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Strongly intuitionistic fuzzy rw-continuous functions

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Abstract
In this paper, we introduce the strongly intuitionistic fuzzy rw-continuous functions and study its properties. Furthermore we introduced perfectly intuitionistic fuzzy rw-continuous functions, intuitionistic fuzzy rw-irresolute, and intuitionistic fuzzy T_{rw}-space in intuitionistic fuzzy topological spaces.

Keywords: Intuitionistic fuzzy rw-closed sets, intuitionistic fuzzy rw-open sets, intuitionistic fuzzy clopen sets, intuitionistic fuzzy rw-continuous functions and intuitionistic fuzzy gpr-continuous functions

1. Introduction

After the introduction of fuzzy sets by Zadeh [22] in 1965 and fuzzy topology by Chang [3] in1967, several researches were conducted on the generalizations of the notions of fuzzy sets andfuzzy topology. The concept of intuitionistic fuzzy sets (IFS) was introduced by Atanassov [1] as a generalization of fuzzy sets. In the last 25 years various concepts of fuzzy mathematics have been extended for IFS. In 1997 Coker [4] introduced the concept of IF topological spaces. Recently many fuzzy topological concepts such as fuzzy compactness [6], fuzzy connectedness [20], fuzzy separation axioms [2], fuzzy continuity [7], fuzzy g-closed sets [10], fuzzy g-continuity [11], fuzzy rg-closed sets [12] have been generalized for IF topological spaces. Recently authors of this paper introduced the concept of IF w-closed sets [16] in IF topology. S.S. Thakur and Jyoti Pandey Bajpai [17] introduced intuitionistic fuzzy rw-continuity. In the present paper we introduced strongly intuitionistic fuzzy rw-continuous functions and also introduced perfectly intuitionistic fuzzy rw-continuous functions. We obtain some of their characterization and properties.

2. Preliminaries

Let X is a nonempty fixed set. An intuitionistic fuzzy set A [1] in X is an object having the form $A = \{<x, \mu_A(x), \gamma_A(x)> : x \in X\}$, where the functions $\mu_A: X \to [0,1]$ and $\gamma_A: X \to [0,1]$ denotes the degree of membership $\mu_A(x)$ and the degree of non-membership $\gamma_A(x)$ of each element $x \in X$ to the set A respectively and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$. The intuitionistic fuzzy sets $\tilde{0} = \{< x, 0, 1> : x \in X\}$ and $\tilde{1} = \{<x, 1, 0> : x \in X\}$ are respectively called empty and whole intuitionistic fuzzy set on X. An intuitionistic fuzzy set
$A = \{<x, \mu_A(x), \gamma_A(x)> : x \in X\}$ is called a subset of an intuitionistic fuzzy set
$B = \{<x, \mu_A(x), \gamma B(x)> : x \in X\}$ (for short $A \subseteq B$) if $\mu_A(x) \leq \mu_A(x)$ and $\gamma_A(x) \geq \mu_A(x)$ for each $x \in X$. The complement of an intuitionistic fuzzy set
$A = \{<x, \mu_A(x), \gamma_A(x)> : x \in X\}$ is the intuitionistic fuzzy set $A^c = \{<x, \gamma_A(x), \mu_A(x) >: x \in X\}$. The intersection (resp. union) of any arbitrary family of intuitionistic fuzzy sets $A_i = \{< x, \mu_A(x), \gamma_A(x) > : x \in X, (i \in \wedge)\}$ of X be the intuitionistic fuzzy set $\cap Ai = \{< x, \mu_A(x), \vee \gamma_{Ai}(x) > : x \in X\}$ (resp. $\cup Ai = \{<x, \vee \mu_A(x), \wedge \gamma_{Ai}(x) >: x \in X\}$). Two intuitionistic fuzzy sets $A = \{<x, \mu_A(x), \gamma_A(x)> : x \in X\}$ and $B = \{<x, \mu_B(x), \gamma_B(x)> : x \in X\}$ are said to be q-coincident (AqB for short) if and only if $\exists$ an element $x \in X$ such that $\mu_A(x) > \gamma_B(x)$or $\gamma_A(x) < \mu_B(x)$. A family τ of intuitionistic fuzzy sets on a non-empty set X is called an intuitionistic fuzzy topology [3] on X if the intuitionistic fuzzy sets $\tilde{o}$, $\tilde{1} \in \tau$, and $\Im$ is closed under arbitrary union and finite intersection. The ordered pair (X, τ) is called an intuitionistic fuzzy topological space and each intuitionistic fuzzy set in τ is called an intuitionistic fuzzy open set.

The compliment of an intuitionistic fuzzy open set in X is known as intuitionistic fuzzy closed set. The intersection of all intuitionistic fuzzy closed sets which contains A is called the closure of A. It denoted by cl (A). The union of all intuitionistic fuzzy open subsets of A is called the interior of A. It is denoted by
int (A) [4].

Definition 2.1 [5]: Let X is a nonempty set and c∈X a fixed element in X. If α∈(0,1] and β∈[0,1) are two real numbers such that α+β≤1 then :

a) c(α,β) = < x,c_{α}, $c_{1-\beta}$ > is called an intuitionistic fuzzy point in X, where α denotes the degree of membership of c(α,β), and β denotes the degree of non-membership of c(α, β).
b) c(β) = < x,0, 1-$c_{1-\beta}$ > is called a vanishing intuitionistic fuzzy point in X, where β denotes the degree of non-membership of c(β).

Definition 2.2 [6]: A family {Gi : i∈Λ} of intuitionistic fuzzy sets in X is called an intuitionistic fuzzy open cover of X if ∪{Gi : i∈Λ} = ĩ and a finite subfamily of an intuitionistic fuzzy open cover {Gi: i∈Λ}of X which also an intuitionistic fuzzy open cover of X is called a finite sub cover of {Gi: i∈Λ}.

Definition 2.3 [6]: An intuitionistic fuzzy topological space (X,τ) is called intuitionistic fuzzy compact if every intuitionistic fuzzy open cover of X has a finite sub cover.

Definition 2.4: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X, τ) is called:

a) Intuitionistic fuzzy g-closed [10] if cl(A) ⊆ O whenever A ⊆ O and O is intuitionistic fuzzy open in X.
b) Intuitionistic fuzzy sg-closed [15] if scl(A) ⊆ O whenever A ⊆ O and O is intuitionistic fuzzy semi open in X.
c) Intuitionistic fuzzy rg-closed [12] if cl(A) ⊆ O whenever A ⊆ O and O is intuitionistic fuzzy regular open in X.
d) Intuitionistic fuzzy αg-closed [8] if αcl(A) ⊆ O whenever A ⊆O and O is intuitionistic fuzzy open in X.
e) Intuitionistic fuzzy gsp-closed [9] if spcl(A) ⊆ O whenever A ⊆O and O is intuitionistic fuzzy open in X.
f) Intuitionistic fuzzy w-closed [16] if cl(A) ⊆ O whenever A ⊆ O and O is intuitionistic fuzzy semi open in X.
g) Intuitionistic fuzzy rw-closed [17] if cl(A) ⊆ O whenever A ⊆ O and O is intuitionistic fuzzy regular semi open in X.
h) Intuitionistic fuzzy gpr-closed [18] if pcl(A) ⊆ O whenever A ⊆ O and O is intuitionistic fuzzy open in X.
i) Intuitionistic fuzzy rgα-closed [19] if αcl(A) ⊆ O whenever A ⊆O and O is intuitionistic fuzzy regular α-open in X.

The complements of the above mentioned closed sets are their respective open sets.

Definition 2.5 [10]: An intuitionistic fuzzy topological space (X, τ) is said to be intuitionistic fuzzy- $T_{1/2}$ if every intuitionistic fuzzy g-closed set in X is intuitionistic fuzzy closed in X.

Definition 2.6 [19]: An intuitionistic fuzzy topological space (X, τ) is said to be intuitionistic fuzzy rgα- $T_{1/2}$ if every intuitionistic fuzzy rgα-closed set in X is intuitionistic fuzzy closed in X.

Definition 2.7 [7]: Let (X, τ) and (Y, σ) be two intuitionistic fuzzy topological spaces and let *f* : X→Y be a function. Then *f* is said to be

a) Intuitionistic fuzzy continuous if the pre image of each intuitionistic fuzzy open set of Y is an intuitionistic fuzzy open set in X.
b) Intuitionistic fuzzy open if image of each intuitionistic fuzzy open set in X is intuitionistic fuzzy open in Y.
c) Intuitionistic fuzzy closed if image of each intuitionistic fuzzy closed set in X is intuitionistic fuzzy closed in Y.

Definition 2.8: Let (X, τ) and (Y, σ) be two intuitionistic fuzzy topological spaces and let *f*: X→Y be a function. Then *f* is said to be:

a) Intuitionistic fuzzy g-continuous if pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy g –closed in X [11].
b) Intuitionistic fuzzy sg-continuous if pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy sg –closed in X [16].
c) Intuitionistic fuzzy αg-continuous if pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy αg –closed in X [8].
d) Intuitionistic fuzzy rg-continuous if pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy rg –closed in X [13].
e) Intuitionistic fuzzy rg-irresolute if pre image of every intuitionistic fuzzy rg-closed set in Y is intuitionistic fuzzy rg –closed in X [14].
f) Intuitionistic fuzzy w-continuous if pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy w –closed in X [16].
g) Intuitionistic fuzzy gpr-continuous if pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy gpr –closed in X [18].

Remark 2.1

a) Every intuitionistic fuzzy continuous mapping is intuitionistic fuzzy w-continuous [16].
b) Every intuitionistic fuzzy w-continuous mapping is intuitionistic fuzzy g-continuous [16].
c) Every intuitionistic fuzzy g-continuous mapping is intuitionistic fuzzy rg-continuous [13].
d) Every intuitionistic fuzzy g-continuous mapping is intuitionistic fuzzy gpr-continuous [18].
e) Every intuitionistic fuzzy rg- continuous mapping is intuitionistic fuzzy gpr-continuous [18].

The converse of above statements may be false.

3. Strongly Intuitionistic Fuzzy rw-Continuous functions

Definion: 3.1

A function f: (X,τ)→(Y, σ) is called strongly instuitionistic fuzzy rw-continuous in IFTS if f^{-1}(V) is instuitionistic fuzzy closed in X for every instuitionistic fuzzy rw- closed set V in Y.

Example 3.1

Let X={a,b},Y={x,y} and instuitionistic fuzzy sets U and V are defined as follows,
U=(<a,0,0.2>,<b,0.4,0.2>)
V=(<a,0.4,0.1>,<b,0.6,0.1>)
and W=(<x,0.2,0>,<y,0.2,0.4>)
Let τ={õ,ĩ,U,V} and σ ={õ,ĩ,W} be instuitionistic fuzzy topologies on X and Y respectively. Then the mapping f:

(X,τ)→(Y, σ) defined by f(a)= x and f(b)= y is strongly instuitionistic fuzzy rw-continuous.

Remark 3.1
Every instuitionistic fuzzy rw-continuous functions are not strongly instuitionistic fuzzy rw-continuous functions.

Example 3.2
Let X={a,b},Y={x,y} and instuitionistic fuzzy sets U and V are defined as follows,
U=(<a,0.7,0.2>,<b,0.6,0.3>)
V=(<x,0.7,0.2>,<y,0.8,0.1>)
Let τ={õ,ĩ,U} and σ ={õ,ĩ,V} be instuitionistic fuzzy topologies on X and Y respectively. Then the mapping f: (X,τ)→(Y, σ) defined by f(a)= x and f(b)= y is instuitionistic fuzzy rw-continuous but not strongly instuitionistic fuzzy rw-continuous functions.

Theorem 3.1
Every strongly instuitionistic fuzzy rw-continuous functions are instuitionistic fuzzy rw-continuous.
Proof: obvious.

Example 3.3
Let X={a,b},Y={x,y} and instuitionistic fuzzy sets U and V are defined as follows,
U=(<a,0,0.2>,<b,0.4,0.2>)
V=(<a,0.4,0.1>,<b,0.6,0.1>)
and W=(<x,0.2,0>,<y,0.2,0.4>)
Let τ = {õ,ĩ,U,V} and σ = {õ,ĩ,W} be instuitionistic fuzzy topologies on X and Y respectively. Then the mapping f: (X,τ)→(Y, σ) defined by f(a) = y and f(b)= x is strongly instuitionistic fuzzy rw-continuous and also instuitionistic fuzzy rw-continuous.

Theorem 3.2
Every strongly instuitionistic fuzzy rw-continuous is instuitionistic fuzzy gpr-continuous.

Proof
Let f:(X,τ)→(Y,σ) is called strongly instuitionistic fuzzy rw-continuous function. Since every strongly instuitionistic fuzzy rw-continuous function is instuitionistic fuzzy rw-continuous function and instuitionistic fuzzy rw-continuous function is instuitionistic fuzzy gpr-continuous function.

Remark 2.1
Every strongly instuitionistic fuzzy rw-continuous is instuitionistic fuzzy rg-continuous,but the converse may not be true.

Example 3.4
Let X={a,b,c,d},Y={p,q,r,s} and instuitionistic fuzzy sets O,U,V,W,T are defined as follows,
O=(<a,0.9,0.1>,<b,0,1>,<c,0,1>,<d,0,1>)
U=(<a,0,1>,<b,0.8,0.1>,<c,0,1>,<d,0,1>)
V=(<a,0.9,0.1>,<b,0.8,0.1>,(<c,0,1>,<d,0,1>)
W=(<a,0.9,0.1>,<b,0.8,0.1>,<c,0.7,0.2>,<d,0,1>)
and T=(<p,0,1>,<q,0,1>,<r,0.7,0.2>,<s,0,1>)
Let τ={õ,ĩ,O,U,V,W,} and σ={õ,ĩ,T} be instuitionistic fuzzy topologies on X and Y respectively. Then the mapping f: (X,τ)→(Y, σ) defined by f(a)=p and f(b)=q,f(c)=r,f(d)=s is instuitionistic fuzzy rg-continuous but not strongly instuitionistic fuzzy rw-continuous.

Theorem 3.4
If f: (X,τ)→(Y, σ) is called strongly instuitionistic fuzzy rw-continuous and A is fuzzy closed subset of X,then the f/A:A→Y instuitionistic is strongly fuzzy rw-continuous.

Proof
Let V be any rw-closed set of Y.Since f is strongly rw-continuous,then $f^{-1}(V)$ is instuitionistic fuzzy closed in (X, τ).since A is instuitionistic fuzzy closed in X,
$(f/A)^{-1}(V)=A \cap f^{-1}(V)$ is instuitionistic fuzzy closed in A.hence f/A is strongly fuzzy rw-continuous.

Theorem 3.5
If f: (X,τ)→(Y,σ) is called strongly instuitionistic fuzzy rw-continuous then it is continuous

Proof
Obvious.

Definion 3.2
A function f: (X,τ)→(Y, σ) is called perfectly instuitionistic fuzzy rw-continuous in IFTS if $f^{-1}(V)$ is instuitionistic fuzzy clopen in X for every instuitionistic fuzzy rw- closed set V in Y.

Example 3.5
Let X={a,b},Y={x,y} and instuitionistic fuzzy sets U,V and W are defined as follows,
U = (<a,0,0.2>,<b,0.4,0.2>)
V = (<a,0.4,0.1>,<b,0.6,0.1>)
and W = (<x,0.2,0>,<y,0.2,0.4>)
Let τ={õ,ĩ,U,V} and σ ={õ,ĩ,W} be instuitionistic fuzzy topologies on X and Y respectively. Then the mapping f:(X,τ)→(Y, σ) defined by f(a)= x and f(b)=y is perfectly instuitionistic fuzzy rw-continuous.

Theorem 3.6
If f:(X,τ)→(Y, σ) is called perfectly instuitionistic fuzzy rw-continuous then f is strongly rw- continuous.

Proof
Let F be any rw-open set of (Y, σ).By assumption, we get that $f^{-1}(F)$ is clopen in (X,τ), which implies that $f^{-1}(F)$ is open and closed in (X,τ).Hence f is strongly rw- continuous.

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