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Applications of manpower levels for business using six and eight point state space in the stochastic models

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Abstract
The present study aim is to compare the steady rate of crisis and steady state of probabilities under varying conditions which are manpower, under irregular conditions of full availability and nil availability in the case of business and manpower. Six and eight point state space has been compared under the different assumption that the transition from one state to another in both business and manpower occur in exponential time with different parameters.

Keywords: Markov chain, Steady state, Crisis rate

1. Introduction
Nowadays we start that labor has turn into a buyers' market as well as sellers' market. Any business which generally runs on commercial base wishes to keep only the optimum level of all resources necessary to meet company's responsibility at any time during the course of the business and manpower is not an exemption. This is spelt in the sense that a company may not want to maintain manpower more than what is needed. Therefore, retrenchment and recruitment are general and persistent in most of the companies now. If trained laborers and technically capable persons leave the business the seriousness is most awfully felt and the company has to assign paying deep price or pay overtime to employees. Approach to manpower problems have been dealt in many different ways as early as 1947 by Vajda [11] and others. Manpower planning models have been dealt in depth in Grinold & Marshal [3], Barthlomew [1] and Vajda [11]. The methods to compute wastages (Dismissal, resignation and death) and promotion intensities which make the proportions corresponding to some selected planning proposals have been dealt by Lesson [4]. Markov model are designed for wastages and promotion in manpower system by Vassilou [12]. Subramaniam [11]. For an application of Markov chains in a manpower system with efficiency and seniority and Stochastic structures of graded size in manpower planning systems one may refer to Setlhare [9]. A two unit stand by system has been studied by Chandrasekar and Natrajan [2] with confidence limits under steady state. For n unit standby system may refer to Ramanarayanan and Usha [8]. Yadhavalli and Botha [13] have observed the same for two unit system with an introduction of preparation time for the service facility and the confidence limits for stationary rate of disappointment of an intermittently used system. For three characteristics system involving machine, manpower and money one may refer to C. Mohan and R. Ramanarayanan [6]. The study of Semi Markov Models for Manpower planning one may refer to the paper by Sally Meclean [5]. Stochastic Analysis of a Business with Varying Levels in Manpower and Business has been studied by C. Mohan and P. Selvaraju [7]. Applications of manpower with various stages in Business using stochastic models R. Arumugam and M. Rajathi [14].

2. Markov chain model with four and six point state space
In this paper we consider two characteristics namely business and manpower. We compare steady state probabilities and steady state rate of crisis in both six and eight point state space. The situations may be that the manpower may be fully available or hardly available and business may fluctuate between complete availability to nil availability. It goes off when the manpower becomes nil.

This is so because the experts may take the business along with them or those who have brought good will to the concern may bring the client's off the concern. The business depends on steady state probabilities and the availability of manpower. The steady state probabilities of the continuous Markov chain connecting the transitions in different states are identified for presenting the cost analysis. Numerical illustrations are also given.

3. Assumptions
1. There are two levels of manpower namely manpower is full and manpower is nil.
2. There are two levels of business namely, (a) business is fully available (b) business is nil.
3. The time T- during which the manpower remains continuously filled and becomes nil has exponential distribution with parameter α_{10} and the time R- required to stop full recruitment from zero stage is exponentially distributed with parameter β_{01}.
4. The fully available and zero periods of the business are exponentially distributed with parameters 'λ' and 'μ' respectively.
5. The time T'' - during which the business is continuously full becomes moderate has an exponential distribution with parameter α_{21} and the time R''- required to stop full recruitment from moderate level is exponentially distributed with parameter β_{12}. The time T''' - during which the business is moderately.
6. While manpower becomes zero, the business is vanished and becomes nil.
7. T- and R- are independently distributed random variables.

4. System Analysis
4.1 Six Point state space
The time for which the manpower remains continuously full and enters a state of reduced strength follows exponential distribution with parameter α_1. The other two phases have exponential distribution with means $1/\beta_1$ and $1/\gamma_1$. The overall time for the process of recruitment time is then hypo-exponential. Since the sojourn time is in the down state is two - stage hypo-exponentially distributed, the system being modeled in a semi Markov process. The busy and lean periods of the business are exponentially distributed with parameters a and b. The Stochastic Process X(t) describing the state of the system is Markov chain continuous time with six points state space as given below in the order of manpower and business

$$S = \{(0\ 0), (0\ 1), (0\ 2), (1\ 0), (1\ 1), (1\ 2)\} \tag{1}$$

The steady state probabilities of Manpower are,

$$\pi_{0M} = \frac{1}{1 + \frac{\alpha_1}{\gamma_1} + \frac{\alpha_1}{\beta_1}}; \quad \pi_{1M} = \frac{\alpha_1}{\beta_1\left(1 + \frac{\alpha_1}{\gamma_1} + \frac{\alpha_1}{\beta_1}\right)}; \quad \pi_{2M} = \frac{\alpha_1}{\gamma\left(1 + \frac{\alpha_1}{\gamma_1} + \frac{\alpha_1}{\beta_1}\right)}$$

$$\left.\begin{aligned}
&\pi_{00} = \frac{1}{Y'}\frac{a}{X'}; \quad \pi_{01} = \frac{\alpha_1}{\gamma_1 Y'}\frac{a}{X'}; \quad \pi_{02} = \frac{\alpha_1}{\beta_1 Y'}\frac{a}{X'};\\
&\pi_{10} = \frac{1}{Y'}\frac{b}{X'}; \quad \pi_{11} = \frac{\alpha_1}{\gamma_1 Y'}\frac{b}{X'}; \quad \pi_{12} = \frac{\alpha_1}{\beta_1 Y'}\frac{b}{X'};
\end{aligned}\right\} \tag{2}$$

Where $X' = (b + a)$ and $Y' = \left(1 + \frac{\alpha_1}{\beta_1} + \frac{\alpha_1}{\gamma_1}\right)$

The rate of Crisis in steady state conditions is,

$$C_\infty = \alpha_1 \pi_{00} + a\pi_{11} = \frac{\alpha_1}{Y}\frac{b}{\beta}\frac{a}{X} \tag{3}$$

Eight Point state space

The Stochastic Process X(t) describing the state of the system is Markov chain continuous time with four points state space as given below in the order of manpower and business

$$S = \{(0\ \ 0),\ (0\ \ 1),\ (0\ \ 2),\ (1\ \ 0),\ (1\ \ 1),\ (1\ \ 2),\ (2\ \ 0),\ (2\ \ 1)\} \tag{4}$$

Where,
1. Refers to shortage/non availability of manpower and business. The system is in state (1, j).
   When the manpower is in state 1 and business is in state for (1, j) = (1, 1) or (1 0) or (0 0) and there is not the state (0 1) as the business is misplaced while the manpower goes off.

2. Refers to moderate availability of business and it refers to full availability of manpower in the case of business.
3. Refers to full availability in the case of business.

The continuous time markov chain of the state space is given below which is a matrix of order eight.

$$Q=\begin{array}{cc} MP/B & \begin{array}{cccccccc} (0\ 0) & (0\ 1) & (0\ 2) & (1\ 0) & (1\ 1) & (1\ 2) & (2\ 0) & (2\ 1) \end{array} \\ \begin{array}{c} (0\ 0) \\ (0\ 1) \\ (0\ 2) \\ (1\ 0) \\ (1\ 1) \\ (1\ 2) \\ (2\ 0) \\ (2\ 1) \end{array} & \begin{bmatrix} \gamma_1 & \beta_{01} & \beta_{02} & \mu & 0 & 0 & 0 & \beta_{21} \\ \alpha_{10} & \gamma_2 & \beta_{12} & 0 & \mu & 0 & \beta_{21} & 0 \\ \alpha_{20} & \alpha_{21} & \gamma_3 & 0 & 0 & \mu & 0 & \beta_{01} \\ \lambda & 0 & 0 & \gamma_4 & \beta_{01} & \beta_{02} & \beta_{12} & 0 \\ 0 & \lambda & 0 & \alpha_{10} & \gamma_5 & \beta_{12} & 0 & \beta_{21} \\ 0 & 0 & \lambda & \alpha_{20} & \alpha_{21} & \gamma_6 & \beta_{10} & 0 \\ 0 & \alpha_{12} & 0 & \alpha_{21} & 0 & \alpha_{01} & \gamma_7 & \beta_{10} \\ \alpha_{12} & 0 & \alpha_{10} & 0 & \alpha_{21} & 0 & \alpha_{01} & \gamma_8 \end{bmatrix} \end{array} \tag{5}$$

Where, $\gamma_1 = -(\beta_{01} + \beta_{02} + \mu + \beta_{21})$; $\gamma_2 = -(\alpha_{10} + \beta_{12} + \mu + \beta_{21})$

$\gamma_3 = -(\alpha_{20} + \alpha_{21} + \mu + \beta_{01})$; $\gamma_4 = -(\lambda + \beta_{01} + \beta_{02} + \beta_{12})$

$\gamma_5 = -(\lambda + \alpha_{10} + \beta_{12} + \beta_{21})$; $\gamma_6 = -(\lambda + \alpha_{20} + \alpha_{21} + \beta_{10})$

$\gamma_7 = -(\alpha_{12} + \alpha_{21} + \alpha_{01} + \beta_{10})$; $\gamma_8 = -(\alpha_{12} + \alpha_{10} + \alpha_{21} + \alpha_{01})$ (6)

The occurrences of transition in both business and manpower are independent, the individual infinitesimal generator of them are given by

Let $\pi = [\pi_{21}\ \pi_{20}\ \pi_{12}\ \pi_{11}\pi_{10}\ \pi_{02}\ \pi_{01}\ \pi_{00}]$ be the steady state probability vector of the matrix Q, then

$\pi Q = 0, \quad \pi e = 1$

Using (5), we get the steady state probabilities:

(i) The infinitesimal generator of manpower of order two is as follows

$$M = \begin{array}{cc} MP & \begin{array}{cc} 1 & 0 \end{array} \\ \begin{array}{c} 1 \\ 0 \end{array} & \begin{bmatrix} -\lambda & \lambda \\ \mu & -\mu \end{bmatrix} \end{array}$$

The steady state probabilities are $\pi_{M1} = \dfrac{\mu}{\mu + \lambda}$ and $\pi_{M0} = \dfrac{\lambda}{\mu + \lambda}$

(ii) The infinitesimal generator of business is given by the matrix of order three.

$$M = \begin{array}{cc} MP & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{bmatrix} -(\beta_{02} + \beta_{01}) & \beta_{01} & \beta_{02} \\ \alpha_{10} & -(\alpha_{12} + \alpha_{10}) & \alpha_{12} \\ \alpha_{20} & \alpha_{21} & -(\alpha_{20} + \alpha_{21}) \end{bmatrix} \end{array}$$

The steady state probabilities of manpower are

$$\pi_{M2} = \frac{d_2}{d_0 + d_1 + d_2}; \quad \pi_{M1} = \frac{d_1}{d_0 + d_1 + d_2}; \quad \pi_{M0} = \frac{d_0}{d_0 + d_1 + d_2}$$

Where, $d_0 = \beta_{12}\alpha_{20} + \alpha_{10}\alpha_{20} + \alpha_{10}\alpha_{21}$; $d_1 = \beta_{01}\alpha_{20} + \beta_{01}\alpha_{21} + \beta_{02}\alpha_{21}$

$d_2 = \beta_{02}\alpha_{10} + \beta_{02}\beta_{12} + \beta_{01}\beta_{12}$

The steady state probability vectors are $\underline{\pi} Q = 0 \quad and \quad \underline{\pi} e = 1$

$$\pi_{00} = \frac{d_0 \lambda}{z \sum t d_i}; \quad \pi_{01} = \frac{d_1 \lambda}{z \sum t d_i}; \quad \pi_{02} = \frac{d_2 \lambda}{z \sum t d_i}; \quad \pi_{10} = \frac{d_0 \mu}{z \sum t d_i}; \quad \pi_{11} = \frac{d_1 \mu}{z \sum t d_i}; \quad \pi_{12} = \frac{d_2 \mu}{z \sum t d_i};$$

$$\pi_{20} = \frac{d_0 \beta_{21}}{z \sum t d_i}; \quad \pi_{21} = \frac{d_1 \beta_{21}}{z \sum t d_i} \tag{7}$$

where $\sum_{i=0}^{2} d_i = [d_0 + d_1 + d_2]$ and $Z = [b + a]$.

While manpower is available business is fully available or nil. Manpower is inadequate or nil will lead to crisis state.
The only crisis states are {(1 1), (1 2), (2 1)} and the crisis arise when there is full business or moderate business also the manpower is moderate or full. Now the rate of crisis in the steady state is given by

$$C_\infty = \beta_{01} \pi_{10} + \beta_{12} \pi_{11} + \lambda_{10} \pi_{12} + \lambda_{01} \pi_{20}$$

Using steady state probabilities, we obtain

$$C_\infty = \frac{\lambda}{Z \sum_{i=0}^{2} t d_i} [\mu (d_0 \beta_{01} + d_1 \beta_{12} + d_2 \lambda_{10}) + d_0 \beta_{21} \lambda_{01}] \tag{8}$$

5. Numerical illustration and steady state cost calculation

5.1 Six Point state space

The steady state probabilities and the rate of crises are measured by using the formulas (2) and (3) respectively. Taking, a = 25, b =12, $\alpha_1 = 8$, $\beta_1 = 10$, $\gamma_1 = 9$, we get $\pi_{00} = 0.25129$, $\pi_{01} = 0.22337$ $\pi_{02} = 0.20103$ $\pi_{10} = 0.12062$ $\pi_{11} = 0.107218$ $\pi_{12} = 0.09649$

Table 1: Relationship between a and C_∞

a	65	36	22	13	7
C - infinity	8.97949	5.87017	4.36912	3.40415	2.76085

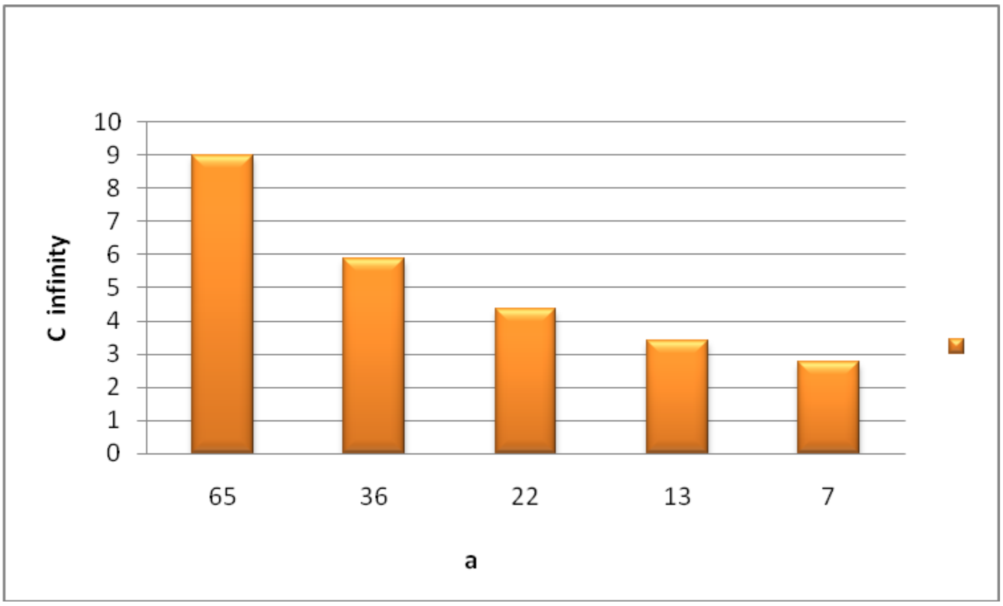

Fig 1: Relationship between a and C_∞

5.2 Eight Point state space

Now taking the values of the parameters in the model as below, we can find the steady state probabilities and the rate of crisis using (7) and (8).

$\alpha_{10} = 8, \ \alpha_{20} = 10, \ \alpha_{21} = 6, \ \beta_{01} = 7, \ \beta_{02} = 8, \ \beta_{12} = 15, \ \beta_{21} = 10, \ \lambda = 15 \ and \ \mu = 18$

Table 2: The value of Steady state probability

Steady state probability	π_{00}	π_{01}	π_{02}	π_{10}	π_{11}	π_{12}	π_{20}	π_{21}
Value	0.2209	0.1494	0.2297	0.2651	0.1793	0.2756	0.1473	0.0996

Now assigning the values of $b = 3, 6, 10, 20 \ and \ 33$. We calculate the corresponding rate of crisis and it is tabulated below:

Table 3: Relationship between λ and C_∞

λ	7	10	13	22	29
C_∞	143.5	102.51	78.39	70.47	55.05

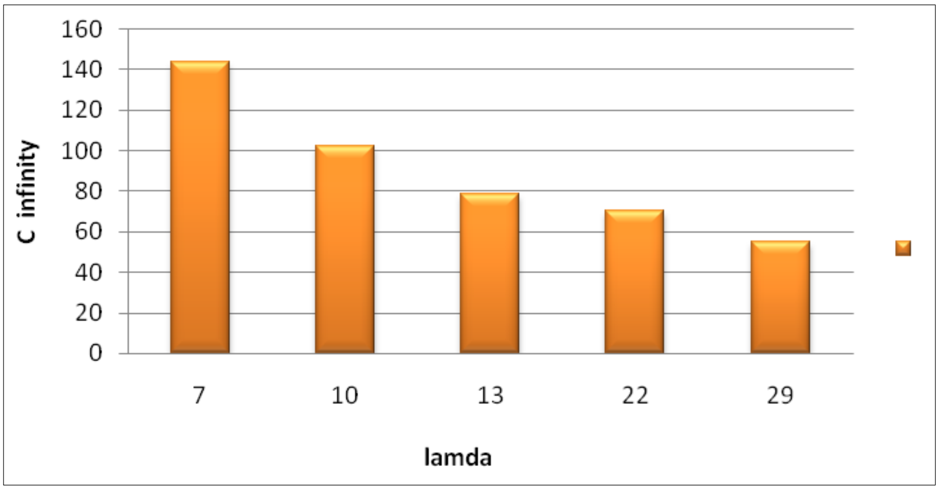

Fig 2: Relationship between λ and C_∞

6. Conclusion

From the above concept it is found that the value of λ increases and the corresponding crisis rate decreases in both six and eight point state space. Also it is observed that if there is full business but there is no manpower, under such situations labour has to be paid a lot. When the manpower is full, there is a chance of the business getting into crisis state if manpower leaves particularly experts and experienced people leave the concern. The same holds in the case of business are moderate while the manpower may be full or inadequate.

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