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# A mathematical model for multi product-multi supplier-multi period inventory lot size problem (MMMILP) with supplier selection in supply chain system

**Binay Kumar and Anchala Kumari**

**Abstract**
Supplier selection is one important decision factors in the supply chain management. Suppliers are necessary entities to any business, however wrong selection may affect the whole business processes; therefore the process of selecting suppliers is extremely important. The success of a supply chain is dependent on selection of better suppliers. Decision makers and managers always face challenges to select suppliers, for procurement of raw material and components for their manufacturing process. In this paper we develop a Mixed Integer Programming Model for Multi product-Multi supplier -Multi period inventory lot size problem (MMMILP) with supplier selection. The solution of the optimal mathematical model is obtained in terms of determining (i) which products should be ordered to which supplier. (ii) The optimal quantity to be ordered and the time of placing orders. We introduce budget constraint, storage capacity constraint and quantity discount approach. A numerical case study is solve with Genetic algorithms.

**Keywords:** supplier selection, mixed integer programming, supply chain, genetic algorithm

## 1. Introduction

Supplier selection is defined as the process of finding the right suppliers, at the right price, at the right time, in the right quantities, and with the right quality Ayhan, (2013b). It is recorded that, 70% of total production cost is composed by the purchases of goods and services Ghodsypour & O'Brien, (1998). Hence, selecting the right supplier will result in reducing operational costs, increasing profitability and quality of products, improving competitiveness in the market and responding to customers' demands rapidly Abdollahi, Arvan, & Razmi, 2015; Onut, Kara, & Isık, (2009).

In the comparative market, it is necessary having a good production planning and replenishment control through effective inventory management. The single product, multi-period inventory lot-size problem is one of the most Common and basic problems. The present work considers an environment with multiple products-multiple periods and multiple suppliers.

This paper is based on the work of Basnet and Laung (2005) [1] which developed the multi-period inventory lot size with multiple products and multiple suppliers. Woarawichai, Kuruvit & Vashirawongpinyo (2012) has introduced storage capacity limitation with this model. In this paper a supplier selection with multi-period inventory lot size with multiple products and multiple suppliers under budget and storage capacity constraints and all unit quantity discount is developed using GA. Among various algorithms, GAs is develop by Wang, Yung, and Ip (2001), is most suitable for selecting best supplier combination and Hokey, Gengui, Mitsuo and Zhenyu (2005), suggested that GA is the best population-based heuristic algorithm, capable of generating a group of best solutions at once.

Lee *et al*. (2013) introduced a MIP model and genetic algorithm (GA) to solve the lot sizing problem with multiple suppliers. It incorporates the incremental and all-unit quantity discounts and is applicable to determine the replenishment strategy for a manufacturer for multi periods. But it is only suitable for single items.

The paper is organized as follows. Section 1 describes the introduction and literature review and section 2 introduces assumptions and notations and initial analysis of mathematical model has developed on section 3. In section 4 we define the problem with its mathematical formulation and numerical case study has discussed in section 5. Section 6 introduces solve the numerical case study by Genetic algorithm and LINGO. Finally, concluding remarks are given in Section 7.

**2. Assumption & Notation**

In this section, we introduce following the assumptions and notations

**Assumption**

1. Initial inventory of the first period and the inventory at the end of the last period are assumed to be zero.
2. Demand of products in each period is known over a planning horizon.
3. Transportation cost is supplier dependent, but does not depend on the variety and quantity of products involved.
4. Product needs a storage space and available total storage space is limited.
5. Shortage or backordering is not permitted.
6. Holding cost of product per period is product dependent.
7. Budget is fixed for each period.
8. All unit quantity discount is considered.

**Parameter**

$i = 1,\dots, I$ index of products
$j = 1,\dots, J$ index of suppliers
$t = 1,\dots,..T$ index of time periods
k = 1,……K index for shipment time instant
l = 0,1, … L index for Quantity discount break point

**Notations**

$d_{ij}$ = demand of product i in period t

$c_{ij}$ = purchase price of product i from supplier j

$H_i$ = holding cost of product i per period

$V_J$ = transportation cost for supplier j

$w_i$ = storage space product i

S = total storage capacity

$C_t$= budget available for each period

K= Set of shipment time instants & {k□K: k □□t) Set of shipping time instants lower than or equal to t.

$Q_{ijlt}$ = Quantity threshold beyond which a price break become valid at period t for product i from supplier j for lth price break.

$a_{ijlt}$ = Discount factor that is valid if more than $Q_{ijlt}$ unit are purchased, $0 < a_{ijlt} < 1$.

Decision variables:

$x_{ijt}$ = number of product i ordered from supplier j in period t

$Y_{jt}$ = 1 if an order is placed on supplier j in time period t, 0 otherwise

$Z_{ijlt}$ = 1 if $Q_{ijlt} \le x_{ijt} < Q_{ij(l+1)t}$, otherwise 0. This binary variable indicate that order size at period t is larger than $Q_{ijlt}$ then discounted prices for the ordered products.

**Intermediate variable**

$I_{it}$ = Inventory of product *i,* carried over from period *t* to period *t* + 1

**Quantity discount with price break**

As a marketing policy, Suppliers grant discounts to buyers who buy in quantity larger than of minimum acceptable order. We consider all-unit quantity discounts. An all-units quantity discount is a discount given on every unit that is purchased after the purchasing exceeds a given level (breakpoint).
As discussed above, variable specifies the fact that the order size at period t is larger than and therefore results in discounted prices for the ordered quantity and price breaks are defined as:

$$D(l) = \begin{cases} 1 & X_{ijt} < Q_{ij0t} \\ a_{ij1t} & Q_{ij0t} \le X_{ijt} < Q_{ij1t} \\ a_{ij2t} & Q_{ij1t} \le X_{ijt} < Q_{ij2t} \\ \dots\dots & \dots\dots\dots\dots\dots\dots\dots\dots \\ a_{ijLt} & Q_{ij(L-1)t} \le X_{ijt} \end{cases}$$

$$i = 1,2\dots\dots I;\ j = 1,2\dots\dots J;\ l = 0,1\dots\dots\dots L;\ t = 1,2\dots\dots\dots T$$

**3. Initial analysis of Mathematical Model**

All demands must be filled in the period in which they occur: shortage or backordering is not allowed.

$$I_{it} = \sum_{j=1}^{J}\sum_{k=1}^{t} X_{ijk} - \sum_{k=1}^{t} d_{ik} \ \ge 0 \ \forall\, i, j$$

There is not an order without charging an appropriate transaction cost.

$$Y_{jt}\left(\sum_{k=1}^{T} d_{ik}\right) - X_{ijt} \ \ge 0 \ \forall\, i, j \ \&\ t$$

Each product has limited capacity

$$\sum_{i=1}^{I} w_i\left(\sum_{j=1}^{J}\sum_{k=1}^{t} X_{ijk} - \sum_{k=1}^{t} d_{ik}\right) \le S \ \forall\, t$$

Budget is fixed of each product

$$\sum_{i=1}^{I}\sum_{j=1}^{J} X_{iJK} C_{ij} \ \le \mathrm{Ct}\ \forall\, t$$

$$X_{ijt} \ge \sum_{l=0}^{L} Q_{ijlt} Z_{ijlt} \quad \forall i, j, t$$

$$\sum_{l=0}^{L} Z_{ijlt} = 1 \quad \forall i, j, t$$

The objective function consists of three parts: the total cost (TC) of purchase cost of the products, transportation cost for the suppliers, and holding cost for remaining inventory in each period in $t$+1.

$$\sum_{l=1}^{L}\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{t=1}^{T} C_{ij} X_{ijt} Z_{ijlt} a_{ijlt} + \sum_{j=1}^{J}\sum_{t=1}^{T} V_j Y_{jt} + \sum_{i=1}^{I}\sum_{t=1}^{T} H_i \left[ \sum_{j=1}^{J}\sum_{k=1}^{t} X_{ijk} - \sum_{k=1}^{t} d_{ik} \right]$$

Decision variables

$Y_{jt} = 0\, and\, 1 \quad \forall i, j$ is binary variable 0 or 1

$X_{ijk} \geq 0 \quad \forall i, j, t$ is non-negative restrictions

$Z_{ijlt} = 0 \;\; and \;\; 1 \quad \forall i, j, i, t$ is binary variable 0 or 1.

### 4. Formulation of Mathematical model

The objective function of the Mathematical model

$$\sum_{l=1}^{L}\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{t=1}^{T} C_{ij} X_{ijt} Z_{ijlt} a_{ijlt} + \sum_{j=1}^{J}\sum_{t=1}^{T} V_j Y_{jt} + \sum_{i=1}^{I}\sum_{t=1}^{T} H_i \left[ \sum_{j=1}^{J}\sum_{k=1}^{t} X_{ijk} - \sum_{k=1}^{t} d_{ik} \right]$$

Subject to constraints

$$I_{it} = \sum_{j=1}^{J}\sum_{k=1}^{t} X_{ijk} - \sum_{k=1}^{t} d_{ik} \;\; \geq 0 \;\forall\, i, t$$

$$Y_{jt}\left(\sum_{k=t}^{T} d_{ik}\right) - X_{ijt} \;\geq\; 0 \;\forall\, i, j \;\&\; t$$

$$\sum_{i=1}^{I} w_i \left( \sum_{j=1}^{J}\sum_{k=1}^{t} X_{ijk} - \sum_{k=1}^{t} d_{ik} \right) \leq S \;\forall\, t$$

$$\sum_{i=1}^{I}\sum_{j=1}^{J} X_{iJK} C_{ij} \;\leq Ct \;\forall\, t$$

$$X_{ijt} \geq \sum_{l=0}^{L} Q_{ijlt} Z_{ijlt} \quad \forall i, j, t$$

$$\sum_{l=0}^{L} Z_{ijlt} = 1 \quad \forall i, j, t$$

Decision variables

$Y_{jt} = 0\, and\, 1 \quad \forall\, i, j$

$X_{ijk} \;\geq\; 0 \quad \forall\, i, j, t$

$Z_{ijlt} = 0 \;\; and \;\; 1 \quad \forall i, j, i, t$

### 5. Numerical case study

We consider a scenario with three products over a planning horizon of five periods whose requirements are as follows: demands of three products over a planning horizon of five periods are given in Table 1. There are three suppliers and their prices and transportation cost, holding cost and storage space are show in Table 2 and Table 3, respectively.

| **Product** | **Planning Horizons (five periods)** | | | | |
|---|---|---|---|---|---|
| | **1** | **2** | **3** | **4** | **5** |
| A | 12 | 15 | 17 | 20 | 13 |
| B | 20 | 21 | 22 | 23 | 24 |
| C | 20 | 19 | 18 | 17 | 16 |
| Budget | 1820 | 2000 | 3500 | 3000 | 3500 |

**Table 1:** Demands of three products over a planning horizon of five periods (*dit*) and budget for each period.

| **Products** | **Price** | | |
|---|---|---|---|
| | **X** | **Y** | **Z** |
| A | 30 | 33 | 32 |
| B | 32 | 35 | 30 |
| C | 45 | 43 | 45 |
| Transportation Cost | 110 | 80 | 102 |

**Table 2:** Price of three products by each of three suppliers X, Y, Z (C*ij*) and transportation cost of them (*Vj*).

| **Product / Cost** | **A** | **B** | **C** |
|---|---|---|---|
| Holding Cost | 1 | 2 | 3 |
| Storage Space | 10 | 40 | 50 |

**Table 3:** Holding cost of three products A, B, C (*Hi*) and storage space of item (*wi*).

All unit quantity discount as follow:

$$D(l) = \begin{cases} 1 & X_{ijt} < 15 \\ 0.15 & 15 \leq X_{ijt} < 35 \\ 0.20 & 35 \leq X_{ijt} < 55 \end{cases}$$

### 6. Solution of Numerical case study

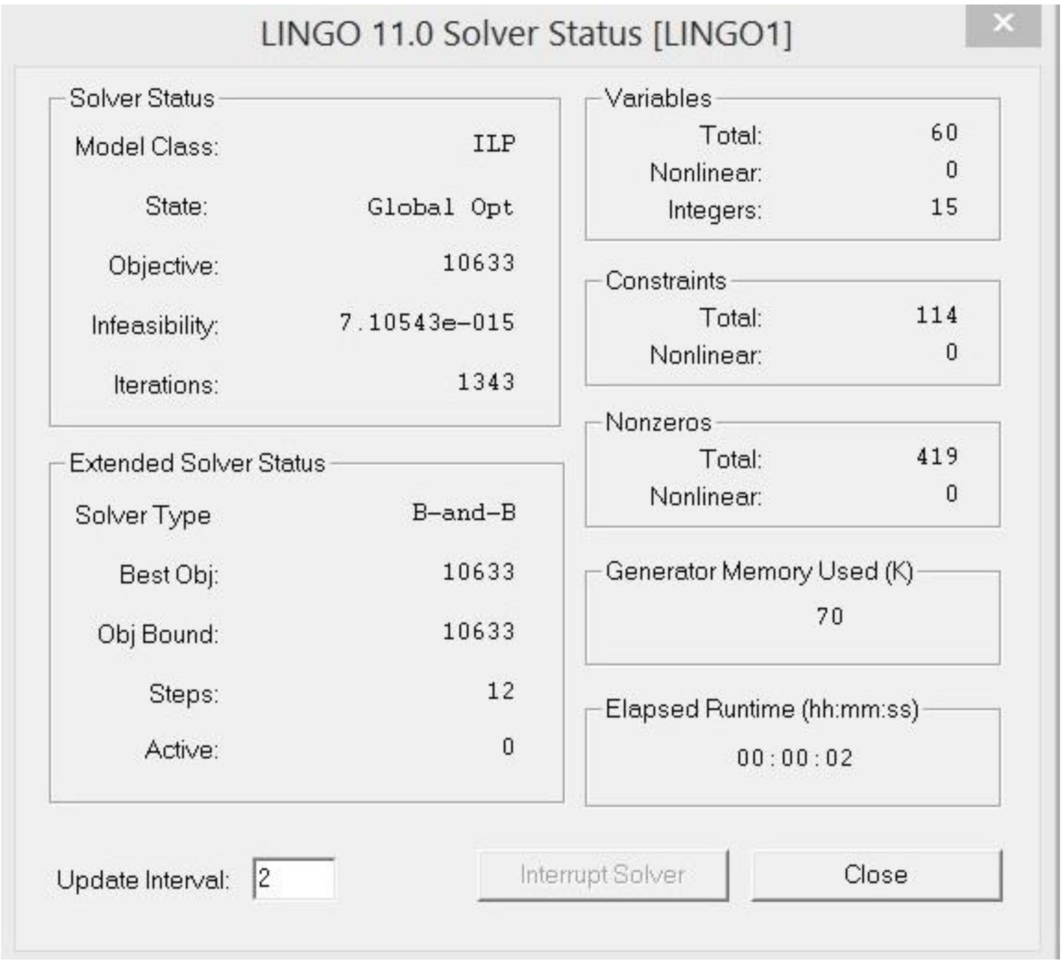
We applied Lingo and GAs approach to solve this numerical case study. We use LINGO 11.0 and MATLAB R2013 software and experiments are conducted on a personal computer equipped with an Intel Core 2 duo 2.00GHz, CPU speeds, and 2 GB of RAM.

Lingo is Optimization Modeling Software for Linear, Nonlinear, and Integer Programming. We use this software for solution of numerical case study. We find Lingo result of this case study in the following table.

**Table 4:** Order of three products over a planning horizon of five periods ($X_{ijt}$)

| **Products** | **Planning Horizon (Five Periods)** | | | | |
|---|---|---|---|---|---|
| | **1** | **2** | **3** | **4** | **5** |
| A | $X_{111} = 12$ | $X_{132} = 15$ | $X_{113} = 37$ | - | $X_{135} = 13$ |
| B | $X_{231} = 20$ | $X_{222} = 21$ | $X_{213} = 22$ | $X_{234} = 23$ | $X_{235} = 24$ |
| C | $X_{321} = 20$ | $X_{332} = 19$ | $X_{313} = 18$ | $X_{334} = 17$ | $X_{335} = 16$ |

| | | | | | |
|---|---|---|---|---|---|
| Global optimal solution found. | | | Infeasibilities | : | 0.7105427E-14 |
| Objective value | : | 10633.00 | Extended solver steps | : | 12 |
| Objective bound | : | 10633.00 | Total solver iterations | : | 1343 |




Genetic Algorithm approach is based on a natural selection process that mimics biological evolution. It belongs to the larger class of evolutionary algorithms (EA). Gas code is developed in MATLAB. The transportation costs are generated from *int* [50; 200], a uniform integer distribution including 50 and 200. The prices are from *int* [20; 50], the holding costs from *int* [1; 5], the storage space from *int* [10; 50], the quantity discount for products are from *int* [15; 55] and the demands are from *int* [10; 200]. A problem size of *I; J; T* indicates number of suppliers = *I*, number of products = *J*, and number of periods = *T*. Computation time limit is set at 120 minutes. We obtain same result as LINGO and solution time is 0.01 minutes.

## 7. Conclusion

In this paper, we present genetic algorithms (GAs) and LINGO applied to the multi-product, multi supplier and multi-period inventory lot-sizing problem with supplier selection under budget constraint and all unit quantity discount with maximum storage space for the decision maker in each period is considered. The decision maker needs to determine what products to order in what quantities with which suppliers in which periods. The mathematical model is formulated as a mixed integer programming and illustrated though a numerical case study. The numerical case study is solved with LINGO and the GAs. GAs provides better solution than LINGO that are close to Optimum in a very short time.

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