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A study of mathematical modeling and predicting the current trends of human population growth in India

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Abstract
The Logistic Growth Model is most effective and plausible tool for data analysis in mathematical modeling, an advanced field of mathematical science. Using this model the purpose of my study is focused on exploring and determining the predict of population growth of India since 2021-2051.

Keywords: Logistic growth model, growth rate, population size

Introduction
Mathematics has an important role in real life [3]. It is an exact science that becomes a basis for other sciences and correlate themselves [1]. One of the benefits of differential equations in the field of demography is to calculate the population in a particular area. So that, need to be tested to see the accuracy of the results [5, 8]. Benefits Logistics model is usually used to predict human population [9], with the projection (prediction) of the population useful in decision-making for socio-economic and demographic development [4]. Previous research describes that predator and prey logistic growth models can be used as a solution population model equilibrium [6], the population model can be modeled using the Verhulst model and its limits to determine unlimited population growth [7], as an alternative computationally reliable for problem population solving [2].

In this study the logistic growth model was used in modeling the India population growth, employing the data during 1991 – 2011. From the study, it is calculated that the carrying capacity for the population of India is $2051 \cdot 06 \times 10^{6}$ and the growth rate is $3 \cdot 57\%$ per annum. The study was presented to explore and apply logistic equation as a mathematical model.

Mathematical Model
The population growth not only depend how to increase of our population size but also on how this size is far from its upper limit.

Let $N(t)$ be the population for size at time t. Then the population growth model will be given by

$\frac{dN}{dt} \propto \mathrm{N(t)}\left(1-\frac{\mathrm{N}}{\mathrm{n}}\right)$,$\mathrm{N(o)} = \mathrm{No}$

$$\frac{dN}{dt} = r(t)N(t)\left(1-\frac{N(t)}{n}\right)$$

Where $r(t) > 0$ is the growth rate $n > ois$ the maximum susistainable population.

$$\frac{dN}{N-\frac{1}{n}N^2} = r\,dt$$

$$\frac{dN}{N\left(1-\frac{1}{n}N\right)} = r\,dt$$

$$\int_{N}^{N(t)} \frac{dN}{N\left(1-\frac{1}{n}N\right)} = \int_{t0}^{t} r\,dt$$

$$\int_{0}^{t} r\,dt = \int_{N0}^{N(t)} \left[\frac{1}{N} + \frac{\frac{1}{n}}{1-\frac{1}{n}N}\right] dt$$

$$t = \left[\lg N - \lg\left(1-\frac{1}{n}N\right)\right]_{N0}^{N}$$

$$rt = \left[\lg\left(1-\frac{1}{n}N\right)\right]_{N0}^{N}$$

$$e^{rt} = \left(\frac{N}{1-\frac{N}{n}}\right)\frac{\left(1-\frac{No}{n}\right)}{No}$$

$$e^{rt} = \frac{N}{No}\frac{\left(1-\frac{No}{n}\right)}{\left(1-\frac{N}{n}\right)}$$

$$= \frac{N}{No}\frac{(n-No)}{(n-N)}$$

$$\left(\frac{n-N}{N}\right) = \left(\frac{n-No}{No}\right)e^{-rt}$$

$$\frac{n}{N} - 1 = \left(\frac{n-No}{No}\right)e^{-rt}$$

$$\frac{n}{N} = 1 + \left(\frac{n}{No} - 1\right)e^{-rt}$$

$$N = \frac{n}{1+\left(\frac{n}{No}-1\right)e^{-rt}}$$

$$N(t) = \frac{n}{1+\left(\frac{n}{No}-1\right)e^{-rt}}$$

$$\lim_{t\to\infty} N(t) = \lim_{t\to\infty} \frac{n}{1+\left(\frac{n}{No}-1\right)e^{-rt}} = n$$

$$N_{max} = \lim_{t\to\infty} N(t) = n$$

t	0	T	$2T$	$3T$	$4T$	$5T$
$N(t)$	N_0	N_T	N_{2T}	N_{3T}	N_{4T}	N_{5T}

$$e^{-rT} = \frac{N_0}{N_T}\frac{(N_{2T}-N_T)}{(N_T-N_0)}$$

$$r = \frac{1}{T}\ln\frac{N_{2T}}{N_0}\frac{(N_T-N_0)}{(N_{2T}-N_T)}$$

$$n = \frac{N_T(N_0N_T - 2N_0N_{2T} + N_TN_{2T})}{N_T^2 - N_0N_{2T}} = N_{max}$$

$$\frac{d^2N}{dt^2} = n\frac{dN}{dt}\left(1-\frac{2N}{n}\right)$$

$\frac{d^2N}{dt^2} = 0$, for $n = \frac{n}{2}$ and implies $t = \frac{1}{n}\ln\left(\frac{n}{N_0} - 1\right)$

$\frac{d^2N}{dt^2} > 0$for $N < \frac{n}{2}$

$\frac{d^2N}{dt^2} < 0$for $N < \frac{n}{2}$

In other words $N(t)$ increases at increasing rate when $N(t) < \frac{N}{2}$ and it is morcase but at dicrasing rate when $N(t) > \frac{N}{2}$ and here in a part of inflection when $N(t) = \frac{N}{2}.t = \frac{1}{r}\ln\left(\frac{n}{N_0} - 1\right)$

Analysis

The data [10] consider for the study based on the population (10) from 1991 to 2011 as shown in table [1].

Table 1: Population of India

Year	Actual Population (Million)	$N(t)$
1991	$846 \cdot 6$	N_0
2001	$1028 \cdot 7$	N_T
2011	$1210 \cdot 8$	N_{2T}

Table 2: Logistic Model Parameters

r	$\cdot 0357808$
n	$2051 \cdot 06 \times 10^6$

$$N(t) = \frac{2051 \cdot 06}{1 + (1 \cdot 423)(0 \cdot 6982)^t}$$

Using mathematical Logistic model, be population of India was projected for 2021 and 2051. The projections are shown in table [3] followed by a graphical representation in figure 1.

Table 3: Estimated Population of India

Year	Actual Population of India (Million)	Projected Population (million)
1991	$846 \cdot 6$	$846 \cdot 446$
2001	$1028 \cdot 7$	$1027 \cdot 445$
2011	$1210 \cdot 8$	$1211 \cdot 065$
2021		$1381 \cdot 836$
2031		$1607 \cdot 548$
2041		$1659 \cdot 299$
2051		$1760 \cdot 808$

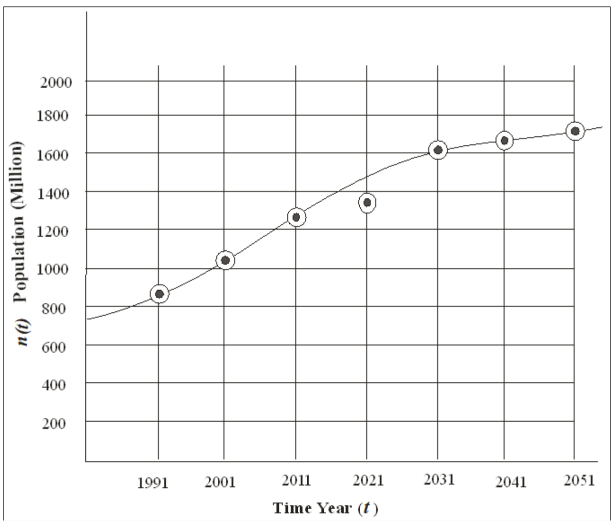

Fig 1: Estimated Population of India

Conclusion

By mathematical analysis using Logistic Growth Model, I have tried to calculate the population of India since 2021-

2051. I have found that this model predicts a carrying capacity for the population of India to be $2051 \cdot 06 \times 10^6$ with growth rate approximately $3 \cdot 57\%$ per annum. Thus, this study provides a significant key role for prediction of the future population size of any country. Results obtained in mathematical analysis showed that the logistic equation can also be used to predict some business and economic system as well as describing the effects of measuring employees working in different public sectors as problems associated with the areas of concern.

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