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A competitive mathematical modeling of technological innovation diffusion

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Abstract
Technological innovation in developed countries are very fast. A technology appears in the market before it is penetrating the whole market another better technology comes and begins to complete for adoption by non-adopter in fast some those who have adopted the first one also switch over to the newer technology. This studies help in process taking decisions about the launching of a new products in the market.
I have concluded this study, when there is only one technology in the market number of adopter increases with time. When new technology is coming old becomes stagnant during same period and there after it loses the market and new one influencing the market and number of adopter increases the same process is repeated when the new technology is coming up.

Keywords: Diffusion model, Fisher-Pray Model.

Introduction

The diffusion of an innovation has traditionally been defined as the process by which an innovation is communicated through certain channels over time among the members of a social system. There are four elements in the diffusion process; the innovation, channels of communication, time and the social system. Technology Diffusion is understood as a process of which a new technology or an innovation is propagated through certain channels over time among the units of system. Schumpeter (1939) [5] sees diffusion as the final stage of the technology development. Rogers (1962) [4] describes diffusion of new product as a five-stage process- awareness, interest, evaluation, trial, and adoption, Grubler (1998) [2] describes as widespread adoption of technologies over time, in space and between social strata. The elements of technology diffusion comprise of innovation, strategies to commercialize technologies is expepropagation, time, and units of social system (Narayanan, 2001) [3]. Scted to follow a path way set in stages of imagining, incubating, demonstrating, promoting and sustaining technologies (Vedpuriswar, 2003) [6]. Diffusion is considered as the stage after invention and innovation of a technology. The diffusion process passes through filtering, tailoring and acceptance of a technology. Many inventions may or may not reach the stage of diffusion. The diffusion processes in general follow as S curve (Figure 1). The curve generally comprises of three distinct phases:

i) An initial slow growth,
ii) A rapid take-off period and
iii) A flattening of growth, signifying a near completion of diffusion.

There are diverse examples for depicting this S shaped pattern in the natural growth of many phenomena including diffusion of Cistercian monasteries in Europe one thousand year ago and life expectancy of creative geniuses. Diffusion curve-cumulative adoption vs time diffusion modeling captures the diffusion process or behavior in a mathematical form that allows quantifying the diffusion parameters for further diffusion analysis. Models can be used to explain the diffusion rates and estimate parameters that measure the coefficients of diffusion in a given context. Different diffusion models have been used, particularly since the 1960s to capture this diffusion trend in the form of mathematical equations.

These models have been applied to study various diffusion processes that include population of cars, television, computers, consumer goods, etc. as well as frequency of economic booms and busts, number of fatal car accidents, incidence of major nuclear accidents, technological change in the computer industry and number of deaths from AIDS.

Modeling technology diffusion processes was initially derived from the theory of growth of a colony of biological cell in a medium. Since the growth of a cell would be limited due to limited nutrients or space, it would slow down and saturate resulting in an S-curve pattern. Similarly, technology diffusion models assume that the growth of technology or an innovation is dependent on the total potential adopters and the rate of increase in represented by the following fundamental diffusion equation referred to as the internal influence diffusion model.

Diffusion Model

We have to identify the variables and parameters of innovation diffusion model. The number of adopters of adopters of new innovation, n (t) till time t is a variable and the total number of potential adopter N is parameter. We assume that the innovation spreads by word of mouth that is communication of information about the product from those who have adopted it to those who have not yet adopted.

At any time t n(t) is number of adopters and N-n(t) is non-adopters. If the number of successful adopters, who can communicate the new innovation in efficient manner, is large, the greater will be the number who can possibly adopt it and larger will be the rate of change n(t) so that we assume (JN Kapur, 1992)[1]

$$\frac{dn}{dt} = c\, n(t)[N - n(t)] \quad (1)$$

c is constant. This gives the first mathematical model.

Let $\frac{n(t)}{N} = f(t)$

(1) reduces to

$$\frac{df}{dt} = c' f(`1 - f) \quad (2)$$

we investigate this model

since $n(t) \le N(t), f(t) \le 1, \frac{df}{dt} \ge 0$

$$\frac{d^2f}{dt^2} = c'(1-2f)\frac{df}{dt} \quad (3)$$

$\frac{d^2f}{dt^2} > 0$ for $f < \frac{1}{2}$

$\frac{d^2f}{dt^2} = 0$ for $f = \frac{1}{2}$

$\frac{d^2f}{dt^2} < 0$ for $f > \frac{1}{2}$

$\frac{df}{dt}$ increases when $f > \frac{1}{2}$

$\frac{df}{dt}$ decreases when $f > \frac{1}{2}$

In other words $f(t)$ or $n(t)$ increases at increasing rate when $n(t) < \frac{N}{2}$ and it increases but at decreasing rate when $n(t) > \frac{N}{2}$ and there is a point of inflexion when $f(t) = \frac{1}{2}$ or $n(t) = \frac{N}{2}$. This statement can be represented by graph (1).

From (2)

$$\log\frac{f}{1-f} = \log\frac{f_0}{1-f_0} + c't \text{ Or } \frac{f}{1-f} = \frac{f_0}{1-f_0}e^{c't}$$

f_0 is the value f at $t = 0$.

$$f = \frac{\frac{f_0}{1-f_0}e^{c't}}{1 + \frac{f_0}{1-f_0}e^{c't}} = \frac{1}{1 + \frac{1-f_0}{f_0}e^{-c't}}$$

Thus $f(t) \le 1$

and $f(t) \to 1$, when $t \to \infty$

or $n(t) \to N$ when $t \to \infty$,

That is innovation has to wait very long time till all the potential adopters it. In the fast growing technological age adopters have always possibilities to adopt new innovation hence old one will not be adopted by all the potential adopters. This also clear from the asymptotic behavior of the curve.

The model (2) is called Fisher-Pry model(1971)[7] and is very successful model in the study of innovation diffusion. In this model point of inflexion is $n = \frac{N}{2}$ while in real life situation it can occur before or after $\frac{N}{2}$.

Competition model for new technology

Technological innovation in developed countries are very fast. A technology appears in the market before it is penetrating the whole market another better technology comes and begins to complete for adoption by non-adopter in fact some those who have adopted the first one also switch over to the newer technology. Similarly the fourth superior comes and influences the adopters and so many technologies are in competition in the market. We want to model this situation.

Let there are five technologies in the market and $n_1(t), n_2(t), n_3(t), n_4(t), n_5(t)$ are the number of adopters at time 't' and

$$N - n_1(t) - n_2(t) - n_3(t) - n_4(t) - n_5(t)$$

are number of those who have not adopted any one of the five till time 't'

In the first time interval (O, T_1), where there is one technology the model is

$$\frac{dn_1}{dt} = c\, n_1(t)[N - n_1(t)],\ o \leq t \leq T_1$$

In the second time interval (T_1, T_2) where there are two technologies, the model is

$$\frac{dn_1}{dt} = c\, n_1(t)[N - n_1(t) - n_2(t)] - c_1 n_1(t) n_2(t)$$
$$\frac{dn_2}{dt} = c_2 n_2(t)[N - n_1(t) - n_2(t)] + c_1 n_1(t) n_2(t)$$

In the third interval (T_2, T_3) there are three technologies, the model is

$$\frac{dn_1}{dt} = dn_1(t)[N - n_1(t) - n_2(t) - n_2(t)] -$$
$$-d_1 n_1(t) n_2(t) - d_2 n_1(t) n_3(t)$$
$$\frac{dn_2}{dt} = d_4 n_2(t)[N - n_1(t) - n_2(t) - n_3(t) +$$
$$+d_1 n_1(t) n_2(t) - d_3 n_2(t) n_3(t)$$
$$\frac{dn_3}{dt} = d_5 n_3(t)[N - n_1(t) - n_2(t) - n_3(t)] +$$
$$+d_2 n_1(t) n_3(t) + d_3 n_2(t) n_3(t)$$

In the fourth time interval(T_3, T_4) there are four technologies, the model is,

$$\frac{dn_1}{dt} = kn_1(t)][N - n_1(t) - n_2(t) - n_3(t) - n_4(t)] -$$
$$-k_1 n_1(t) n_2(t) - k_2 n_1(t) n_3(t) - k_3 n_1(t) n_4(t)$$
$$\frac{dn_2}{dt} = k_6 n_2(t)[N - n_1(t) - n_2(t) - n_3(t) - n_4(t)] +$$
$$+k_1 n(t) n_2(t) - k_4 n_2(t) n_3(t) - k_5 n_2(t) n_4(t)$$
$$\frac{dn_3}{dt} = k_8 n_3(t)[N - n_1(t) - n_2(t) - n_3(t) - n_4(t)] +$$
$$+k_2 n_1(t) n_3(t) + k_4 n_2(t) n_3(t) - k_7 n_3(t) n_4(t)$$
$$\frac{dn_4}{dt} = k_9 n_3(t)[N - n_1(t) - n_2(2) - n_3(t) - n_4(t)] +$$
$$+k_3 n_1(t) n_4(t) + k_5 n_2\,(t) n_4(t) + k_7 n_3(t) n_4(t)$$

In the fifth time interval (T_4, T_5) there are five technologies, the model is,

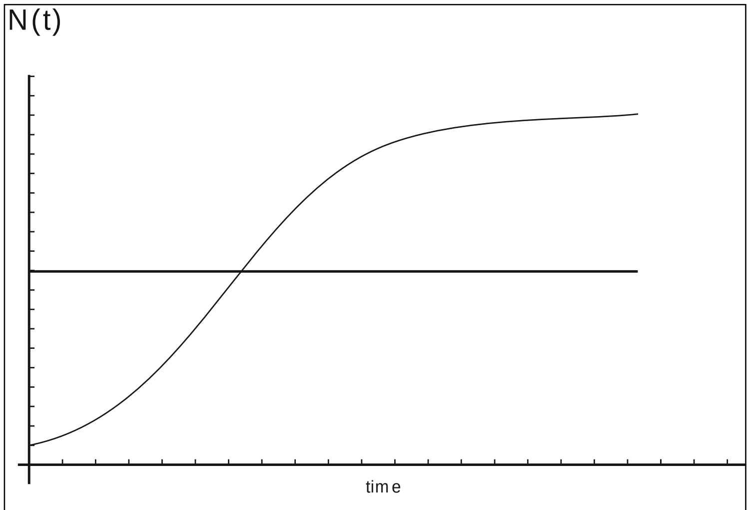
$$\frac{dn_1}{dt} = \lambda n_1(t)][N - n_1(t) - n_2(t) - n_3(t) - n_4(t) - n_5(t)] -$$
$$-\lambda_1 n_1(t) n_2(t) - \lambda_2 n_1(t) n_3(t) - \lambda_3 n_1(t) n_4(t) - \lambda_4 n_1(t) n_5(t)$$
$$\frac{dn_2}{dt} = \lambda_{11} n_2(t)[N - n_1(t) - n_2(t) - n_3(t) - n_4(t) - n_5(t)] +$$
$$+\lambda_1 n(t) n_2(t) - \lambda_5 n_2(t) n_3(t) - \lambda_6 n_2(t) n_4(t) - \lambda_7 n_2(t) n_5(t) \qquad \ldots(4)$$
$$\frac{dn_3}{dt} = \lambda_{12} n_3(t)[N - n_1(t) - n_2(t) - n_3(t) - n_4(t) - n_5(t)] +$$
$$+\lambda_2 n_1(t) n_3(t) + \lambda_4 n_2(t) n_3(t) + \lambda_7 n_3(t) n_4(t) - \lambda_8 n_2(t) n_4(t)$$
$$\frac{dn_4}{dt} = \lambda_{13} n_3(t)[N - n_1(t) - n_2(2) - n_3(t) - n_4(t) - n_5(t)]$$
$$+\lambda_3 n_1(t) n_4(t) + \lambda_5 n_2\,(t) n_4(t) + \lambda_7 n_3(t) n_4(t) - \lambda_9 n_4(t) n_5(t)$$
$$\frac{dn_5}{dt} = \lambda_{14} n_3(t)[N - n_1(t) - n_2(2) - n_3(t) - n_4(t) - n_5(t)]$$
$$+\lambda_3 n_1(t) n_5(t) + \lambda_5 n_2\,(t) n_5(t) + \lambda_7 n_3(t) n_5(t) + \lambda_{10} n_4(t) n_5(t)$$

Mathematical model represented by equation (4) is called competition model between five technologies. N is taken constant but increase in the population size and the economic prosperity of society changes the value of N with time. If we substitute in the above model N' as function of time we get very complicated competition model. So for the present purpose N is independent of time. When there is only one technology in the market number of adopter increases with time. When new technology is coming old becomes stagnant during same period and there after it loses the market and new one influencing the market and number of adopter increases the same process is repeated when the new technology is coming up.

Results

We have studied the models of innovation diffusion starting from simple case and taken into account influences which are coming from real life situation. These models and such type of studies help in process of talking decision about the launching of new products in the market, making right kind of investment in the product as it influences the number of potential adopters.

This study reveals that, when there is only one technology in the market number of adopter increases with time. When new technology is coming old becomes stagnant during same period and there after it looses the market and new one influencing the market and number of adopter increases the same process is repeated when the new technology is coming up.



Figure(1)

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