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## Bayesian analysis of dynamic correlation -multivariate stochastic volatility (DC-MSV) model

Lameck Ondieki Agasa, George Otieno Orwa and Joel Kibiwot Koima

### Abstract

The forex exchange rate is the most volatile aspect in financial studies. This study uses a bayesian approach to estimate dynamic correlation multivariate stochastic volatility, a case of the Kenya stocks. Data was obtained from the Central Bank of Kenya website depicting the daily exchange rates for a period of 12 years (2003-2015). Multivariate Stochastic Volatility (MSV) model was fitted and its residuals exhibited volatility clustering hence the use Dynamic Correlation Multivariate Stochastic Volatility (DC-MSV) was applied to address these characteristics. Using Akaike Information Criterion (AIC) and Deviance Information Criterion (DIC) found that the returns are leptokurtic and have fat tails. The study estimated posterior parameters of the model by using of Markovian chain Monte Carlo (MCMC) iterations that worked well. Empirical results suggest that the best specifications are those that allow for time-varying correlation coefficients. The dynamic correlation multivariate stochastic volatility was found ideal in estimating volatility in stock exchanges.

**Keywords:** Volatility, Bayesian analysis, dynamic correlation, multivariate stochastic, Markovian chain Monte Carlo

### 1. Introduction

Multivariate volatility models have proved to be a powerful inferential tool (William J. McCausland \*Universit'e de Montr'eal, CIREQ and CIRANO, 2012). They are characterized by different types of dynamic cross-sectional dependence among multiple assets returns.

Research has revealed that asset return volatility varies over time, changing in response to views and revised expectations of future performance. It tends to cluster, so that large price changes tend to be followed by other large changes. Volatility is not independent across markets and assets, and this cross-sectional dependence is time-varying. Cross-sectional correlations increase substantially in periods of high market volatility, especially in bear markets. The distribution of returns is heavy tailed. There is an asymmetric relation between price and volatility changes known as the "leverage effect": increases in volatility are associated more with large decreases in price than with large increases.

Multivariate volatility models that can capture these stylized facts are in high demand in finance given their many important applications, especially in modern portfolio management and exchange rate forecasting. Learning about the joint distribution of asset returns is a key element for the construction, diversification, and evaluation and hedging of portfolios. Accurate estimation of the covariance matrix of multiple asset returns allows the investor to timely identify opportunities or risks associated with a particular portfolio. It is important to track changes in Correlations to assess the risk of a portfolio, especially during periods of market stress. Financial crises usually have a strong impact on correlation. Markets tend to behave as one during big crashes — as the risk of some assets increases, investors wish to sell other risky investments. The result is more highly correlated returns. The pessimistic conclusion is that diversification is least effective at reducing risk at the very times when risk is highest. An awareness of this fact avoids false optimism. Exchange rate influences, interest rate charged, inflation, imports and exports. Kenya is among many countries that embrace the free floating exchange rate system where demand and supply forces of market determine the daily value of one currency against another. The exchange rate affects the prices of directly imported goods as well as and indirectly through the local goods that are under competitive

pressure from imported goods. Thus there are high rates of volatility exhibited by exchange rates. An investor with different asset returns will wish to look unto volatilities in the market before investing.

## 2. Literature Review

Volatility modeling is understood to have a constant volatility thus a poor fit of time series data while dynamic structures have provided a more realistic approach to variance modelling of returns.

Several approaches have been advanced to model time – varying data. There are two main types of multivariate volatility models: observation-driven and parameter-driven. In observation-driven models, volatility is a deterministic function of observed variables, which allows straightforward evaluation of the likelihood function like multivariate GARCH type models, which are observation driven, Bauwens, Laurent, and Rombouts (2006) <sup>[4]</sup>. This study focuses on parameter-driven volatility models, also called stochastic volatility (SV) models; volatility is a latent stochastic process. Jacquier, Polson, and Rossi (1994) <sup>[14]</sup> and Geweke (1994) <sup>[10]</sup> give evidence suggesting that Stochastic Volatility (SV) models are more realistic. Stochastic processes differ with GARCH type models in terms of the structure of the model. The stochastic process depends on previous realized stocks in the series while GARCH assumes that it follows its own stochastic process with potential dependence.

Bates (1996) give an overview of how to estimate the stochastic volatility models which the research will use to model the multivariate stochastic process as developed by (Harvey *et al*, 1994) <sup>[13]</sup>, who came up with multivariate stochastic models that later led to development of multifactor models for multivariate series (Hopes 1999, Aqillar and West 2000, and Chib *Et al* 2002 <sup>[1-8]</sup>). Engle (1995) <sup>[9]</sup> and Shephard (2005) <sup>[25]</sup> comprehensively reviewed the stochastic models and GARCH. Meyer and Yu (2006) <sup>[27, 28]</sup> realized that SV models are superior to GARCH type models in time series modelling. Aydemi (1998) observed that we often observe movements between markets or stocks or sectors in the exchange rates. He attributed this to a set of assets being formally linked together due to being influenced by common unobserved factors.

A study by Ntawihebasenga, Mwita, P. and Mungatu, J. (2014) on modeling volatility in exchange rate used Generalized Autoregressive Conditional Heteroscedasticity(GARCH) approach to model volatility in Rwanda Exchange rate returns. The Autoregressive (AR) model with GARCH errors was fitted to the daily exchange rate returns using Quasi-Maximum Likelihood Estimation (Q-MLE) method to get the current volatility. Asymptotic consistency and asymptotic normality of estimated parameters were given. The study showed the model fits Rwanda exchange rate returns data well.

A study by Mumo, P.M. (2016) on effects of macroeconomic volatility on stock prices in Kenya. The study exploited the presence of unit roots of order 1(1) on the data set to apply the Johansen procedure and the Vector Error Correction Model (VECM) for data analysis. It was realized that both a long-run equilibrium relationship between stock prices and the macroeconomic variables and between inflation and other macro variables. The results suggested a negative long-run equilibrium relationship between money supply and stock prices. Inflation shows negative but insignificant relationship. Exchange rates and interest rates show a positive relationship. The short-term dynamics from the VECM support earlier documented evidence, implying the earlier evidence reflect short-run and not long-run dynamics.

There are also more natural discrete time representations of the continuous time models upon which much of modern finance theory is based. Unfortunately, computation of the likelihood function, which amounts to integrating out latent states, is difficult. However, since the introduction of Bayesian Markov chain Monte Carlo (MCMC) methods by Jacquier, Polson, and Rossi (1994) <sup>[14]</sup> for univariate SV models, inference for these models has become much more feasible. These methods require the evaluation of the joint density of returns, states and parameters, which is straightforward. In addition, simulation methods for Bayesian inference make exact finite sample inference possible.

Tabak and Guerra (2002) examined the relationship between stock returns and volatility over the period of June 1990 to April 2002. The relationship between stock returns and volatility is tested using seemingly unrelated regressions methods and AR (1)-GARCH (1, 1) estimation. They conclude that using both a seemingly unrelated regressions (SUR) methodology and an AR (1)-GARCH (1, 1) estimation changes in volatility are negatively related to stock returns. Assuming that  $N \times 1$  vectors of returns  $y_t = (y_{1t}, \dots, y_{Nt})$ ,  $t=1, \dots, T$  with corresponding covariance matrix  $E_t$ . Further letting  $r_t$  second order stationary moments  $E(E_t) = E$  exists. Efforts by various scholars to unveil the parsimonious structures that capture dynamic behavior of  $E_t$ , Meyer and Yu (2006) <sup>[27, 28]</sup> and Asai, McAleer and Yu (2006) <sup>[3]</sup> attempted to give review on the parsimonious structure.

Meyer and Yu (2000) illustrated the ease of implementing Bayesian estimation of univariate SV models based on purpose-built MCMC software called BUGS (Bayesian analysis using Gibbs sampler) developed by Spiegelhalter *et al.* (1996) <sup>[26]</sup>. Univariate SV models have been used in a number of studies (Berg *et al.*, 2004; Lancaster, 2004; Meyer *et al.*, 2003; Seljuk, 2004; Yu, 2005) <sup>[5, 24]</sup>. Furthermore, Berg *et al.* (2004) <sup>[5]</sup> showed that model selection of alternative univariate SV models is easily performed using the deviance information criterion (DIC), which is computed by BUGS. Its from univariate SV models, that we build multivariate stochastic volatility models. However according Chanet *et al.*, 2005 multivariate stochastic volatility models still pose significant computational challenges to applied researchers.

Bayesian statistics has proved to be of vital interest in the modern statistics. It's due to this that Bayesian inference is regarded superior in drawing conclusion on a phenomenon. Scenarios arise when the posterior distribution is intractable i.e  $\pi(y/\theta)$  or  $\pi(\theta|f(y/\theta))$  cannot be evaluated point wise. Intractability arises due to emergence of intractable likelihood given by  $f(y/\theta)$  which cannot be evaluated (Everett, Johasem, Rowing and Hogan, 2015).

The use of big data sets where  $f(y/\theta)$  consists of a product of large number of terms. Existence of a large number of latent variables such that  $f(y/\theta)$  is a high dimensional integral  $f(y/\theta)$ . The difficulties in parameter estimation due to intractable likelihood can be overcome by using Bayesian approach via Markov Chain Monte Carlo (MCMC) methods (Meyer and Yu (2000).

### 3. Model Development

The research questions were answered by utilizing the data of daily average exchange rate of Kenya Shilling versus US dollar, Japanese Yen, SA Rand, Sterling pound and Euro. There were 2750 trading days.

According to Yu and Meyer, 2006<sup>[27, 28]</sup> they introduced dynamic-MSV as:

Let the observed (mean-centered) log-returns at time to be denoted by  $(y_{1t}, y_{2t})$  for  $t=1, \dots, T$ . let

$$\Sigma_t = (\Sigma_{1t}, \Sigma_{2t}), \Omega_t = (\Omega_{1t}, \Omega_{2t}), \mu = (\mu_1, \mu_2), h_t = (h_{1,t}, h_{2,t}), \Omega_t = \text{diag}(\exp(\frac{h_t}{2}))$$

$$\Phi = \begin{pmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{pmatrix}, \Sigma_t = \begin{pmatrix} 1 & \rho_\epsilon \\ \rho_\epsilon & 1 \end{pmatrix},$$

$$\Sigma_n = \begin{pmatrix} \sigma_{n_1}^2 & \rho_n \sigma_{n_1} \sigma_{n_2} \\ \rho_n \sigma_{n_2} \sigma_{n_1} & \sigma_{n_2}^2 \end{pmatrix}$$

Letting Basic MSV model

$$y_t = \Omega_t \Sigma_t \text{ in } \Sigma_t \xrightarrow{\text{yields}} N(0,1)$$

$$h_{t+1} = \mu + \text{diag}(\Phi_{11}, \Phi_{22})(h_t - \mu) + \Omega_t$$

$\Omega_t \xrightarrow{\text{yields}} N(0, \text{diag}(\sigma_{n_1}^2, \sigma_{n_2}^2))$  with  $h=0$ . This model is equivalent to stacking two basic univariate SV models together. Clearly, this specification does not allow for correlation across the returns or across the volatilities. However, it does allow for leptokurtic return distributions and volatility clustering.

Developing the model from Meyer and Yu (2005) Dynamic correlation-MSV or (DC-MSV)

$$Y_t = \Omega_t \Sigma_t, \Sigma_t / \Omega_t \xrightarrow{\text{yields}} N(0, \Sigma_{\Sigma_t})$$

$$\Sigma_{t,t} = \begin{pmatrix} 1 & P_t \\ P_t & 1 \end{pmatrix}$$

$$h_{t+1} = \mu + \text{diag}(\Phi_{11}, \Phi_{22})(h_t - \mu) + \Omega_t \Omega_t \xrightarrow{\text{yields}} N(0, \text{diag}(\sigma_{n_1}^2, \sigma_{n_2}^2))$$

$$q_{t+1} = \varphi_0 + \varphi(q_t - \varphi_0) + \sigma_p V_t V_t \xrightarrow{\text{yields}} N(0, 1)$$

$$R_t = \frac{\exp(q_t) - 1}{\exp(q_t) + 1}$$

Letting  $h_0 = \mu, q_0 = \varphi_0$

This model has time varying volatilities and correlation coefficients.

It has to be bounded by -1 and 1 for  $\Sigma_+$  to be a well-defined correlation matrix. This is achieved by using Fisher transformation (Tsay 2002), Christodoulakis and Satchell (2002)

In higher dimensional cases, generating the model is not easy.

To allow for time-varying correlations in an N-dimensional setting with  $N > 2$ , one can follow Engle (2002) by constructing a sequence of matrices  $\{Q_t\}$  according to

$$Q_{t+1} = S + B_o(Q_t - S) + A_o(V_t V_t^1 - S)$$

$$= (U^1 - A - B) \circ S + B \circ Q_t + A \circ V_t V_t^1$$

$V_t \sim N(0, 1), *1$  is a vector of ones and  $\circ$  is the Hadamard product. According to Ding and Engle (2001) and Engle (2003), as long as A, B and  $U^1 - A - B$  are positive semidefinite,  $Q_t$  will be positive semidefinite. As a result we obtain  $Q^{-1}$  and its choleski decomposition  $Q_t^{-\frac{1}{2}}$  (defined by  $Q_t^{-\frac{1}{2}}(Q_t^{-\frac{1}{2}}) = Q_t^{-1}$ ).

Finally, a sequence of covariance matrices for  $\Sigma_t$  is constructed according to

$$\Sigma_{\epsilon,t} = \text{diag}(Q^{-\frac{1}{2}}) Q_t \text{diag}(Q_t^{-\frac{1}{2}}).$$

By construction of all elements in  $\Sigma_{\epsilon,t}$  are bounded -1 and 1 and all the main diagonal elements in  $\Sigma_{\epsilon,t}$  is positive semidefinite. As a result,  $\Sigma_{\epsilon}$  is well defined correlation matrix. It seems that the specification of time varying correlation is convoluted in this model. Also, it is not easy to interpret  $Q_t$ . An alternative way of specifying dynamic correlation is an Asai and Mc Aleer (2005).

#### 3.1 Bayesian Approach on the model

The dynamic correlation-Multivariate Stochastic volatility model is complete when we specify a prior distribution for all unknown parameters  $b = (b_1 \dots b_p)$ . Bayesian inference was drawn based on the joint posterior distribution of all unobserved quantities  $\theta$  in the model. The vector comprises the unknown parameters and the vector log-volatilities (Meyer and Yu, 2006)<sup>[27, 28]</sup>. The vector  $\theta$  consists of unknown parameter s and vectors of volatilities

$$\Theta = (b, h_1, \dots, h_T)$$

Letting  $p(\cdot)$  denote the probability density function of a random variable. We use independent priors for the parameters and successive conditioning on the sequence of latent states, the joint prior.

Density of  $\theta$  is estimated as shown below. Building our case from basic MSV model where  $p=6$  and vector  $b=(\mu_1, \mu_2, \phi_{11}, \phi_{22}, \sigma_{n_1}^2, \sigma_{n_2}^2)$

$$P(a)p(h_0) \prod_{t=1}^T p(h_t/b)$$

$$p(\mu_1)p(\mu_2)p(\phi_{11})p(\phi_{22})p(\sigma_{n_1}^2)p(\sigma_{n_2}^2)p(h_0) \prod_{t=1}^T p(h_t/b)$$

joint posterior density is updated to joint posterior density of unknown quantities  $p(\theta/y)(y=(y_1, \dots, y_T))$ . Following Bayes theorem, we multiply  $p(\theta)$  and the likelihood  $p(y/\theta)$

$$p(\theta/y) \propto p(\theta)p(y/\theta) \propto p(h_0) \prod_{t=1}^T p(h_t/b) \prod_{t=1}^T p(h_t/y_t)$$

The marginal posterior distribution is calculated for the parameters of interest  $p(b/y)$  that requires  $(P+2T)$ -dimensional integration to find the normalization constant.

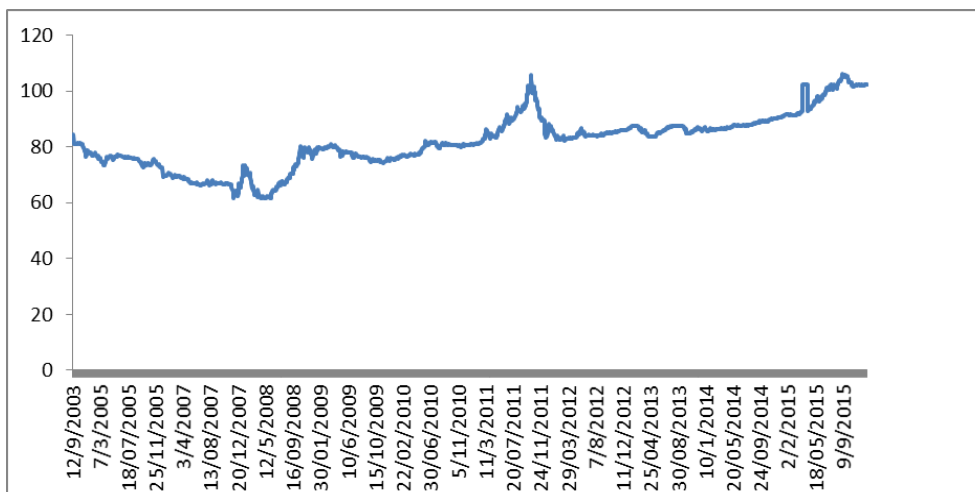
$$\text{It follows then: } p(b/y) = \int_{h_1} \dots \int_{h_T} p(b, h_1, \dots, h_T) dh_T \dots dh_1$$

Since the equation is of a high-dimensional distribution it is usually not possible to integrate it (Liesenfeld and Richard, 2003 and Durham, 2005) [18]. Yu and Meyer, 2006 [27, 28] suggested the use of MCMC techniques to overcome this problem by constructing a Markov chain with stationary distribution equal to the target density  $p(\theta / y)$  and simulate from this Markov chain. The study utilized the Markov chain that run long enough to reach equilibrium, the samples in each iteration were regarded (dependent) samples from  $p(\theta / y)$ . By the ergodic theorem, sample averages are still consistent estimates of the population quantities.

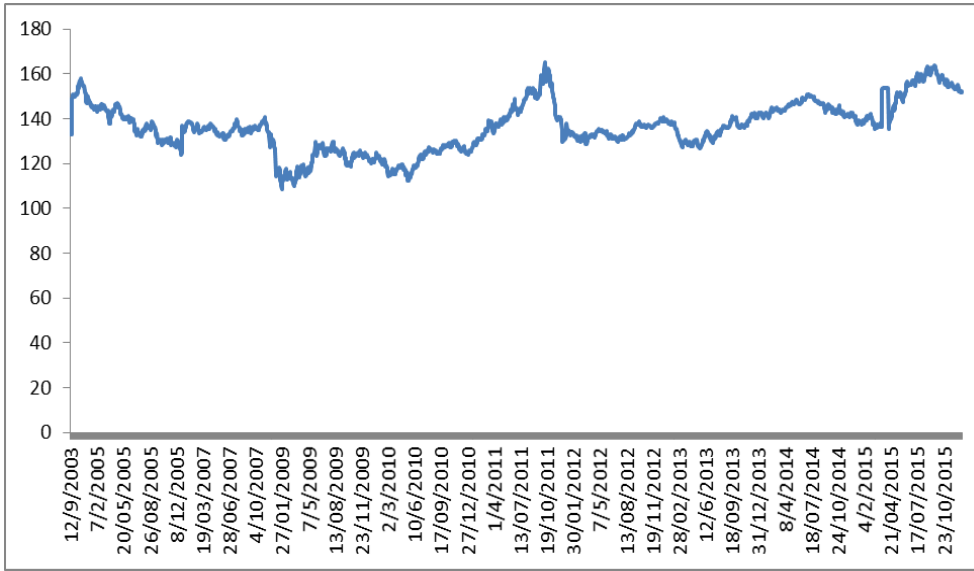
Meyer and Yu (2000) described the use of BUGS for Bayesian posterior computation in univariate SV models and emphasized the ease with which BUGS can be used for the exploratory phase of model building, as any modifications of a model, including changes of priors and sampling error distributions, are readily realized with only minor changes of the code. A BUG automates the calculation of the full conditional posterior distributions that are needed for Gibbs sampling using a model representation by directed acyclic graphs. It contains an expert system for choosing an effective sampling method for each full conditional

**4. Results and Discussion**

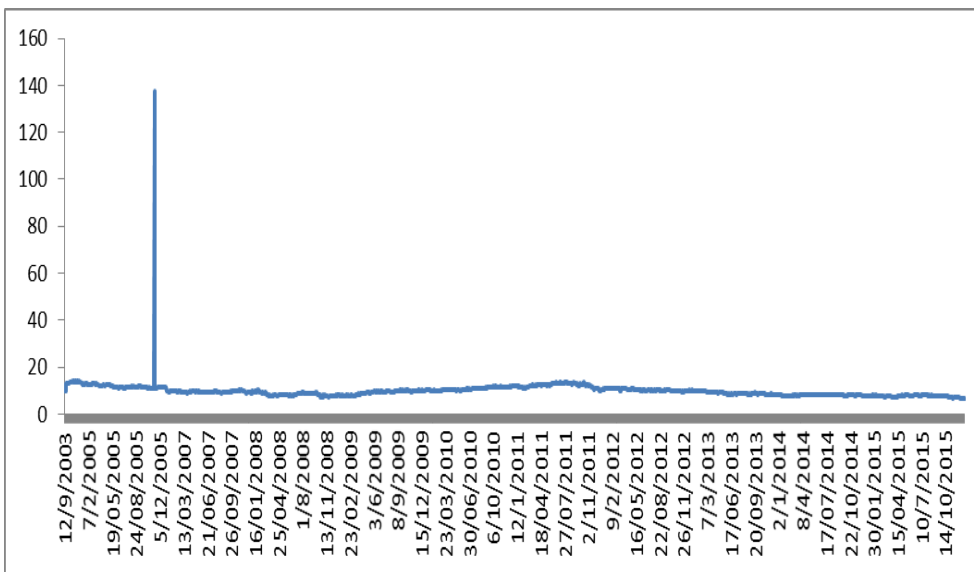
The study utilized data consisting of daily exchange rates as they trade off in Nairobi Stock Exchange (NSE). The Kenya shilling was compared against the US Dollar (USD), Euro, South Africa Rand (SA Rand) Sterling pound and Japanese Yen (100). According to Central Bank of Kenya the daily exchange rate is considered to be an average of buying and selling rates of commercial banks spot exchange rates as traded in stock exchange before close of business. In computation of the exchange rates, we base the computation on foreign exchange transactions of Kenyan commercial banks which participate in the foreign exchange market. The sample runs from September 1, 2003 until December 22, 2015.



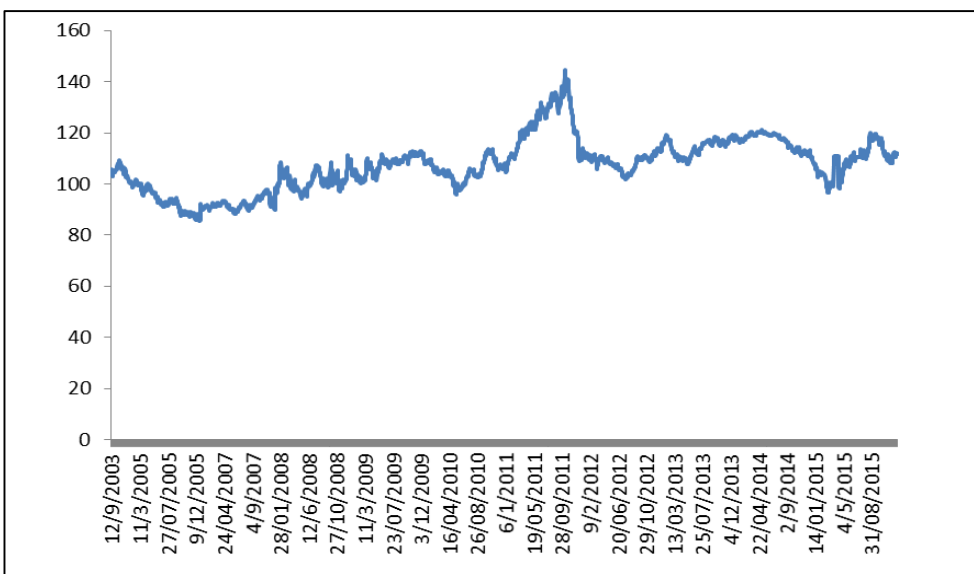
**Fig 1:** A time Series Graph showing the exchange rate of USD/Kes



**Fig 2:** A time Series Graph showing the exchange rate of Sterling pound/Kes



**Fig 3:** A time Series Graph showing the exchange rate of SA Rand/KES



**Fig 4:** A time Series Graph Showing the Exchange Rate of Euro/Kes

Figure 1, 2, 3 and 4 shows the volatility of Kenyan shilling against other currencies from 2003-2015. Its clear that the Kenyan shilling has not been stable for that period.

**Prior Distributions:** The model parameters were given as: Parameter in the mean equation ( $\rho_\varepsilon, v$ ), in the variance equation ( $\phi_{11}, \phi_{22}, \phi_{21}, \mu_1, \mu_2, \sigma_{\eta 1}, \sigma_{\eta 2}$ ) and in the correlation equation ( $\psi_0, \psi_1, \sigma_p$ ). we assumed the parameters are mutually independent. The prior distribution is specified:

- $\rho_\varepsilon \sim U(-1, 1)$
- $v^* \sim \chi^2_{(4)}$  where  $v^* = v/2$
- $\mu_1 \sim N(0, 30)$
- $\mu_2 \sim N(0, 30)$
- $\phi_{11} \sim \text{beta}(20, 1.5)$  where  $\phi_{11} = (\phi_{11} + 1)/2$
- $\phi_{22} \sim \text{beta}(20, 1.5)$  where  $\phi_{22} = (\phi_{22} + 1)/2$
- $\phi_{21} \sim U(0, 10)$
- $\rho_{\eta} \sim U(-1, 1)$
- $\sigma_{\eta 1} \sim \text{inverse gamma}(3.0, 0.030)$
- $\sigma_{\eta 2} \sim \text{inverse gamma}(3.0, 0.030)$
- $\psi_1 \sim \text{beta}(20, 1.5)$  where  $\psi_1 = (\psi_1 + 1)/2$
- $\psi_0 \sim N(0.7, 10)$
- $\sigma_p^2 \sim \text{inverse-gamma}(3.0, 0.03)$

**Table 1:** Mean and standard deviation of prior distributions for parameters

Parameter	$\rho_\varepsilon$	$v$	$\phi_{11}$	$\phi_{22}$	$\phi_{21}$	$\mu_1$	$\mu_2$	$\rho_{\eta}$	$\sigma_{\eta 1}$	$\sigma_{\eta 2}$	$\psi_1$	$\psi_0$	$\sigma_p^2$
Prior mean	0	12	0.76	0.76	0	0	0	0	0.12	0.12	0.8	0.76	0.12
Prior SD	.76	6	.14	.14	0.33	5	5	0.76	0.05	0.05	3.4	0.11	0.05

**Table 2:** Posterior quantities for parameters

parameter	$\mu_1$	$\mu_2$	$\phi_{11}$	$\phi_{22}$	$\psi$	$\psi_0$	$\sigma_p^2$
mean	0.0123	-0.76	0.98	0.99	0.98	1.945	0.1124
SD	2.262	0.35	0.0135	0.0052	0.0122	0.281	0.31
95%CI	-6.50, 2.45	-1.45, -0.101	0.944, 0.996	0.980, 0.999	0.950, 0.997	1.39, 2.52	0.065, 0.189

The correlation process is highly persistent with a posterior mean of  $\psi$  being 0.98. This signifies the importance of time varying correlation. Parameters  $\phi_{11}, \phi_{22}, \psi, \psi_0, \sigma_p^2$  have positive posterior means with the lower limit of the 95% posterior credibility interval being greater than zero. This suggests that the volatility of Kenyan shilling causes volatility in dollar, EURO and Yen consistent with the expectation.

**Table 3:** DCI for the model

Value	$D^-$	$p_D$
2756.03	2741.2	53.52

DIC indicates an almost trivial to compute and particularly suited to compare Bayesian models when posterior distributions have been obtained using MCMC simulation. In the table above its clear that the data fits to the model well.

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