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Extended vertex edge additive cordial (EVEAC) labeling-a new labeling method for graphs

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Abstract

In this paper we discuss a new type of graph labeling Extended Vertex Edge Additive Cordial (eveac) labeling and show that path Pn, $K_{1,n}$, $K_{2,n}$, $K_{3,n}$, $K_{4,n}$ are eveac graphs.

Keywords: cordial, edge, vertex, label, extended vertex

1. Introduction

I. Cahit [5] introduced the notion of cordial labeling a variation of both graceful and harmonious labelings. He defined a cordial function f which restricts the vertices of graph to {0, 1}. Afterwards many types of cordial labelings were defined [2, 9, 10] etc. We introduce a new type of graph binary labeling. We allow vertices to take values from 0 to |V|-1 and restrict the edge labels to 0 and 1.Only. Such labels which take value 0 or 1 only are many times refered as binary labeling. All graphs that we consider are simple, finite and undirected. For terminology and definitions we refer Harary [8] and J. A. Gallian [7]. Let G be a (p, q) graph. Define a function f:V→{0, 1, 2, p-1}which introduces edge label function given by f*:E→{0, 1}such that edge(uv)ϵE(G) then f*(uv) = f(u) + f(v) (mod 2). Further the condition to be satisfied is that $|e_f(0)-e_f(1)|\leq 1$.where $e_f(i)$ is the number of edges labeled with i=0,1. Then f is called as Extended Vertex Edge Additive Cordial (eveac) labeling The graph that admits eveac labeing is called as eveac graph We show that path Pn, $K_{1,n}$, $K_{2,n}$, $K_{3,n}$ and pathunions Pn(C_3), Pn(C_4) are eveac graphs.

2. Definitions

Definition 2.1 A star graph $K_{1,n}$ is obtained by attaching n pendent edges to a single point.It has one vertex of degree n and all other veretices of degree 1.

Definition 2.2 A bistar graph or $K_{2,n}$ is obtained by joining the n-degree vertices of two copies of star graphs $K_{1,n}$ by an edge. It has two vertices of degree n+1 and have 2n pendent edges.

Defination 2.3 A tristar $K_{3,n}$ has three star graphs, each of it joined by an edge at n-degree vertex. It has three adjacent vertices of degree n+1 each and in all it has 3n pendent vertices

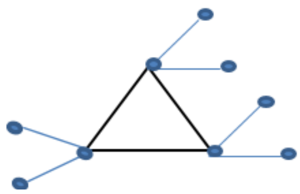


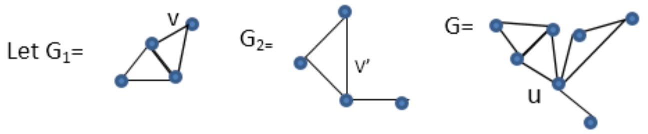
Fig 3: $K_{3,2}$

Definition 2.4 A pathunion Pn(G) essentially consists of a path P_n At every vertex of Pn a copy of G is fused with a fixed vertex of G. Thus a Pn(G) has |V| = n|V(G)| and |E| = n|E(G)| + (n-1)

Definition 2.5: Fusion of vertices: Let v ϵV(G_1), v' ϵ V(G_2) where G_1 and G_2 are two graphs. We fuse v and v' by replacing them with a single vertex say uand all edges incident with v in

G_1 and that with v' in G_2 are incident with u in the new graph $G=G_1FG_2$.The concept of fusion of vertex is explained in [6] by J.Clark and Holtan.

Deg $_G$u=deg$_{G1}$ (v) $_+$ deg$_{G2}$(v') and $|V(G)| =|V(G_1)|+|V(G_2)|-1$, $|E(G)|=|E(G_1)|+|E(G_2)|$



3. Theorems with Proofs

Theorem 3.1 A star $K_{1,n}$ is eveac

Proof: Let the n degree vertex be called as a, the central vertex and the adjacent vertices to x as v_1, v_2, v_n.

Define f: V→{0, 1, 2, p-1} as

f(a) =0,f(v_i) = i for all i = 1, 2, n

$e_f(0) = e_f(1) = \frac{n}{2}$ for n is even and $e_f(0) = \frac{n-1}{2}$; $e_f(1)=\frac{n+1}{2}$ otherwise. #

Theorem 3.2 A bistar graph $K_{2,n}$ is eveac.

Proof: We call two vertices with degree (n+1) as a and b respectively. The pendent vertices adjacent to vertex a and b are v^a_i, and v^b_i respectively i = 1, 2. n. Let p = 2n+2, the number of vertices. Define a function f as

f:V→{0, 1, 2, p-1} as

f(a)=2,f(b)=0,f(v^a_1)=1,f(v^b_1)=3;

f(v^a_i)=2i ; f(v^b_i) = 2i + 1, i= 2, 3, n

Note that the edge label numbers are; $e_f(0)$=i and $e_f(1)$ = i+1#

Theorem 3.3 A tristar graph $K_{3,n}$ is eveac.

Proof: Let three corner vertices of $K_{3,n}$ be x,y,z. The pendent edges at a= x,y,z be e^a_1,e^a_2,e^a_3, e^a_n corresponding to vertices v^x_i, v^z_i, v^y_i (i = 1,2, n) respectively. Define a eveac function f as follows.

f:V→{0, 1, 2, p-1} given by f(x) =0,f(y)= 1,f(z)=2

f(v^x_i) = 3i for all i = 1….n

f(v^z_i) =1+3i, for all i = 1….n

f(v^y_i)= 2+3i for all i = 1….n

We have $e_f(1)$ = 2, $e_f(0)$= 1 for n =0. In this case there is only C_3 and no pendent edges attached to vertices of C_3.

$e_f(1) = 2+3\frac{n}{2}$, $e_f(0)= 1+3\frac{n}{2}$ for n = 2, 4, 6

$e_f(1) = \frac{3(n+1)}{2} = e_f(0)$ for n = 1, 3, 5#

Theorem 3.4 Path union $Pn(C_3)$ is eveac graph

Proof: To define $Pn(C_3)$ we start with a path Pn= ($v_1e_1v_2e_2…e_{n-1}v_n$). A copy of C_3 fused at vertex i of Pn is C^i_3 = C^i given by (u^i_1 =v_i,e^i_1,u^i_2, e^i_2,u^i_3, e^i_3,v_i)

When n =1. We have $Pn(C_3)$ as C_3.Label the three vertices on C_3 as 0,1,2 respectively. We have $e_f(0)$ = 1 and $e_f(1)$ = 2

Case n = 2 Fig 1 gives the details.

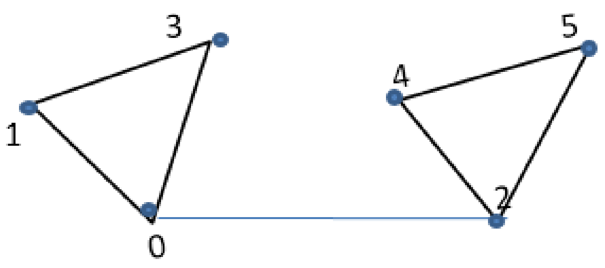


Fig 1: $P_2(C_3)$

Case n>2

Define f: V(Pn(C_3))→{0, 1, p-1} as follows for n =2 the fig 2 gives the required labeling. When n >2 we have following scheme.

f(v_i) = 3(i-1) for i is odd number.

f(vi) =3(i-1)+1 for i is even number.

f(u^i_2)=3(i-1)+1 for odd i

f(u^i_3)=3(i-1)+2 for odd i

f(u^i_2)=3(i-1) for even i

f(u^i_3)=3(i-1)+2 for even i

Note that on every copy of C_3 there is only one edge with even number as label. And all edges on path Pn are even edges. Thus we get $e_f(0)$ = 2n-1,$_f$(1)= 2n#

Theorem 3.5 Path union G=Pn(C_4) is eveac graph.

Proof: To define Pn(C_4) we start with a path Pn= (v_1,e_1,v_2,e_2,…,e_{n-1},v_n). A copy of C_4 fused at vertex i of Pn is C^i_4 = C^i given by (u^i_1 =v_i,e^i_1,u^i_2, e^i_2,u^i_3, e^i_3,u_4,e^i_4,v_i) i = 1, 2, n.

For n=1 label the four vertices as 0,1,3,2 in this order.

For n>1

f:V(G) --.{0, 1, 2, p-1} as

Label the vertices on path as follows

f(v_n) = 4n-3 for n ≡ 3 (mod4); f(v_n) =4n-4 for n≡0,1,2(mod 4);

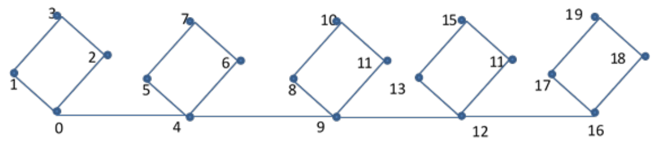


Fig 2: P_5(C4)

The cycle vertices (not on path Pn) are labeled as follows:

f(u^i_3)= f(v_i)+1 for i ≡ 3(mod4);

f(u^i_4)= f(v_i)+2 for i ≡ 3(mod4);

f(u^i_2)= f(v_i)-1 for i ≡ 3(mod4);

f(u^i_2)= f(v_i)+1 for i ≡ 0,1,2(mod4);

f(u^i_3)= f(v_i)+3 for i ≡ 0,1,2(mod4);

f(u^i_4)= f(v_i)+2 for i ≡ 0,1,2(mod4);

Note that Label numbers are $e_f(0)$ = 2n+2x= $e_f(1)$ for n ≡ 1 (mod4). n = 4x+1; x = 0, 1, 2.

$e_f(0)$ = 2n+2x+1; $e_f(1)$ =2n+2x for n ≡ 2 (mod4). n = 4x+2 ; x = 0, 1, 2, $e_f(0)$ = 2n+2x+1; $e_f(1)$ =2n+2x+1 for n ≡ 3 (mod4). n = 4x+3; x = 0, 1, 2, $e_f(0)$ = 2n+2x ;$e_f(1)$ =2n+2x+1 for n ≡ 0(mod4). n = 4x+2; x = 0, 1, 2

4. Challenges Ahead

1. It is necessary to investigate eveacl for $K_{m,n}$. We conjecture that Km,n is eveac graph for all m and n.
2. It is necessary to investigate eveacl for path unions Pn(C_m).We conjecture that Pm(C_n) is eveac graph for all m and n.

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