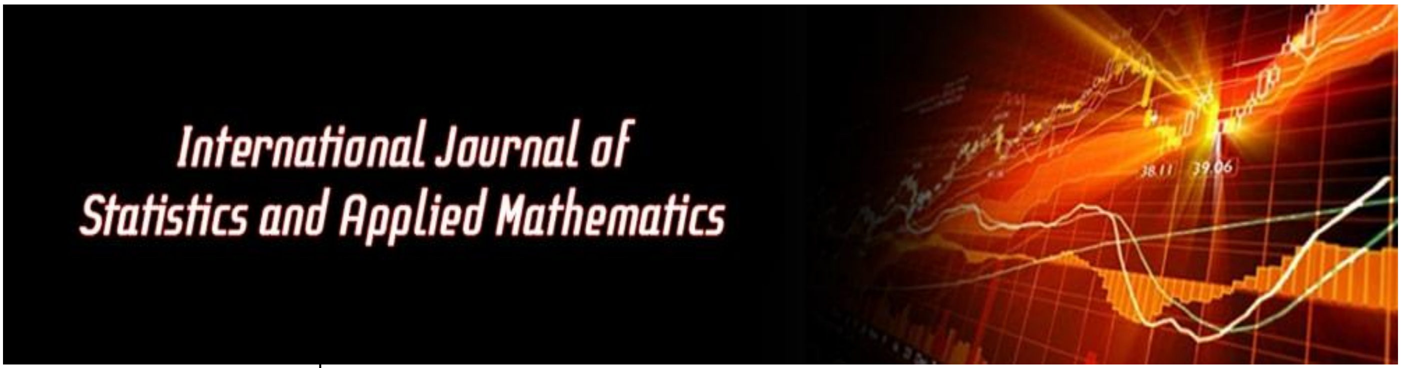

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Maximum probability estimation for an autoregressive process

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Abstract
We observe X_i, …., X_n, where $X_i = \theta_i X_{i-2}$ x Y_i, ---------- (1)
Where X_θ is defined as zero, and Y_i,…., Y_n are unobservable random variables, independent, each normal with mean zero and variance θ_2. θ_1 and θ_2 are both unknown and are to be estimated by using Maximum Probability Estimation. It is shown that the maximum likelihood estimators of the parameters have certain optimal properties.

Keywords: Maximum probability estimation, autoregressive process

1. Introduction

Maximum probability estimators were decolorized in Weiss and Wolfowitz (2007, 2008, 2009, 2010) [3-6]. They represented an attempt to explain why maximum likelihood estimators work so well in many cases. The brief description of this asymptotic. A brief description of this asymptotic theory follows. There is an index *n*, which takes positive integral values, and we examine what happens as *n* increases. For each *n*, X(n) is the random vectors to be observed. The joint density function for the components of X(n) is fn(X $(n)_I$ θ_i, …, θ_m), where θ_1, ….., θ_m are unknown parameters to be (θ_1, ….., θ_m), R_n (θ_1, …., θ_m) is a bounded measurable subset of m – dimensional space, containing the m – dimensional origin $(\theta_1, \dots \theta)_i$ and as *n* increases, R_n (θ_1,…. θ_m) shrinks towards this origin. For any given value, D_1, …., D_m let R_n^*(D_1, …., D_m) denote the set of vectors(θ_1, …, θ_m) such that the vector ($D_1 - \theta_1$, …., D_m - θ_m) is in R_n (θ_1, …., θ_m). let θ_1(n) ….., θ_m(n) denoteteh values of D_1, …., D_m which maximize the integral

∫..∫ fn 1 X (n)i θ_1, …., θ_m) dθ_1, …, dθ_m ---------- (2)
Rn (D_1,... D_m)

$\hat{\theta}_1$ (n) …, $\hat{\theta}_m$(n) are called "*Maximum probability estimators with respect to Rn(θ_1, ….., θ_m)*" because if θ_1, …, θ_m can be estimated with increasing accuracy as *n* increases, then among all estimators satisfying a reasonable regularity condition, ($\hat{\theta}_1$(n), …., $\hat{\theta}_m$(n)) gives the highest asymptotic probability that the vector (estimator of $\theta_i - \hat{\theta}_1$) …. estimator of $\theta_m - \hat{\theta}_m$) falls in R_n(θ_1, … θ_m).

In Weiss (2004) [2], small-sample properties of maximum probability estimators were investigated. Now, we apply the theory of maximum probability estimators to the following estimation problem. Y_1, …., Yn are independent and identically distributed, each normal with mean zero and unknown positive variance θ_2. We do not observe Y_1, …. Yn, what we observe are X_i, …., Xn, where $X_i = \theta_1$ x $_{i-1} + Y_i$ ---(3) with X_o defined as zero. θ_1 is an unknown parameter. The problem is to estimate θ_1 and θ_2.

The joint density function for
X_1, …., X_n is $(2\pi\ \theta_2)^{-n/2}{}_{exp}[(-1/2\ \theta_2)\sum^n_{i=1}(X_i - \theta_1 X_{i-1})^2]$ -------- (4)
and the maximum likelihood estimators are ;

$$\hat{\theta}_1 = \frac{\sum^n_{i=1} X_i X_{i-1}}{\sum^n_{i=1} X_i X_{i-1}} \quad (5)$$

$$\hat{\theta}_2 = 1/n \sum^n_{i=1} (X_i - \hat{\theta}_1 X_{i-1})^2 \quad (6)$$

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In Weiss (2004) [2] and Weiss and Wolfowitz (1968), this process was studied for the ease where θ_2 was known to be equal to one, and so only θ_1 had to be estimated.

2. The Case / θ_1/ < 1

If / θ_1/<1, it is easily verified that the conditions of Weiss (2003) [1] are satisfied, and therefore the maximum likelihood estimators have the following properties.

$n^{1/2}$ ($\theta_1 - \hat{\theta}_1$) and $n^{1/2}$ ($\theta_2 - \hat{\theta}_2$) have asymptotically a bivariate normal distribution with zero means, respective variances $1 - \theta_1^2$, $2\ \theta_2^2$, and covariance zero. Furthermore, $\hat{\theta}_1$, $\hat{\theta}_2$ have the same asymptotic properties as maximum probability estimators with respect to any convex region symmetric about the origin (0, 0)

3. The Estimation of θ_1

Known θ_2 is of no advantage in estimating θ_1. That is, a statistician who does not know θ_2 can estimate θ_1 as well as statistician who know the value of θ_2. To see this, imagine that unknown to us, each Y_1 is multiplied by a nonzero value c. this replaces θ_2 by $C^2\ \theta_2$, and multiplies each X_1 by C, but does not change either $\hat{\theta}_1$ or θ_1. The fact that knowing θ_2 is of no relevance for estimating θ_1 explains the form of asymptotic convenience matrix in section 2.

In Weiss (2004) [2], the problem of estimating θ_1 when θ_2 is known to be one and θ_1 is completely unknown as studied. In this case, the maximum likelihood estimator $\hat{\theta}_1$ is also the maximum probability estimator with respect to any region $R_n(\theta_1)$ symmetric around zero, and it was shown that $\hat{\theta}_1$ is uniformly arbitrarily close to a Bayes estimator. For each n > 2. From the discussion above, it follows that these desirable properties of $\hat{\theta}_1$ also hold if θ_2 is unknown.

4. Asymtotic Properties Of $\hat{\theta}_2$ When / θ_1/ >, 1

In this section, we show that if / θ_1/>,1, the asymptotic distribution of θ_2 is the same as that of the maximum likelihood estimator of θ_2 which would be available to somebody who knew the value of θ_1. Denote by θ_2^* the maximum likelihood estimator of θ_2 when θ_1 is known.

$$\theta_2^* = 1/n \sum^n_{i=1} (X_i - \theta_{i-1})^2 \text{ -----} \quad (7)$$

Since $n\theta_2^*/\theta_2$ has a chi-square distribution with n degrees of freedom, it follows that $(n/2\theta_2^2)^{1/2}$ $(\theta_2^x - \theta_2)$ is asymptotically standard normal.

Let R denote $\sum^n_{i=1} X_i - {}_1 Y_1$ and let S denote $\sum^n_{i=1}, X^2_{i=1}$. The following relations are easily verified:

$$\hat{\theta}_1 = \theta_1 + {}^R/_S \text{ ----------} \quad (8)$$

$$\hat{\theta}_2 - \theta_2^* = -(\hat{\theta}_1 - \theta_1)^2 S \text{ --------} \quad (9)$$

U

We will show that if $/\theta_1/\geq 1$, then $(\hat{\theta}_{1\,-}\ \theta_1)^2\ {}^S/n^{1/2}$ converses to as n increases: considering (9) and the asymptotic distribution of θ_2^*, this will prove that the asymptotic distribution of $({}^n/_2\ \theta_2^2)^{1/2}\ (\hat{\theta}_2 - \theta_2)$ is standard normal.

5. Discussion

First, we discuss the case where $/\theta_1/>1$. Since $X_1 = \sum^n_{i-1} \theta_1^{i-1}\ Y_i$ for i > 1. X_1 has a normal distribution with mean zero and variance $[\theta_2^2/(1 - \theta_1^2)]\ [n - 1 - (\theta_1^2 - \theta_1^2) / (1 - \theta_1^2)]$. It is easily shown that;

$$S = (1 - \theta_1^2)\ 1 - X_n^2 + \sum^n_{i-1} Y_i^2 + 2\theta_1 R] \text{ -------} \quad (10)$$

Thus, R is Op $(/\theta_1/^n)$. X_n is Op $(/\theta_1/^{2n})$, and $\sum^n_{1=1} Y_i^2$ is Op $(/\theta_1/^{*n})$. Then $(\hat{\theta}_1 - \theta_1)^2\ S/n^{1/2}$ converses to zero in probability as n increases.

Now we look at the case where / θ_1/ = 1. We carry out the details for the case $\theta_1 = -1$ being essentially the same. In this case R has mean $\theta_2 n\ (n - 1)/2$. In computing the variance of S, we use the fact that if j > i, then

$X_i = X_i + (Y_i + {}_1\ Y\ {}_{.......+}\ Y_1)$, and $(Y_i + {}_1\ Y\ {}_{.......+}\ Y_1)$ is independent of straightforward calculation then shows that the variance of S is $\theta_2^2\ n^4/_{12} + \theta_2^2\ O(n^3)$. We can write R as θ_2/n (n – 1) 12 + $12R^1$ where R^i is Op (1), and we can write S as $\theta_2 n$ (n – 1) 12 + $\theta_2 n^2 S^1/\ (12)^{1/2}$ where S^1 is Op (1). It follows directly that $\hat{\theta}_i - \theta_1 = R/S$ is Op $({}^1/_n)$, and then $(\hat{\theta}_i - \theta_1)^2\ S/n^{1/2}$ = Op $(n^{1/2})$. This completes the demonstration.

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