_1 \subseteq \tau_2$ it follows that G is τ_2-rg-open set containing A and τ_2-cl(τ_1-int(A)) ⊆ τ_2-cl(τ_1-int(A)). Then τ_2-cl(τ_1-int(A)) ⊆G and A is (τ_1, τ_2)-RMG-closed.

Theorem 3.33: Let (i, j)∈{1, 2}. For each point x of (X, τ_1, τ_2), a singleton {x} is τ_i-rg-closed or $\{x\}^c$is (τ_i, τ_j)-RMG-closed.

Proof: Suppose {x} is not τ_i-rg-closed. Then $\{x\}^c$ is not τ_i-rg-open. Therefore a τ_i-rg-open set containing $\{x\}^c$ is X only. Also τ_j-cl(τ_i-int($\{x\}^c$)) ⊆X. Hence $\{x\}^c$ is {i, j}-RMG-closed.

Theorem 3.34: If a subset A of X is ((i, j)-RMG-closed in (X, τ_i, τ_j)if and only if τ_j-cl(τ_i-int(A)-A does not contain any non-empty τ_i-rg-closed set in (X, τ_i, τ_j).

Proof: Suppose that A is a (i, j)-RMG-closed set in (X, τ_i, τ_j). We prove the the result by contradiction. Let U be τ_i-rg-closed set such that U⊂ τ_j-cl(τ_i-int(A))-A and U≠ ∅. Then U⊂ τ_j-cl(τ_i-int(A))∩ A^c. Therefore U⊂ τ_j-cl(τ_i-int(A)) and U⊂ A^c is τ_i-rg-closed set and A is (i, j)-RMG-closed, τ_j-cl(τ_i-int(A))⊆ U^c. That is U⊆ $[\tau_j - \mathrm{cl}(\tau_i - \mathrm{int}(A))]^c$.Hence U⊆ τ_j-cl(τ_i-int(A))∩

$[\tau_j - cl(\tau_i - int(A))]^c = \emptyset$ That is U= $\emptyset$. This is a contradiction. Hence τ_j-cl(τ_i-int(A))-A does not contain any non-empty τ_i-rg-closed set in (X, τ_i, τ_j).

Conversely, assume that τ_j-cl(τ_i-int(A))-A contains no non empty τ_i-rg-closed set. Let A⊆U, U is a τ_i-rg-closed set. Suppose that τ_j-cl(τ_i-int(A)) is not contained in U. Then τ_j-cl(τ_i-int(A))∩ U^c is a non empty τ_i-rg-closed set and contained in τ_j-cl(τ_i-int(A))-A, which is contradiction. Therefore τ_j-cl(τ_i-int(A))⊆U. Hence A is (i, j)-RMG-closed in (X, τ_i, τ_j).

Theorem 3.35: In a bitopological space (X, τ_1, τ_2), RGO(X, τ_i) ⊂{F ⊂X:$F^c \in \tau_j$} if and only if every subset of (X, τ_i, τ_j) is a (i, j)-RMG-closed set.

Proof: Suppose that RGO(X, τ_i) ⊂{F ⊂X:$F^c \in \tau_j$}. Let A be any subset of X. G∈ RGO(X, τ_i) be such that A⊂G. Then τ_j-cl(τ_i-int(G))⊂ τ_j-cl(A)=G. Also τ_j-cl(τ_i-int(A)) ⊂ τ_j-cl(τ_i-int(G))=G. That is τ_j-cl(τ_i-int(A)) ⊂G. Therefore A is (i, j)-RMG-closed set.

Conversely, suppose that every subset of (X, τ_1, τ_2) is (i, j)-RMG-closed set. Let G∈ RGO(X, τ_i). Since G⊂G and G is(i, j)-RMG-closed, we have τ_j-cl($\tau_i$int(G))⊂G.Thus τ_j-cl($\tau_i$int(G))⊂ τ_j-cl(A) =G and so G is τ_j-closed. That is G∈ (F ⊂X:$F^c \in \tau_j$}. Hence RGO(X, τ_i) ⊂{F ⊂X:$F^c \in \tau_j$}.

Theorem 3.36: If A is a (i, j)-RMG-closed set and A⊂B⊂ τ_j-cl(τ_i-int(A)), Then B is (i, j)-RMG-closed.

Proof: Let U be a τ_j-rg-open set such that B⊂U. As A is (i, j)-RMG-closed set and A⊂G, we have τ_j-cl(τ_i-int(A))⊂U. Now B⊂ τ_j-cl(τ_i-int(A)) which implies, τ_j-cl(τ_i-int(B))⊂ τ_j-cl(τ_i-int{τ_j-cl(τ_i-int(A))})= τ_j-cl(τ_i-int(A))⊂U. Thus τ_j-cl(τ_i-int(B))⊂U. Therefore B is a (i, j)-RMG closed set.

Theorem 3.37: Let A⊂Y⊂X and suppose that A is (i, j)-RMG-closed in (X, τ_1, τ_2). Then A is (i, j)-RMG-closed relative to Y provided Y is τ_i- open set.

Proof: τ_{i-Y} be the restriction of τ_i to Y. Let G be a τ_{i-Y}-open set such that A⊂G. Since A⊂Y⊂X and Y is τ_i-open. By the Lemma 3.26 [12], G is τ_i-rg-open. Since A is (i, j)-RMG-closed,
τ_j-cl(τ_i-int(A))⊂G. That is Y∩ τ_j-cl(τ_i-int(A))⊂Y∩G=G. Also Y∩ τ_j-cl(τ_i-int(A))= τ_{j-Y}-cl(τ_{i-Y}-int(A)). Thus τ_{j-Y}-cl(τ_{i-Y}-int(A))⊂G. Hence A is (i, j)-RMG-closed in Y.

Theorem 3.38: In a bitopological space (X, τ_1, τ_2), if RGO(X, τ_i)={X, $\emptyset$}, Then every subset of (X, τ_1, τ_2) is (i, j)-RMG-closed.

Proof: Let RGO(X,τ_i)={X, $\emptyset$} in a bitopological space (X, τ_1, τ_2). Let A be any subset of X. To prove that A is an (i, j)-RMG-closed. Suppose A≡ $\emptyset$, Then A is (i, j)-RMG-closed. Suppose A≠ $\emptyset$, then X is only τ_i-rg-open set and τ_j-cl(τ_i-int(A))⊂X. Hence A is a (i, j)-RMG-closed set.

4.(τ_i, τ_j)-Rmg-Open Sets And Some Of Their Properties.

In this section, we introduce (i, j)-RMG-open sets in bitopological spaces and study some of their properties.

Definition 4.1: Let i, j∈{1, 2} be fixed integers. In a bitopological space (X, τ_1, τ_2), a subset A⊂X is said to be (τ_i, τ_j)-Regular Mildly Generalized open (briefly, (i, j)-RMG-open) if A^c is (i, j)-RMG-closed. We denote the family of all (i, j)-RMG-open sets in a bitopological space (X, τ_1, τ_2) by $D^*_{RMG}(\tau_i, \tau_j)$ or D^*_{RMG}(i, j)

Theorem 4.2: In bitopological space (X, τ_1, τ_2), we have the following.

(i) Every (i, j)-pre-open set is (i, j)-RMG-open set but not conversely.

(ii) Every (i, j)-RMG-open set is (i, j)-mildly-g-open set but not conversely.

(iii)Every τ_j-open set is (i, j)-RMG-open set but not conversely.

(iv) Every (i, j)-RMG-open set is (i, j)-wg-open set but not conversely.

Proof: The proof follows from the Theorems 3.3, 3.5, 3.7 and 3.19.

Example 4.3: Let X={a, b, c, d}, τ_1 ={X, $\emptyset$, {a}, {b, c}, {a, b, c}} and τ_2 ={X, $\emptyset$, {a}, {b}, {a, b}, {a, b, c}}. Then the subsets {c} and {b, c} are (1, 2)-RMG-open sets, but not (1, 2)-pre-open sets in the bitopological space (X, τ_1, τ_2).

Example 4.4: Let X={a, b, c, d}, τ_1 ={X, ∅, {a}, {b}, {a, b}, {a, b, c}} and τ_2 ={X, ∅, {a}, {b, c}, {a, b, c}}. Then the subsets {c} and {a, c} are (1, 2)-mildly-g-open sets, but not (1, 2)-RMG-open sets in the bitopological space (X, τ_1, τ_2).

Example 4.5: Let X={a, b, c}, τ_1 ={X, ∅, {a}} and τ_2 ={X, ∅, {a}, {b}, {a, b}}. Then the subsets {b}, {a, b} and {a, c} are (1, 2)-RMG-open sets, but not τ_2-open sets in the bitopological space (X, τ_1, τ_2).

Example 4.6: Let X={a, b, c, d}, τ_1 ={X, ∅, {a}, {b}, {a, b}, {a, b, c}} and τ_2 ={X, ∅, {a, b}, {c, d}}. Then the subsets {a, c}, {a, d}, {b, c} and {b, d} are (1, 2)-wg-open sets, but not (1, 2)-RMG-open sets in the bitopological space (X, τ_1, τ_2).

Remark 4.7: The intersection of two (i, j)-RMG-open sets is generally not an (i, j)-RMG-open sets as seen from the following example.

Example 4.8: Let X={a, b, c, d}, τ_1 ={X, ∅, {a}, {b, c}, {a, b, c}} and τ_2 ={X, ∅, {a}, {b}, {a, b}, {a, b, c}}. Then the subsets {a, b, d} and {a, c, d} are (1, 2)-RMG-open sets, but {a, b, d}∩{a, c, d}={a, d} is not (1, 2)-RMG-open set in the bitopological space (X, τ_1, τ_2).

Remark 4.9: The union of two (i, j)-RMG-open sets is generally not an (i, j)-RMG-open sets as seen from the following example.

Example 4.10: Let X={a, b, c, d}, τ_1 ={X, ∅, {a}, {b}, {a, b}, {a, b, c}} and τ_2 ={X, ∅, {a, b}, {c, d}}. Then the subsets {a} and {c} are (1, 2)-RMG-open sets, but {a}∩{c}={a, c} is not (1, 2)-RMG-open set in the bitopological space (X, τ_1, τ_2).

Remarks 4.11: The family $D^*_{RMG}(X, \tau_1, \tau_2)$ is generally not equal to the family $D^*_{RMG}(X, \tau_2, \tau_1)$ as seen from the following example.

Example 4.12: Let X={a, b, c},τ_1={X, ∅,{b},{c},{b, c}}, τ_2={X, ∅,{a},{c}, {a, c}}. Then $D^*_{RMG}(X, \tau_1, \tau_2)$ ={X, ∅, {b}, {c}, {a, c}, {b, c}} and $D^*_{RMG}(X, \tau_2, \tau_1)$ ={X, ∅, {a}, {b}, {c}, {a, c}, {b, c}}.Therefore $D^*_{RMG}(X, \tau_1, \tau_2) \neq D^*_{RMG}(X, \tau_2, \tau_1)$.

Theorem 4.13: A subset A of (X, τ_1, τ_2) is (i, j)-RMG-open if and only if F⊆ τ_j-int(τ_i-cl(A)), where F is τ_i-rg-closed set and F⊆A.

Proof: Suppose that F is τ_i-rg-closed set, F⊆A and F⊆ τ_j-int(τ_i-cl(A)). Let G be a τ_i-rg-open set and A^c ⊆G. Then G^c ⊆A and G^c is τ_i-rg-closed. Thus G^c ⊆ τ_j-int(τ_i-cl(A)) and $[\tau_j - \mathrm{int}(\tau_i - \mathrm{cl}(A))]^c$ ⊆G. It follows that $\tau_j - \mathrm{int}(\tau_i - \mathrm{cl}(A^c))$ ⊆G and hence A^c is (i, j)-RMG-closed. Hence A is {i, j)-RMG-open.
Conversely, Suppose that A is (i, j)-RMG-open, F⊆A and F is τ_i-rg-closed set. Then F^c is τ_i-rg-open and $A^c \subseteq F^c$. Therefore $\tau_j - \mathrm{cl}(\tau_i - \mathrm{int}(A^c)) \subseteq F^c$ and hence $[\tau_j - \mathrm{int}(\tau_i - \mathrm{cl}(A))]^c \subseteq F^c$. Thus F⊆ τ_j-int(τ_i-cl(A)), since $\tau_j - \mathrm{cl}(\tau_i - \mathrm{int}(A^c)) = [\tau_j - \mathrm{int}(\tau_i - \mathrm{cl}(A))]^c$.

Theorem 4.14: Let A and G be two subsets of a bitopological space (X, τ_1, τ_2). If the set A is (i, j)-RMG-open, then G=X whenever G is τ_i-rg-open and τ_j-int(τ_i-cl(A))∪ A^c ⊆G.

Proof: Let A be (i, j)-RMG-open. G be the τ_i-rg-open and τ_j-int(τ_i-cl(A))∪ A^c ⊆G.Then $G^c \subseteq [\tau_j - \mathrm{int}(\tau_i - \mathrm{cl}(A)) \cup A^c]^c = [\tau_j - \mathrm{int}(\tau_i - \mathrm{cl}(A))]^c$- A^c. Since A^c is (i, j)-RMG-closed and G^c is τ_i-rg-open, by the Theorem 3.34, it follows that G^c = ∅.Therefore G=X.

The converse of the above theorem need not be true as seen from the following example.

Example 4.15: Let X={a, b, c, d}, τ_1 ={X, ∅, {a}, {b}, {a, b}, {a, b, c}} and τ_2 ={X, ∅, {a}, {b, c}, {a, b, c}}. If A={a, c} then only τ_1-rg-open set containing τ_2-int(τ_1-cl(A))∪ A^c is X. But A is not (1, 2)-RMG-open set in (X, τ_1, τ_2).

Theorem 4.16: If a subset A of (X, τ_1, τ_2) is (i, j)-RMG-closed, then τ_j-cl(τ_i-int(A))-A is (i, j)-RMG-open.

Proof: Let A be a (i, j)-RMG-closed subset in (X, τ_1, τ_2). Let F be a τ_i-rg-open set such that F⊆ τ_j-cl(τ_i-int(A))-A. By Ttheorem 3.34, F= ∅. Therefore F⊆ τ_j-int (τ_i-cl(τ_j-cl(τ_i-int(A))-A)) and by Theorem 4.13, τ_j-int(τ_i-cl(A)) is (i, j)-RMG-open.
The converse of the above theorem need not be true as seen from the following example.

Example 4.17: For a subset A={b} in X. X={a, b, c, d}, τ_1 ={X, ∅, {a}, {b}, {a, b}, {a, b, c}} and τ_2 ={X, ∅, {a, b}, {c, d}}. Then τ_j-cl(τ_i-int(A))-A={a, b}-{b}={a} is (1, 2)-RMG-open but A={b} is not (1, 2)-RMG-closed.

Theorem 4.18: If τ_j-int(τ_i-cl(A))⊂ B ⊂ A and A is (i, j)-RMG-open in (X, τ_1, τ_2), Then B is (i, j)-RMG-open.

Proof: Let F be τ_i-rg-open such that F⊂B. Now F⊂ B ⊂ A. That is F ⊂ A. Since F is (i, j)-RMG-open, by Theorem 4.13, F⊂ τ_j-int(τ_i-cl(A)). By hypothesis τ_j-int(τ_i-cl(A))⊂ B. Therefore τ_j-int(τ_i-cl(τ_j-int(τ_i-cl(A)))) ⊂ τ_j-int(τ_i-cl(B)). That is τ_j-int(τ_i-cl(A))⊂ τ_j-int(τ_i-cl(B)) and hence F⊂ τ_j-int(τ_i-cl(B)). Again by Theorem 4.13, B is (i, j) is RMG-open set in (X, τ_1, τ_2).

Corollary 4.19: Let A and B be subsets of a space (X, τ_1, τ_2). If B is (i, j)-RMG-open and A⊃ τ_j-int(τ_i-cl(B)), Then A∩B is (i, j)-RMG-open.

Proof: Let B be (i, j)-RMG-open and A⊃ τ_j-int(τ_i-cl(B)). That is τ_j-int(τ_i-cl(B))⊂A. Then τ_j-int(τ_i-cl(B))⊂A∩B. Also τ_j-int(τ_i-cl(B))⊂A∩B⊂B and B is (i, j)-RMG-open. By Theorem 4.18, A∩B is also (i, j)-RMG-open.

Theorem 4.20: Every singleton point set in a space (X, τ_1, τ_2) is either (i, j)-RMG-open or τ_i-rg-closed.

Proof: Let (X, τ_1, τ_2) be a bitopological space. Let x∈X. To prove that {x} is either (i, j)- RMG-open or τ_i-rg-closed. That is to prove X-{x} is either (i, j)- RMG-closed or τ_i-rg-closed, Which is follows from the Theorem 3.33.

5.(τ_i, τ_j)-RMG-Neighbourhoods and some of their properties.

Definition 5.1: In a bitopological space (X, τ_1, τ_2). A subset N of X is said to be (τ_i, τ_j)-RMG-neighbourhood (briefly, (τ_i, τ_j)-RMG-nhd) of a point x ϵ X if ∃ a (τ_i, τ_j)-RMG-open set G such that x ∈ G ⊆ N.

Example 5.2: X={a, b, c, d},τ_1={X, ∅,{a},{b},{a, b}, {a, b, c} and τ_2 ={X, ∅,{a},{b, c},{a, b, c}},then (τ_i, τ_j)-RMG-closed sets are X, ∅, {c},{d}, {a, d}, {c, d},{a, c, d},{b, c, d} and (τ_i, τ_j)-RMG-open sets are X, ∅,{a},{b}, {a, b}, {b, c},{a, b, d}, {a, b, c}}.Let c∈{b, c}∈{a, b, c} where {b, c} is (τ_1, τ_2) RMG-open.
∴ {a, b, c} is a (τ_i, τ_j)-RMG-neighbourhood of c.
The collection of all (τ_i, τ_j)-RMG- neighbourhoods of x ∈ X is denoted by (τ_i, τ_j)-RMG-N(x).

Theorem 5.3: If N be a subset of a bitopological space(X, τ_1, τ_2) is (τ_i, τ_j)-RMG-open set, then N is(τ_i, τ_j)-RMG-nhd of each of its points.

Proof: In a bitopological space(X, τ_1, τ_2). Let N be a (τ_i, τ_j)-RMG-open set.
⇒ ∀ x ∈ N there exists a (τ_i, τ_j)-RMG-open set N such that x ∈ N ⊆ N.
⇒ N is a (τ_i, τ_j)-RMG-neighbourhood of x.

Remark 5.4: The (i, j)-RMG-nhd of point x∈X need not be a (i, j)-nhd of x in X as seen from the following example.

Example 5.5: Let X={a, b, c, d}, τ_1 ={X, ∅, {a}, {b}, {a, b}, {a, b, c}} and τ_2 ={X, ∅, {a, b}, {c, d}}. $D^*_{RMG}(\tau_i, \tau_j)$={X, ∅, {a}, {b}, {c}, {a, b}, {c, d}, {a, b, c}, {a, b, d}}, the set {b, c} is (i, j)-RMG-nhd of b, since there exists a (i, j)-RMG-open set G={b} such that b∈{b}⊆{b, c}. Also the set the set {b, c} is (i, j)-RMG-nhd of c, since there exists a (i, j)-RMG-open set G={c} such that c∈{c}⊆{b, c}. However {b, c} is not (i, j)-RMG-open set in X.

Theorem 5.6: Let (X, τ_i, τ_j) be a bitopological space.
(i) $\forall$ x ∈ X, $(\tau_i, \tau_j) - \mathrm{RMG} - \mathrm{N(x)} \neq \emptyset$.
(ii)If $\mathrm{N} \in (\tau_i, \tau_j) - \mathrm{RMG} - \mathrm{N(x)}$ and $\mathrm{N} \subseteq \mathrm{M} \Rightarrow \mathrm{M} \in (\tau_i, \tau_j) - \mathrm{RMG} - \mathrm{N(x)}$.
(iii)If $\mathrm{N} \in (\tau_i, \tau_j) - \mathrm{RMG} - \mathrm{N(x)} \Rightarrow \exists\, \mathrm{M} \in (\tau_i, \tau_j) - \mathrm{RMG} - \mathrm{N(x)}$such that $\mathrm{M} \subseteq \mathrm{N}$ and $\mathrm{M} \in (\tau_i, \tau_j) - \mathrm{RMG} - \mathrm{N(y)}\ \forall\, \mathrm{y} \in \mathrm{M}$.

Proof: i) Since X is an (τ_i, τ_j)-RMG-open set, it is a (τ_i, τ_j)-RMG-nhd of ∀x∈X. Hence there exists at least one (τ_i, τ_j)-RMG-neighbourhood G for every x∈X. Therefore (τ_i, τ_j)-RMG-N(x)≠∅.
ii) If N∈(τ_i, τ_j)-RMG-N(x) and N ⊆ M. Then there exists an (τ_i, τ_j)-RMG- open set G such that x ∈ G ⊆ N, since N ⊆ M, x ∈ G ⊆ M and M is (τ_i, τ_j)-RMG-nhd of x. Hence M is a(τ_i, τ_j)-RMG-neighbourhood of x. Therefore M∈(τ_i, τ_j)-RMG-N(x).
(iii) If N∈(τ_i, τ_j)-RMG-N(x), then there exists an (τ_i, τ_j)-RMG-open set M such that x ∈ M ⊆ N. Since M is an (τ_i, τ_j)-RMG-open set, then it is(τ_i, τ_j)-RMG- neighbourhood of each of its points. Therefore M∈(τ_i, τ_j)-RMG-N(y) ∀y∈M.

Theorem 5.7: Every τ_j- neighbourhood of a point x of a bitopological space(X, τ_1, τ_2) is a(τ_i, τ_j) -RMG- neighbourhood of the point x.

Proof: (X, τ_1, τ_2) is a bitopological space and x∈X. N is a τ_j- neighbourhood of x.
⇒ ∃ a τ_j-open set G such that x∈G⊆N.
⇒ ∃ (τ_i, τ_j)-RMG-open set G such that x∈G⊆N.
⇒ N is a (τ_i, τ_j)-RMG-neighbourhood of the point x.
The converse of the above Theorem need not be true as seen from the following example.

Example 5.8: Let X={a, b, c, d},τ_1={X, ∅,{a},{b},{a, b}, {a, b, c} and τ_2 ={X, ∅,{a},{b, c},{a, b, c}}, (τ_i, τ_j)-RMG-closed sets are X, ∅, {c},{d}, {a, d}, {c, d},{a, c, d},{b, c, d} and
(τ_i, τ_j)-RMG-open sets are X, ∅,{a},{b}, {a, b}, {b, c},{a, b, d}, {a, b, c}}. c∈{b, c}∈{b, c, d} where {b, c} is (τ_1, τ_2) RMG-open. Therefore {b, c, d} is a (τ_i, τ_j)-RMG-neighbourhood of c. N={b, c, d} is a(τ_i, τ_j)-RMG-neighbourhood of c. But there does not exist an open set G in τ_2= {X, ∅, {a}, {b, c},{a, b, c}} such that x∈G⊆N. Therefore N is not τ_2- neighbourhood of c.

Theorem 5.9: If F is a (τ_i, τ_j)-RMG-closed subset of a bitopological space(X, τ_1, τ_2), then ∀ x ∈ F^c there exists an (τ_i, τ_j)-RMG-open set N containing x such that N∩F=∅.

Proof: F is a (τ_i, τ_j)-RMG-closed subset of a bitopological space (X, τ_1, τ_2)
⇒ F^c is a (τ_i, τ_j)-RMG-open subset of X.
⇒ F^c is a (τ_i, τ_j)-neighbourhood of each of its points.
⇒ There exists a (τ_i, τ_j)-RMG-open set N such that x ∈ N ⊆ F^c ∀ x∈F.
⇒ There exists a (τ_i, τ_j)-RMG-open set N containing x such that N∩F=∅.

6.(τ_i, τ_j)-Rmg-Closure and Some of Their Properties.

Definition 6.1: Let (X, τ_1, τ_2) be a bitopological space and i, j∈{1, 2} be fixed integers. For each subset E of X, define (τ_i, τ_j)-RMG-cl(E)=∩{A: E⊆A∈ D_{RMG}(i, j)} and is denoted by (i, j)-RMG-cl(E).

Theorem 6.2: Let A and B are subset of (X, τ_1, τ_2). Then
(i) (i, j)-RMG-cl(X)=X and (i, j)-RMG-cl(∅)= ∅.
(ii) A⊆(i, j)-RMG-cl(A).
(iii)If B is any (i, j)-RMG-closed set containing A, Then (i, j)-RMG-cl(A)⊆B.

Proof: Follows from the Definition 6.1.

Theorem 6.3: Let A and B are subset of (X, τ_1, τ_2) and i, j∈{1, 2} be the fixed integers. If A⊆B, then (i, j)-RMG-cl(A)⊆(i, j)-RMG-cl(B).

Proof: Let A⊆B. By Definition 6.1, (i, j)-RMG-cl(B)=∩{F :B⊆F∈ D_{RMG}(i, j)}. If B⊆F∈ D_{RMG}(i, j), since A⊆B, A⊆B⊆F∈ D_{RMG}(i, j), we have (i, j)-RMG-cl(A) ⊆F. Therefore (i, j)-RMG-cl(A)⊆∩{F :B⊆F∈ D_{RMG}(i, j)}=(i, j)-RMG-cl(B). That is (i, j)-RMG-cl(A)⊆(i, j)-RMG-cl(B).

Theorem 6.4: Let A be a subset of (X, τ_1, τ_2). If $\tau_1 \subseteq \tau_2$ and RGO(X, τ_1) ⊆RGO(X, τ_2), then (1, 2)-RMG-cl(A) ⊆(2, 1)-RMG-cl(A).

Proof: By definition 6.1, (1, 2)-RMG-cl(A)=∩{F :A⊆F∈ D_{RMG}(1, 2)}. Since $\tau_1 \subseteq \tau_2$, by Theorem 3.32, $D_{RMG}(2,1) \subseteq D_{RMG}(1,2)$. Therefore ∩{F : A⊆F∈ D_{RMG}(1, 2)}⊆ ∩{F :A⊆F∈ D_{RMG}(2, 1)}. That is (1, 2)-RMG-cl(A) ⊆∩{F :A⊆F∈ D_{RMG}(2, 1)}=(2, 1)-RMG-cl(A). Hence (1, 2)-RMG-cl(A) ⊆(2, 1)-RMG-cl(A).

Theorem 6.5: Let A be a subset of (X, τ_1, τ_2) and i, j∈{1, 2} be fixed integers, then A⊆(i, j)-RMG-cl(A) ⊆ τ_j-cl(A).

Proof: By definition 6.1, it follows that A⊆(i, j)-RMG-cl(A). Now to prove that (i, j)-RMG-cl(A) ⊆ τ_j-cl(A). By definition of closure, τ_j-cl(A)=∩{F⊆X :A⊆F and F is τ_j-closed}. If A⊆F and F is τ_j-closed, then F is (i, j)-RMG closed, as every τ_j-closed set is (i, j)-RMG-closed. Therefore (i, j)-RMG-cl(A)⊆∩{F⊆X :A⊆F and F is τ_j-closed}= τ_j-cl(A). Hence (i, j)-RMG-cl(A) ⊆ τ_j-cl(A).

Remark 6.6: Containment relation in above theorem may be proper as seen from the following example.

Example 6.7: Let X={a, b, c, d},τ_1={X, ∅,{a},{b},{a, b}, {a, b, c} and τ_2 ={X, ∅,{a},{b, c},{a, b, c}}. Then τ_2-closed sets are X, ∅, {d}, {a, d}, {b, c, d} and (1, 2)-RMG-closed sets are X, ∅, {c},{d}, {a, d}, {c, d},{a, c, d},{b, c, d}. Take A={a, c}. Then

τ_2-cl(A)=X and (1, 2)-RMG-cl(A)={a, c, d}. Now A⊆(1, 2)-RMG-cl(A), but A≠(1, 2)-RMG-cl(A). Also (1, 2)-RMG-cl(A)⊆ τ_2-cl(A), but (i, j)-RMG-cl(A) ≠-cl(A).

Theorem 6.8: Let A be a subset of (X, τ_1, τ_2) and i, j∈{1, 2} be fixed integers. If A is (i, j)-RMG-closed, then (i, j)-RMG-cl(A)=A.

Proof: Let A be a (i, j)-RMG-closed subset of (X, τ_1, τ_2). We know that A⊆(i, j)-RMG-cl(A). Also A⊆A and A is (i, j)-RMG-closed. By the Theorem 6.2(iii), (i, j)-RMG-cl(A) ⊆A. Hence (i, j)-RMG-cl(A) =A.

Remark 6.9: The converse of the above Theorem 6.7 need not be true as seen from the following example.

Example 6.10: Let X={a, b, c, d},τ_1={X, ∅,{a},{b},{a, b}, {a, b, c} and τ_2 ={X, ∅,{a, b},{c, d}}. Then (1, 2)-RMG-closed sets are X, ∅, {c},{d}, {a, b}, {c, d}, {a, b, d}, {a, c, d},{b, c, d}. Take A={a}. Now (1, 2)-RMG-cl(A)=X∩{a, b}∩{a, b, d}∩{a, c, d}={a}, but {a} is not (1, 2)-RMG-closed set.

Theorem 6.11: A and B are subsets of a bitopological space (X, τ_1, τ_2), Then (τ_i, τ_j)-RMG-cl(A) ∪ (τ_i, τ_j)-RMG-cl(B) ⊆ (τ_i, τ_j)-RMG-cl(A∪B).

Proof: Let A and B are subset of (X, τ_1, τ_2). Clearly A⊆A∪B and B⊆A∪B
⇒ (τ_i, τ_j)-RMG-cl(A)⊆ (τ_i, τ_j)-RMG-cl(A∪B), (τ_i, τ_j)-RMG-cl(B) ⊆(τ_i, τ_j)-RMG-cl(A∪B)
⇒ (τ_i, τ_j)-RMG-cl(A)∪ (τ_i, τ_j)-RMG-cl(B)⊆ (τ_i, τ_j)-RMG-cl(A∪B).

Remark 6.12: (τ_i, τ_j)-RMG-cl(A∪B) ⊈ (τ_i, τ_j)-RMG-cl(A)∪(τ_i, τ_j)-RMG-cl(A)

Example 6.13: Let X={a, b, c, d},τ_1={X, ∅,{a},{b},{a, b}, {a, b, c} and τ_2 ={X, ∅,{a, b},{c, d}}. Then (1, 2)-RMG-closed sets are X, ∅, {c},{d}, {a, b}, {c, d}, {a, b, d}, {a, c, d},{b, c, d}. Let A={a, b}and A={c}. Then A∪B= {a, b, c}, (τ_i, τ_j)-RMG-cl(A∪B)=X, (τ_i, τ_j)-RMG-cl(A)={a, b} and (τ_i, τ_j)-RMG-cl(B)={c}. Then X ⊈{a, b, c}.

Theorem 6.14: If A is a subset of a bitopological space (X, τ_1, τ_2), then (τ_i, τ_j)-RMG-cl ((τ_i, τ_j)-RMG-cl(A))=(τ_i, τ_j)-RMG-cl(A).

Proof: Since (τ_i, τ_j)-RMG-cl (A) is a (τ_i, τ_j)-RMG-closed set in X, It follows (τ_i, τ_j)-RMG-cl((τ_i, τ_j)-RMG-cl(A))=(τ_i, τ_j)-RMG-cl(A).

Theorem 6.15: If A and B are subsets of a bitopological space (X, τ_1, τ_2), then
(τ_i, τ_j)-RMG-cl(A∩B)⊆ (τ_i, τ_j)-RMG-cl(A)∩(τ_i, τ_j)-RMG-cl(B).

Proof: Let A and B are subsets of (X, τ_1, τ_2). Clearly A∩B⊆A and A∩B⊆B. Therefore (τ_i, τ_j)-RMG-cl(A∩B)⊆ (τ_i, τ_j)-RMG-cl(A), (τ_i, τ_j)-RMG-cl(A∩B)⊆(τ_i, τ_j)-RMG-cl(B).
Therefore (τ_i, τ_j)-RMG-cl (A∩B)⊆ (τ_i, τ_j)-RMG-cl(A)∩(τ_i, τ_j)-RMG-cl(B)

Theorem 6.16: A is a nonempty subset of a bitopological space (X, τ_1, τ_2), x∈X and x∈(τ_i, τ_j)-RMG-cl(A) if and only if V∩A≠∅ ∀ (τ_i, τ_j)-RMG-open set V containing x.

Proof: Le A be a nonempty subset of a bitopological space (X, τ_1, τ_2). Let x∈X and x∈(τ_i, τ_j)-RMG-cl(A). To prove the result by contradiction. Suppose ∃ a (τ_i, τ_j)-RMG-open set V containing x such that V∩A=∅. Then A⊆X-V and X-V is a (τ_i, τ_j)-RMG-closed set and so (τ_i, τ_j)-RMG-cl (A)⊆X-V. This shows that x∉ (τ_i, τ_j)-RMG-cl (A), which is a contradiction. Hence V∩A≠ ∅ for every (τ_i, τ_j)-RMG-open set V containing x.
Conversely, let V∩A≠ ∅ for every (τ_i, τ_j)-RMG-open set containing x. To prove x∈(τ_i, τ_j)-RMG-cl(A).We prove the result by contradiction. Suppose that x∉(τ_i, τ_j)-RMG-cl(A).
Then there exists a (τ_i, τ_j)-RMG-closed subset F containing A such that x∉F. Then x∈X-F and X-F is (τ_i, τ_j)-RMG-open. Also (X-F)∩A= ∅, which is contradiction. Hence x∈(τ_i, τ_j)-RMG-cl(A).

Theorem 6.17: If A is a subset of a bitopological space (X, τ_1, τ_2). Then (τ_i, τ_j)-RMG-cl(A) ⊆ (τ_i, τ_j)-pre-cl(A).

Proof: Let A be a subset of (X,τ_1, τ_2). By the definition of (τ_i, τ_j)-pre-closure, (τ_i, τ_j)-pre-cl (A)=∩{F⊆X: A⊆F is(τ_i, τ_j)-pre-closed set in X}. If A⊆F and F is (τ_i, τ_j)-pre-closed subset of X, then A⊆F∈ D_{RMG}(i, j), Because as every (τ_i, τ_j)-pre-closed set is

(τ_i, τ_j)-RMG-closed set in X, That is (τ_i, τ_j)-RMG-cl(A)⊆F. Therefore {F⊆X: A⊆F, (τ_i, τ_j)-pre-closed}⊆{F⊆X: A⊆F, (τ_i, τ_j)-RMG-closed}. Therefore (τ_i, τ_j)-RMG-cl(A) ⊆(τ_i, τ_j)-pre-cl(A).

Theorem 6.18: If A be subset of a bitopological space (X, τ_1, τ_2).
(i)(τ_i, τ_j)-mildly-g-cl(A) ⊆(τ_i, τ_j)-RMG-cl(A)
(ii)(τ_i, τ_j)-wg-cl(A) ⊆(τ_i, τ_j)-RMG-cl(A)

Proof: i) Let A be a subset of (X, τ_1, τ_2). By the definition of (τ_i, τ_j)-RMG-closure, (τ_i, τ_j)-RMG-cl(A)=∩{F: A⊆F∈ D_{RMG}(i, j)}. If A⊆F∈ D_{RMG}(i, j), then A⊆F∈(τ_i, τ_j)-mildly-g-closed. Because as every (τ_i, τ_j)-RMG-closed set is (τ_i, τ_j)-mildly-g-closed, That is (τ_i, τ_j)-mildly-g-cl(A)⊆F. Therefore (τ_i, τ_j)-mildly-g-cl(A)⊆{F: A⊆F, (τ_i, τ_j)-mildly-g-closed}⊆{F: A⊆F, (τ_i, τ_j)-RMG-closed}. Therefore (τ_i, τ_j)-mildly-g-cl(A) ⊆(τ_i, τ_j)-RMG-cl(A).
ii) Similarly (ii) results may be proved.

7.(τ_i, τ_j)-Rmg-Interior and Some of Their Properties.
Definition 7.1: Let A be a subset of bitopological space (X, τ_1, τ_2). We define the (τ_i, τ_j)-RMG-interior of A to be the union of all (τ_i, τ_j)-RMG-open sets contained in A and is denoted by (τ_i, τ_j)-RMG-interior(A) or (i, j)-RMG-int(A).

Theorem 7.2: Let A and B be a subsets of bitopological space (X, τ_1, τ_2), then
(i) (i, j)-RMG-int(X)=X and (i, j)-RMG-int(∅)= ∅.
(ii) (i, j)-RMG-int(A) ⊆A.
(iii) If B is any (i, j)-RMG- open set contained in A, Then B⊆(i, j)-RMG-int(A).
(iv) If A is (i, j)-RMG-open, Then (i, j)-RMG-int (A)=A.

Proof: (i) Since X and ∅ are (i, j)-RMG-open sets. By the definition 7.1, i, j)-RMG-int(X)=X and (i, j)-RMG-int(∅)= ∅.
(ii) If F={G:G is (i, j)-RMG-open and G⊆A}. then (i, j)-RMG-int(A)=∪ $G_{G\epsilon F}$ and G⊆A, ∀G∈F ∪ $G_{G\epsilon F}$ ⊆A. That is (i, j)-RMG-int (A)⊆A.
(iii) Let B be any (i, j)-RMG-open set such that B⊆A. Let x∈B. Then since B is an (i, j)-RMG-open set contained in A, x is an (i, j)-RMG-interior point of A. That is x∈(i, j)-RMG-int(A). Hence B⊆(i, j)-RMG-int(A).
(iv) If a subset A of space X is (i, j)-RMG-open subset of X. We know that (i, j)-RMG-int(A) ⊆A. Also, A is (i, j)-RMG-open set contained in A. from above result (ii), A⊆(i, j)-RMG-int(A). Hence (i, j)-RMG-int(A)=A.

Theorem 7.3: Let A and B are subsets of bitopological space (X, τ_1, τ_2).
(i) If A⊆B, then (i, j)-RMG-int (A)⊆(i, j)-RMG-int(B).
(ii) (i, j)-RMG-int(A∩B)⊆(i, j)-RMG-int(A)∩(i, j)-RMG-int(B).
(ii) (i, j)-RMG-int(A)∩(i, j)-RMG-int(B) ⊆(i, j)-RMG-int(A∪B).

Proof: (i) Let A and B be are subsets of bitopological space(X, τ_1, τ_2). Then A⊆B
⇒{F:F⊆A and F is (i, j)-RMG-open}⊆{F:F⊆B and F is (i, j)-RMG-open}
⇒∪F∈{F:F⊆A and F is (i, j)-RMG-open}⊆∪F∈{F:F⊆B and F is (i, j)-RMG-open}
⇒ (i, j)-RMG-int(A) ⊆(i, j)-RMG-int(B).
(ii) We know that A∩B⊆A and A∩B⊆B. We have by above result (i), (i, j)-RMG-int(A∩B)⊆ (i, j)-RMG-int(A) and (i, j)-RMG-int(A∩B)⊆ (i, j)-RMG-int(B). This implies that (i, j)-RMG-int(A∩B)⊆(i, j)-RMG-int(A) ∩(i, j)-RMG-int(B).
(iii) Let A and B be are subsets of bitopological space(X, τ_1, τ_2). We know that A⊆A∪B and B⊆A∪B. We have(i, j)-RMG-int(A) ⊆(i, j)-RMG-int(A∪B) and (i, j)-RMG-int(B) ⊆(i, j)- RMG-int(A∪B). This implies that (i, j)-RMG-int(A) ∪(i, j)-RMG-int(B) ⊆(i, j)-RMG-int(A∪B)

Theorem 7.4: If A is a subset of a bitopological space (X, τ_1, τ_2), then
(i) (i, j)-RMG-int(A)⊆ (i, j)-mildly-g-int(A).
(ii) (i, j)-RMG-int(A)⊆(i, j)-wg-int(A).

Proof: (i) A is a subset of a bitopological space(X, τ_1, τ_2).
Let x∈(i, j)-RMG-int(A)⇒ x∈∪ {G⊆X: G is RMG-open, G⊆A}
⇒ there exists a (i, j)-RMG-open set G such that x∈G and G⊆A.
⇒ there exists a (i, j)-mildly-g-open set G such that x⊆G and G⊆A.
⇒ x∈∪{G: G is (i, j)-mildly-g-open, G⊆A}.
⇒ x∈(i, j)-mildly-g-int(A).

Hence (i, j)-RMG-int(A)⊆(i, j)-mildly-g-int(A).
(ii) Similarly these results may be proved.

Theorem 7.5: If A is a subset of a bitopological space (X, τ_1, τ_2), then (i, j)-pre-int(A)⊆ (i, j)-RMG-int(A).

Proof: A is a subset of a bitopological space(X, τ_1, τ_2).
Let x∈(i, j)-pre-int(A)⇒ x∈∪ {G⊆X: G is (i, j)-pre-open, G⊆A}
⇒ there exists a (i, j)-pre-open set G such that x∈G and G⊆A, as every (i, j)-pre-open set is (i, j)-RMG-open set in X.
⇒ there exists a (i, j)-RMG-open set G such that x⊆G and G⊆A.
⇒ x∈∪{G: G is (i, j)-RMG-open, G⊆A}.
⇒ x∈(i, j)-RMG-int(A).
Hence (i, j)-pre-int(A)⊆(i, j)-RMG-int(A).

Theorem 7.6: Let A be any subset of (X, τ_1, τ_2). then
(i) X-(i, j)-RMG-int(A)= (i, j)-RMG-cl(X-A).
(ii) (i, j)-RMG-int(A)= X- (i, j)-RMG-cl(X-A).
(iii) X-(i, j)-RMG-int(X-A)= (i, j)-RMG-cl(A).
(iv) X-(i, j)-RMG-cl(A)= (i, j)-RMG-int-(X-A).

Proof: (i) Let x∈X-(i, j)-RMG-int(A). Then x∉(i, j)-RMG-int(A). That is every (i, j)-RMG-open set U containing x is such that U⊈A. That is every (i, j)-RMG-open set U containing x such that U∩(X-A)≠ ∅. By the Theorem 6.16, x∈(i, j)-RMG-cl(X-A) and Therefore X-(i, j)-RMG-int(A)⊆(i, j)-RMG-cl(X-A).
Conversely, x∈(i, j)-RMG-cl(X-A). Then by Theorem 6.16, every (i, j)-RMG-open set U containing x is such that U∩(X-A)≠ ∅. That is every (i, j)-RMG-open set U containing x such that U⊈A. This implies by Definition of (i, j)-RMG-interior of A, x∉(i, j)-RMG-int(A). That is x∈X-(i, j)-RMG-int(A) and (i, j)-RMG-cl(X-A)⊆X-(i, j)-RMG-int(A). Thus X-(i, j)-RMG-int(A)= (i, j)-RMG-cl(X-A).
(ii) Follows by Taking compliments in (i).
(iv) Follows by replacing A by X-A in (i).
(v) Follows by result (i).

8. References

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