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# A time series evaluation of the asymmetric nature of heteroscedasticity: an EGARCH approach

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**Abstract**
Symmetric GARCH models have provided a rich and useful approach to capturing the conditional variances in stock returns, thereby filling the gap created by the inability of ARIMA models to account for the presence of volatility clustering that leads to the violation of assumption of constant variance. However, symmetric GARCH models are deficient in capturing the leverage effects and their parameters are restricted to ensure that the conditional variance is positive. In order to salvage these deficiencies in the symmetric GARCH model, we employed the Exponential GARCH (EGARCH) model that can capture the asymmetry property of the stock returns (leverage effects) and remove restrictions on parameters by introducing natural logarithm to the conditional variance. We considered the time series on the closing share prices obtained from the Nigerian Stock Exchange on Zenith bank plc spanning from January 4, 2006 to December 30, 2015. ARIMA(1,0,3) model was fitted to the return series. Based on the residuals of the fitted model, the ARCH effect was detected and captured by GARCH(0,1) model while the leverage effect in the return series was captured by EGARCH(0,1) model. Further findings from the EGARCH (0,1) model revealed the asymmetric nature of heteroscedasticity as indicated by the significance of the negative coefficient of the asymmetric parameter. The implication of the findings is that an unexpected decrease in price increases the predictable volatility more than an unexpected increase in price of similar magnitude.

**Keywords:** ARCH Effects, GARCH model, Heteroscedasticity, Leverage Effects, Time series, Volatility.

## Introduction

Linear time series models are not good models for describing certain characteristics of a volatility series since it is assumed that linear dependence is present in such series (Akpan *et al*, 2016; Franses and Van Dijk, 2003) [3, 10]. Also, assumption of homoscedasticity is not appropriate when using financial data. For instasnce, returns typically exhibit linear dependence and as such the ARIMA models are natural candidates for modeling the linear dependence in financial data (Akpan *et al*, 2016; Francq and Zakoian, 2010) [3, 9]. However, financial data frequently exhibit volatility clustering leading to the violation of the assumption of constant variance, thus making a way for the use of non-linear models such as ARCH, GARCH and EGARCH to capture the changing variance of such series.

Despite the apparent success of the GARCH model, it cannot still capture some important features of the financial data. According to Francq and Zakoian (2010) [9], the symmetric GARCH model has an important drawback since the conditional variance only depends on the modulus of the past variables (i.e. past positive and negative innovations have the same effect on the current volatility). This is because the conditional variance must be nonnegative, the coefficients in a GARCH model are often constrained to be nonnegative (Cryer and Chan, 2008) [7]. The most interesting feature not addressed by GARCH model is the leverage effect which occurs when an unexpected drop in price (bad news) increases predictable volatility more than an unexpected increase in price (good news) of similar magnitude (Engle and Ng, 1993: Francq and Zakoian2010) [8, 9]. Also, the nonnegative parameter constraints in GARCH models is necessary to ensure nonnegative conditional variance but may decrease and thwart the dynamical patterns that can be captured in the series (Nelson and Cao, 1992; Tsai and Chan, 2006) [14, 15].

Therefore, this study is built towards handling some weaknesses created when the GARCH model is applied to the returns of a financial series by allowing for the signs of the innovations (returns) to have impact on the conditional variance in addition to the magnitude.

## 2. Materials and Methods

The return series $R_t$ can be obtained given that $P_t$ is the price of a unit share at time, t and $P_{t-1}$ is the share price at time t−1.

$$R_t = log\left( {P_t}/{P_{t-1}} \right) = log\, P_t - log\, P_{t-1} \quad (1)$$

The $R_t$ in equation (1) is regarded as natural logarithm difference of the share price, $P_t$ which is meant to achieve stationarity (Akpan and Moffat, 2015) [2].

### 2.1 Model Selection Criteria

For a given data set, when there are multiple adequate models, the selection criterion is normally based on summary statistics from residuals of a fitted model (Wei, 2006) [17]. There are several model selection criteria based on residuals (see Wei, 2006) [17]. For the purpose of this study, we consider the well-known Akaike's information criterion (AIC), (Akaike, 1973) [1] defined as

$$\text{AIC} = -2\, ln(\text{likelihood}) + 2(\text{number of parameters}) \quad (2)$$

where the likelihood function is evaluated at the maximum likelihood estimates. The optimal order of the model is chosen by the value of the number of parameters, so that AIC is minimum (Wei, 2006) [17].

### 2.2 Diagnostic Checking

**Model Diagnostic Checking**

Ljung and Box Test (1978) [12] is given as

$$Q(m) = T(T+2) \sum_{l=1}^{m} \frac{\hat{\rho}_l^2}{T-l} \quad (3)$$

where T is the number of observations.

The decision rule is to reject $H_0$ if Q(m) $>\chi^2_\alpha$, where $\chi^2_\alpha$ denotes the 100 (1 – $\alpha$)th percentile of a Chi-squared distribution with m – (p + q)degree of freedom (see for example Akpan, Moffat and Ekpo, 2016) [3].

### 2.3 Heteroscedastic Model

The statistical methods for modeling the volatility of a return are referred to as heteroscedastic models (Tsay, 2010) [16]. Let $R_t$ be the return of a share price at time index t. The basic idea behind volatility study is that the series $\{R_t\}$ is either serially uncorrelated or with minor lower-order serial correlations, but it is a dependent series. For the purpose of this study, we consider the GARCH model to account for the ARCH effects (volatility clustering) and the EGARCH model to account for the asymmetric (leverage) effect. To successfully fit the heteroscedastic models, one starts with modeling ARIMA model to remove the linear dependence in the data (see Box, Jenkins and Reinsel, 2008; Cryer and Chen, 2008: Wei, 2006;Fuller, 1996;Brockwell and Davis, 2002) [5, 7, 17, 11, 6]. The residual series of the fitted ARMA model is used to model the GARCH process and then extended to the EGARCH process.

### 2.4 GARCH Model

Although the ARCH model is simple, it often requires many parameters to adequately describe the volatility process of a share price return. Some alternative models must be sought. Bollerslev (1986) [4] proposed a useful extension known as the generalized ARCH (GARCH) model. For a return series, $R_t$, let $a_t = R_t - \mu_t$ be the innovation at time t. Then, $a_t$ follows a GARCH (p, q) model if

$$a_t = \sigma_t e_t,$$

$$\sigma_t^2 = \alpha_o + \sum_{i=1}^{q} \alpha_i a_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \quad (4)$$

where $e_t$ is still a sequence of i.i.d. random variance with mean, 0, and variance, 1, $\alpha_o > 0, \alpha_i \geq 0, \beta_j \geq 0, and \sum_{i=1}^{max(p,q)} (\alpha_i + \beta_i) < 1$ ( see Tsay, 2010) [16].

Here, it is understood that $\alpha_i = 0$,for $i > p$, and $\beta_i = 0$, for $i > q$. The latter constraint on $\alpha_i + \beta_i$ implies that the unconditional variance of $a_t$ is finite, whereas its conditional variance $\sigma_t^2$, evolves over time.

### 2.5 EGARCH Model

The Exponential GARCH (EGARCH) model represents a major shift from ARCH and GARCH models (Nelson, 1991) [13]. Rather than modeling the variance directly, EGARCH models the natural logarithm of the variance, and so no parameter restrictions are required to ensure that the conditional variance is positive. The EGARCH (p, q) is defined as,

$$R_t = \mu_t + a_t, a_t = \sigma_t e_t,$$

$$ln\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \left| \frac{a_{t-i}}{\sqrt{\sigma_{t-i}^2}} \right| + \sum_{k=1}^{r} \gamma_k \left( \frac{a_{t-k}}{\sqrt{\sigma_{t-k}^2}} \right) + \sum_{j=1}^{p} \beta_j\, ln\sigma_{t-j}^2 \quad (5)$$

Alternatively, EGARCH(p, q) model can be represented by

$$ln\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \frac{|a_{t-i}| + \gamma_i a_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^{p} \beta_j\, ln\sigma_{t-j}^2 \quad (6)$$

where again, $e_t$ is a sequence of i.i.d. random variance with mean, 0, and variance, 1, and$\gamma_k$ is the asymmetric coefficient. In the original parameterization of Nelson (1991) [13], p and r were assumed to be equal. The process is covariance stationary if and only if$\sum_{j=1}^{q}\beta_j < 1$.

## 3. Results and Discussion

From the return series comprising2451 observations, the plot of the series presented in figure 1 indicates that the series is not stationary. Also, the ACF of the share price series persists and decays slowly is an indication that the series is not stationary (see Fig. 2).

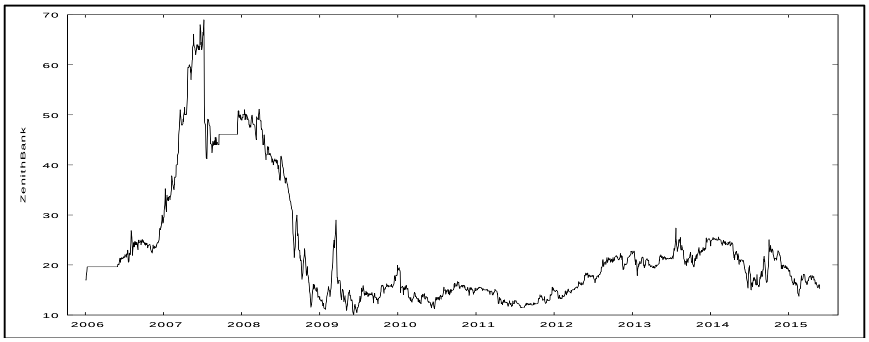


**Fig 1:** Plot of Share price series of Zenith Bank

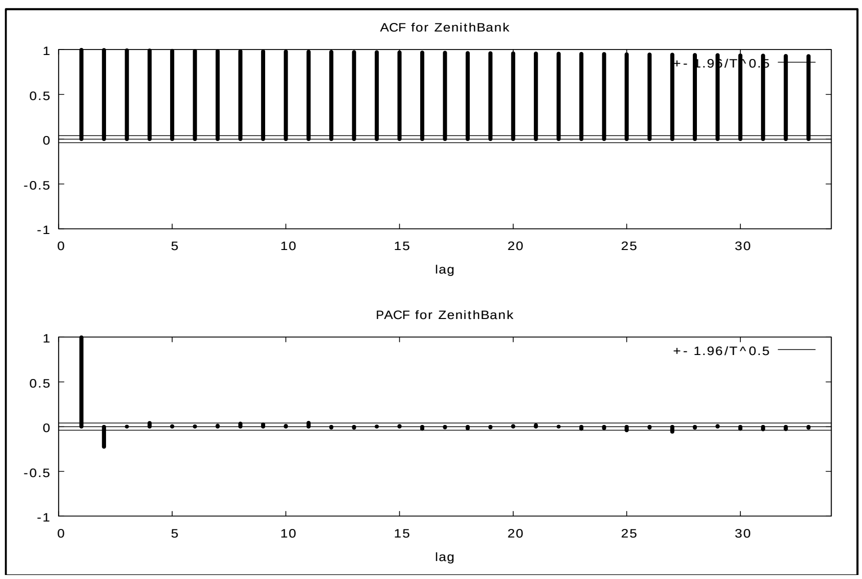


**Fig 2:** ACF and PACF of the Share Price Series

However, achieving stationarity implies ensuring that the mean and the variance of the series are constant. Therefore, taking the natural log difference of the share price series (returns) ensures stationarity and volatility clustering is evident in the returns (see Fig. 3).

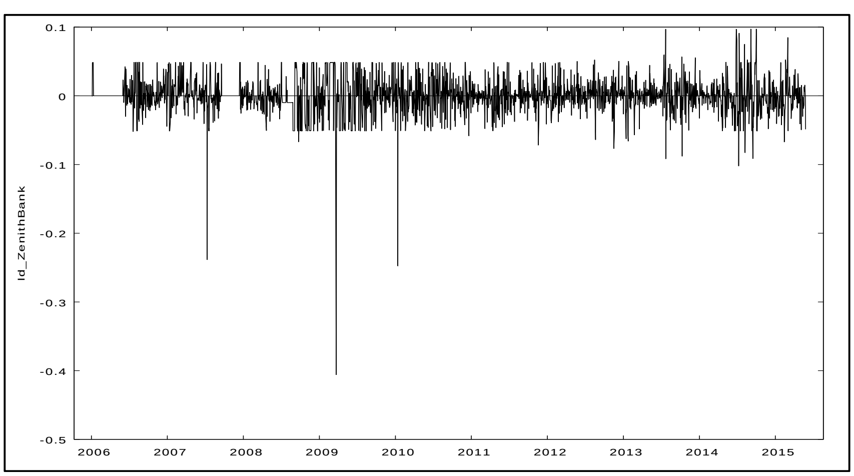


**Fig 3:** Plot of the Return Series

### 3.1 Identification and Estimation of ARIMA Model

In this study, our concentration is on the return series rather than the share price series in that the returns have more attractive statistical properties (volatility clustering and leverage effect) and easy to handle than the share prices. In an attempt to model the linear dependence in the return series, we fit tentatively ARIMA(1,0,1), ARIMA(1,0,2) and ARIMA(1,0,3) models based on the ACF and PACF of the return series in figure 4. The information criteria for the fitted models are presented in Table 1.

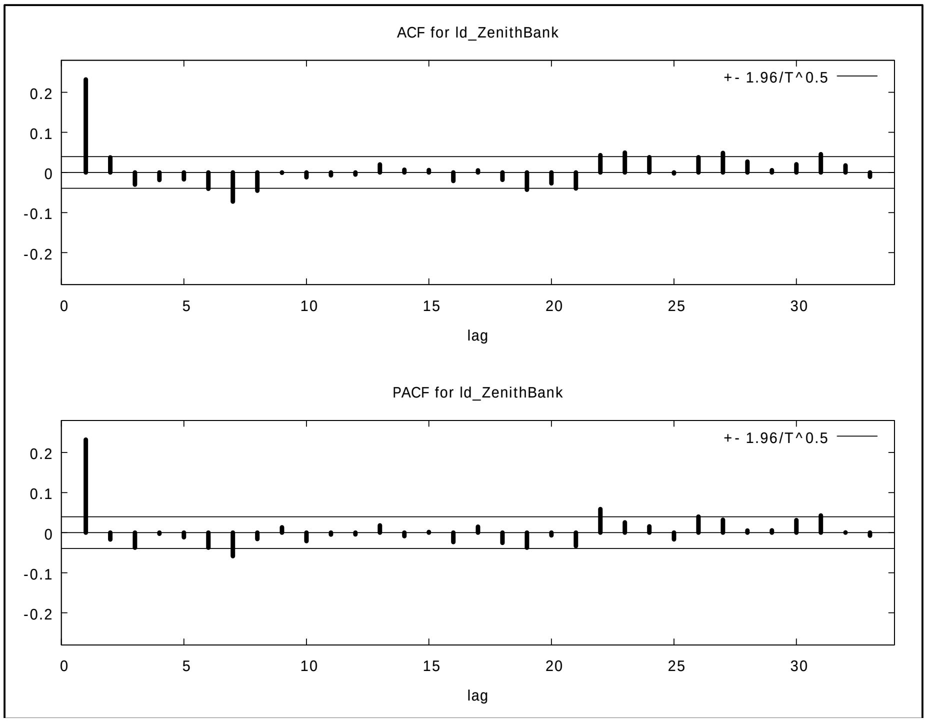


**Fig 4:** ACF and PACF of the Returns of Zenith Bank

**Table 1:** Information Criteria for ARIMA Model

| | ARIMA(1,0,1) | ARIMA(1,0,2) | ARIMA(1,0,3) |
|---|---|---|---|
| AIC | -16613.59 | -16614.67 | -16620.54 |

ARIMA(1,0,3) model has the minimum information criterion and is thus selected as the best fitted model for the return series of the bank. The estimated ARIMA(1,0,3) model is presented in equation (7) below.

$R_t = 0.8289R_{t-1} + \varepsilon_t - 0.5885\varepsilon_{t-1} - 0.1734\varepsilon_{t-2} - 0.0633\varepsilon_{t-3}$ (7)

[Excerpts from table 2 below]

**Table 2:** Output of ARIMA(1,0,3) Model

| Parameter | Estimate | Standard error | z-value | Pr(>\|z\|) |
|---|---|---|---|---|
| ar1 | 0.828927 | 0.065054 | 12.7420 | < 2.2e-16 *** |
| ma1 | -0.588533 | 0.067549 | -8.7127 | < 2.2e-16 *** |
| ma2 | -0.173414 | 0.025665 | -6.7569 | 1.41e-11 *** |
| ma3 | -0.063303 | 0.020043 | -3.1584 | 0.001586 ** |

Signif. codes:0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

The diagnostic checks on the ARIMA(1,0,3) model using Ljung – Box test indicates that the model is a good fit since the null hypothesis of no serial correlations inthe first eight (8) lags of the residuals of the model is not rejected given that the p-value of 0.3062 (associated with the Chi-squared value of 9.4444 with 8 degrees of freedom) is not less than 5% level of significance (see Table 3). Hence, the model is adequate in modeling the dynamic linear dependence of the series.

**Table 3:** Output of Ljung-Box Test

| $\chi$ – squared | Degree of Freedom (df) | p-value |
|---|---|---|
| 9.4444 | 8 | 0.3062 |

### 3.2 Detecting the Presence of Heteroscedasticity

To detect the presence of heteroscedasticity (ARCH effect), we assess the ACF of $\varepsilon_t^2$ of ARIMA(1,0,3) model. If at least one lag term in the squares of residual series ($\varepsilon_t^2$ ) is found to be statistically significant, then heteroscedasticity is said to exist. From figure 5, the first lag term of ACF of the squares of the residual series is significant. Thus heteroscedasticity is said to exist.

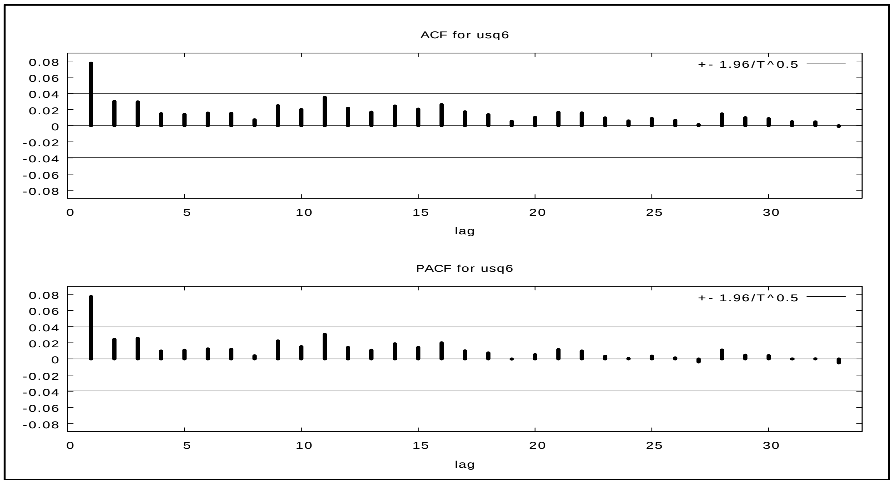


**Fig 5:** ACF and PACF of Squares of Residual Series

Also, according to the Lagrange Multiplier test, the hypothesis of no ARCH effects up to lag 16 is rejected since the Lagrange Multiplier test value of 3215 with corresponding probability (of Chi Square with 16 degree of freedom), $0.000 < 5\%$ level of significance, further confirms the presence of heteroscedasticity.

### 3.3 Estimation of GARCH Model Parameters

Having detected the presence of heteroscedasticity, we moved on to estimate the GARCH model. We fit tentatively GARCH(0,1), GARCH(0,2) and GARCH(0,3) models to the residual series of ARIMA(1,0,3) model. From Table 4, GARCH(0,1) model appears not to have the lowest information criterion amongst the three tentative GARCH models. However, based on the principle of parsimony and the fact that the present conditional variance depends only on the immediate past conditional variance, we select GARCH(0,1) model.

**Table 4:** Information Criteria for GARCH Model

| | GARCH(0,1) | GARCH(0,2) | GARCH(0,3) |
|---|---|---|---|
| AIC | −11417.96 | −11525.57 | −11625.91 |

The estimated GARCH(0,1) model is presented in equation (8) and the values in bracket are the probability values
(p-values) indicating the significance of the parameters in the model. Since all the p-values are not less than 5% level of significance, then, the parameters in the model are said to be significant.

$R_t = 8.91031 \times 10^{-4} + \varepsilon_t$

(0.0255)

$\varepsilon_t = \sigma_t e_t, e_t \sim \boldsymbol{N(0,1)}$

$\sigma_t^2 = 3.38191e^{-4} + 0.631176\varepsilon_{t-1}^2$ (8)

(<0.0001)(<0.0001)

[Excerpts from Table 5]

**Table 5:** Output of GARCH (0,1) Model

| | **Coefficient** | **Std. Error** | **Z** | **p-value** | |
|---|---|---|---|---|---|
| Const | 0.000891031 | 0.000398773 | 2.2344 | 0.0255 | ** |
| alpha(0) | 0.000338191 | 1.47575e-05 | 22.9165 | <0.0001 | *** |
| alpha(1) | 0.631176 | 0.0522735 | 12.0745 | <0.0001 | *** |

| **Mean dependent var** | **−0.000043** | **S.D. dependent var** | **0.025470** |
|---|---|---|---|
| Log-likelihood | 5712.982 | Akaike criterion | −11417.96 |
| Schwarz criterion | −11394.75 | Hannan-Quinn | −11409.53 |

Model 7: GARCH, using observations 2006-01-04:2015-05-26 (T = 2450)
Dependent variable: uhat6
Standard errors based on Hessian

The diagnostic checks on the GARCH(0,1) model indicates that the model is adequate since the Lagrange Multiplier test statistics of 4.372158 with corresponding probability of Chi- Square with lag 8, 0.8221 >5% level of significance (see Table 6).
Thus, the ARCH effects in the return series of Zenith plc appear to be adequately captured by GARCH(0,1) model. The implication of this result is that large (small) absolute returns tend to be followed by large (small) absolute returns of similar magnitude.

**Table 6:** Heteroscedasticity Test: GARCH

| | | | |
|---|---|---|---|
| F-statistic | 0.545482 | Prob. F(8,2432) | 0.8228 |
| Obs*R-squared | 4.372158 | Prob. Chi-Square(8) | 0.8221 |

### 3.4 Estimation of EGARCH Model

In order to model the asymmetric GARCH that would account for the leverage effect in the return series, we extended the GARCH(0,1) model to the EGARCH(0,1) model which is presented in equation (9) below

$R_t = 6.83823 \times 10^{-5} + \varepsilon_t$

$\varepsilon_t = \sigma_t e_t, e_t \sim N(0,1)$

$$ln\sigma_t^2 = -7.84212 + 0.570692 \left(\frac{|\varepsilon_{t-1}| - 0.0808165\varepsilon_{t-1}}{\sigma_{t-1}}\right) \quad (9)$$

[Excerpts from Table 7]

The coefficient of the asymmetric (leverage) effect is $-0.0808165$ with the corresponding p-value of 0.0040 which is less than 5% level of significance. That is, the estimated leverage effect is negative and significant. Thus, the leverage effect in the series appears to be captured by EGARCH(0,1) model. The implication of this result is that an unexpected decrease in price increases predictability of volatility more than an unexpected increase in price of similar magnitude.

**Table 7:** Output of EGARCH(0, 1) Model

| | Coefficient | Std. Error | Z | p-value | |
|---|---|---|---|---|---|
| Const | 6.83823e-05 | 0.000329079 | 0.2078 | 0.8354 | |
| Omega | 7.84212 | 0.0389357 | -201.4 | 0.0000 | *** |
| Alpha | a0.570692 | 0.0403487 | 14.14 | 2.03e-045 | *** |
| Gamma | -0.0808165 | 0.0280480 | -2.881 | 0.0040 | *** |

| | | | |
|---|---|---|---|
| Log-likelihood | 5656.28411 | Akaike criterion | −11304.56822 |
| Schwarz criterion | −11281.35284 | Hannan-Quinn | −11296.13129 |

Model: EGARCH(0,1) [Nelson] (Normal)
Dependent variable: uhat6
Sample: 2006-01-04-2015-05-26 (T = 2450), VCV method: Hessian

The diagnostic checks on the EGARCH(0,1) model shows that the model is adequate, since the Lagrange Multiplier test value of 5.927093 with the p-value of 0.6554 >5% level of significance (see Table 8).

**Table 8:** Heteroskedasticity Test for EGARCH(0,1) Model

| | | | |
|---|---|---|---|
| F-statistic | 0.739952 | Prob. F(8,2432) | 0.6562 |
| Obs*R-squared | 5.927093 | Prob. Chi-Square(8) | 0.6554 |

## 4. Conclusion

From the findings, we found that the assumption of constant variance is being violated as indicated by the presence of ARCH effect in the residuals of ARIMA (1,0,3) model. The ARCH effect (heteroscedasticity) was adequately captured by GARCH(0,1) model. In an attempt to relax the nonnegative constraints in GARCH model, we found that the asymmetric (leverage) effect parameter was negative, significant and adequately captured by EGARCH(0,1) model. Therefore, this study contributes to knowledge as follows; that where the linear models fail to capture changing variance, the symmetric GARCH model provides the basic remedy to the violation of assumption of constant variance. While on the other hand, the asymmetric GARCH model captured the negative asymmetry in the series, thereby overcoming the weaknesses of symmetric GARCH model due to the nonnegative constraints on the parameters.

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