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Stochastic analysis of a reliability model for one-unit system with three types of repair policy

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Abstract
The present paper investigates a one-unit system with two types of repairman and resume/repeat repair policies. On the failure of the unit, it is undertaken by an ordinary repairman with the known fact that he may not be able to do some complex repairs. There may also be the possibility of rather damaging the unit by him during repair resulting it to go into more degraded stage. When the ordinary repairman finds himself unable, an expert repairman comes who first discusses the process of repair done by the ordinary repairman and then adopts either resume repair policy or one of the two types of repeat repair policies.
System is analysed by making use of regenerative point technique and various measures of system effectiveness are obtained. Profit is also evaluated.

Keywords: Stochastic analysis, reliability model, repair policy

Introduction

In the field of reliability, a large number of researchers have studied one-unit/two-unit standby systems under various assumptions. Some authors including [1-3] have taken the assumption that failed unit is undertaken by the perfect repairman whereas some others such as [4-6] assumed that the failed unit is first undertaken by the ordinary repairman and if the ordinary repairman is unable to repair it then an expert comes and starts repairing it. However, after getting some of the repair done by the ordinary repairman**,** the expert repairman may adopt one of the two repair policies-repeat repair policy or resume repair policy. This idea has been incorporated by Taneja and Nanda [1]. There may also be possibility that the ordinary repairman while trying to repair the failed unit may rather damage the failed unit which leads it to go to more degraded state. When repair for such unit is begun again, it is begun from the more degraded stage than the stage it had started earlier. This type of repair policy has not been taken up so far in the literature of reliability.

Keeping this in view, the present chapter aims at studying one-unit system with two types of repairman and introduce three types of repair policy. On the failure of the unit, it is undertaken by an ordinary repairman with the known fact that he may not be able to do some complex repairs. There may also be the possibility of rather damaging the unit by him during repair resulting it to go into more degraded stage. When the ordinary repairman finds himself unable, an expert repairman comes who first discusses the process of repair done by the ordinary repairman. After discussion, if it is found that the process of earlier repair was correct and no mishandling occurred then resume repair policy is adopted whereas if the process was incorrect or there is some mishandling took place, one of the following types of repair policy is to be adopted.

(a) The repair begins at the stage the ordinary repairman had taken over the unit (let us call this as repeat repair policy type-I)

(b) The repair begins from the more degraded stage as the ordinary repairman made some damages (let us call this as repeat repair policy type-II).

Assuming the failure time distribution as exponential and other time distributions as general, the following measures of system effectiveness are obtained:-

- Mean time to system failure (MTSF).
- Steady-state availability of the system.

- Expected busy period per unit time by the ordinary repairman.
- Expected fraction of the time for which the ordinary/expert repairman is busy in repairing the failed unit.
- Expected discussion time.
- Expected number of visits by the ordinary/expert repairman.
- Expected profit incurred to the system.

The system has been analysed by making use of semi-Markov processes and regenerative point technique. Graphical study is made and cut – off points for various rates/costs to study the economic aspect have been obtained.

Notations

p: probability that ordinary repairman is able to repair the failed unit

q: 1 – p, i.e., the probability that the ordinary repairman is unable to repair the failed unit

a: probability that the process of repair done by ordinary repairman was correct

b_1: probability that the process of repair done by the ordinary repairman was incorrect but did not lead to any further damage

b_2: probability that during the repair, the ordinary repairman leads the unit to more degraded stage.

h(t), H(t): p.d.f., c.d.f. of the discussion time

g(t), G(t): p.d.f., c.d.f. of repair time of the ordinary repairman

$g_1(t)$, $G_1(t)$: p.d.f., c.d.f. of the repair time of the expert repairman when resume repair policy is adopted

$g_2(t)$, $G_2(t)$: p.d.f., c.d.f. of the repair time of the expert repairman when repeat repair policy (type I) is adopted

$g_3(t)$, $G_3(t)$: p.d.f. of the repair time of the expert repairman when repeat repair policy (type II) is adopted.

Symbols for the State of the System are

o: operative

F_r: failed unit under repair of ordinary repairman

F_{de}: failed unit under discussion of the expert to know the nature of repair done by ordinary repairman

F_{re_1} : Failed unit under repair of the expert repairman while resume repair policy has been adopted

F_{re_2} : Failed unit under repair of the expert repairman while repeat repair policy (type I) has been adopted F_{re_3} : Failed unit under repair of the expert repairman while repeat repair policy (type II) has been adopted

Transition Probabilities and Mean Sojourn Times

The state transition diagram is shown as in Fig. 1.1. States 1, 2, 3, 4 and 5 are failed states. The epochs of entry into states 0, 1, 2, 3, 4, 5, are regeneration points and thus all the states are regenerative states.

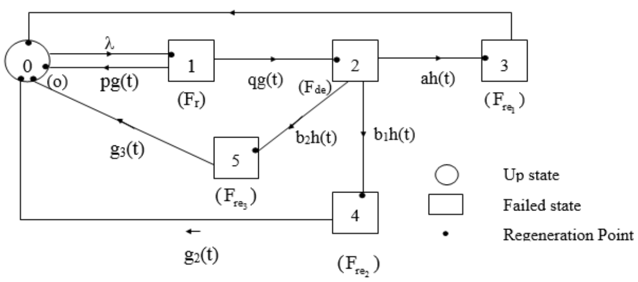

Fig. 1.1

The non-zero elements $p_{ij} = q_{ij}{}^*(s)$ are:

$p_{01} = 1$, $p_{10} = p$, $p_{12} = q$,
$p_{23} = a$, $p_{24} = b_1$, $p_{25} = b_2$
$p_{30} = 1$, $p_{40} = 1$, $p_{50} = 1$

By these probabilities, it can be verified that
$p_{10} + p_{12} = 1$, $p_{23} + p_{24} + p_{25} = 1$

Also μ_i, the mean sojourn time in state i are:

$\mu_0 = \frac{1}{\lambda}$, $\mu_1 = \int_0^\infty \overline{G}(t)\,dt = -g^{*\prime}(0)$, $\mu_2 = \int_0^\infty \overline{H}(t)\,dt = -h^{*\prime}(0)$

$\mu_3 = \int_0^\infty \overline{G_1}(t)\,dt = -g_1{}^{*\prime}(0)$, $\mu_4 = \int_0^\infty \overline{G_2}(t)\,dt = -g_2{}^{*\prime}(0)$

$\mu_5 = \int_0^\infty \overline{G_3}(t)\,dt = -g_3{}^{*\prime}(0)$

The unconditional mean time taken by the system to transit for any state j when it has taken from epoch of entrance into regenerative state i is mathematically stated as

$$m_{ij} = \int_0^\infty t\,dQ_{ij}(t) = -\left[\frac{d}{ds} q_{ij}{}^*(s)\right]_{s=0}$$

Thus

$m_{01} = \mu_0$, $m_{10} + m_{12} = \mu_1$, $m_{23} + m_{24} + m_{25} = \mu_2$
$m_{30} = \mu_3$, $m_{40} = \mu_4$, $m_{50} = \mu_5$

Various measures of system effectiveness obtained for the system are as follows:

Mean Time to System Failure (MTSF) = μ_0

In steady state, the availability of the system is given by

$A_0 = N_1 / D_1$

Where $N_1 = \mu_0$ and $D_1 = \mu_0 + \mu_1 + q(\mu_2 + a\mu_3 + b_1\mu_4 + b_2\mu_5)$

In steady-state, the total fraction of time which the system is under repair of the ordinary repairman, is given by

$B_0 = N_2 / D_1$

Where $N_2 = \mu_1$ and D_1 is already specified.

In steady-state, the total fraction of time for which the system is under repair of an expert repairman, is given by

$B_0^e = N_3 / D_1$

Where $N_3 = p_{12}(\mu_3 p_{23} + \mu_4 p_{24} + \mu_5 p_{25})$
and D_1 is already specified.

In steady state, the total fraction of the discussion time of the expert repairman, is given by

$DT_0 = N_4/D_1$

where $N_4 = \mu_2 p_{12}$
and D_1 is already specified.

In steady-state, the total number of visits by the ordinary repairman per unit time is given by

$V_0 = N_5/D_1$

Where $N_5 = 1$
and D_1 is already specified.

In steady-state, the total number of visits by the expert repairman per unit time is given by

$V_0^e = N_5/D_1$

Where $N_6 = p_{12}$
and D_1 is already specified.

Profit Analysis

The expected total profit incurred to the system in steady-state is given by

$$P_2 = C_0A_0 - C_1B_0 - C_2 B_0^e - C_3DT_0 - C_4V_0 - C_5V_0^e$$

Where

C_0 = revenue per unit up time of the system
C_1 = cost per unit time for which ordinary repairman is busy
C_2 = cost per unit time for which expert repairman is busy in repairing the failed unit
C_3 = cost per unit time for which the expert repairman is busy in discussion
C_4 = cost per visit of the ordinary repairman
C_5 = cost per visit of the expert repairman

Particular Case

For graphical interpretation, the following particular case is considered:

$g(t) = \alpha e^{-\alpha t}$; $g_1(t) = \alpha_1 e^{-\alpha_1 t}$
$g_2(t) = \alpha_2 e^{-\alpha_2 t}$; $g_3(t) = \alpha_3 e^{-\alpha_3 t}$
$h(t) = \beta e^{-\beta t}$

On the basis of the numerical values taken as
$p = 0.5$, $q = 0.5$, $a = 0.2$, $b_1 = 0.7$, $b_2 = 0.1$, $\beta = 10$, $\alpha = 0.25$, $\alpha_1 = 0.4$, $\alpha_2 = 0.35$, $\alpha_3 = 0.2$, $\lambda = 0.005$.
The values of various measures of system effectiveness are obtained as

Mean time to system failure (MTSF) = 200
Availability (A_0) = .9729992
Busy period of ordinary repairman (B_0) = 0.01946
Busy period of expert repairman (B_0^e) = 0.007297
Expected discussion time (DT_0) = 0.00024325
Expected number of visits by the ordinary repairman (V_0) = 0.004865
Expected number of visits by the expert repairman (V_0^e) = 0.0024325

Graphical Interpretation

For the graphical interpretation, the mentioned particular case is considered. Figs. 1.2 and 1.3 shows the behaviour of MTSF and availability respectively with respect to failure rate (λ). It is clear from the graph that the MTSF and the availability both get decrease with increase in the values of failure rate.

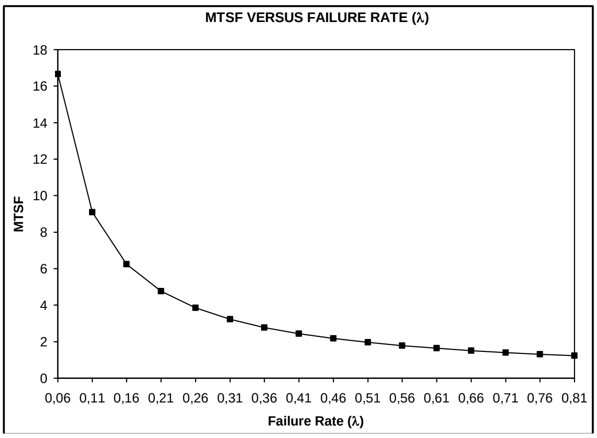

Fig 1.2

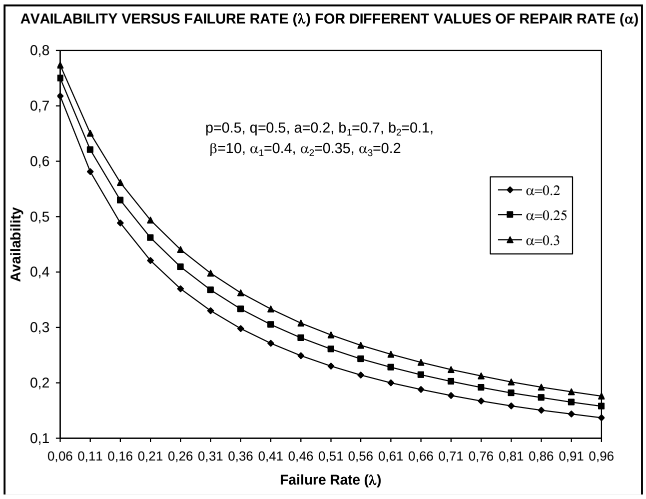

Fig 1.3

Fig. 1.4 reveals the pattern of the profit with respect to failure rate (λ) for different values of repair rate (α). The profit decreases with the increase in the values of failure rate (λ) and is higher for higher values of repair rate (α). Following can also be observed from the graph:

(i) For $\alpha = 0.2$, P_2> or = or < 0 according as λ < or = or > 0.0739. So, the system is profitable only if failure rate is lesser than 0.0739.
(ii) For $\alpha = 0.25$, P_2 is > or = or < 0 according as λ < or = or > 0.085. So, the system is profitable only if failure rate is lesser than 0.085.
(iii) For $\alpha = 0.3$, P_2 is > or = or < 0 according as λ < or = or > 0.093. So, the system is profitable only if failure rate is lesser than 0.093.

So, the companies using such systems can be suggested to purchase only those system which do not have failure rates greater than those discussed in points (i) to (iii) above in this particular case.

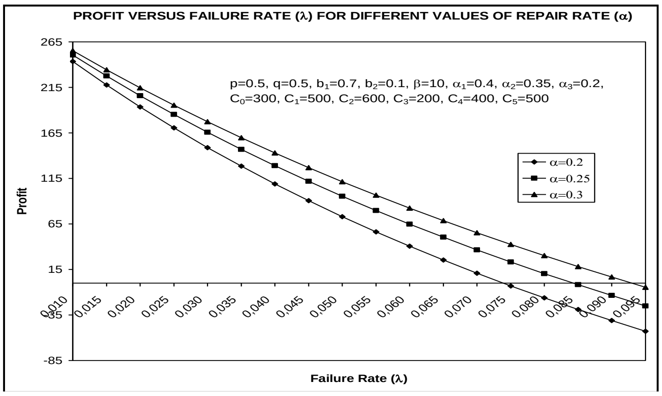

Fig 1.4

Fig. 1.5 shows the behaviour of the profit with respect to revenue per unit time (C_0) for different values of cost (C_2). The profit increases with the increase in the values of revenue (C_0) and becomes lower for higher values of C_2. Following conclusions are drawn:

(i) For $C_2 = 600$, P_2 > or = or < 0 according as C_0 > or = or < 605.2. So, C_0 should be greater than 605.2.
(ii) For $C_2 = 750$, P_2 > or = or < 0 according as C_0 > or = or < 643.5. So, for this case C_0 should be greater than 643.5.
(iii) For $C_2 = 900$, P_2 > or = or < 0 according as C_0 > or = or < 681.7. Therefore, for this case C_0 should be greater than 681.7.

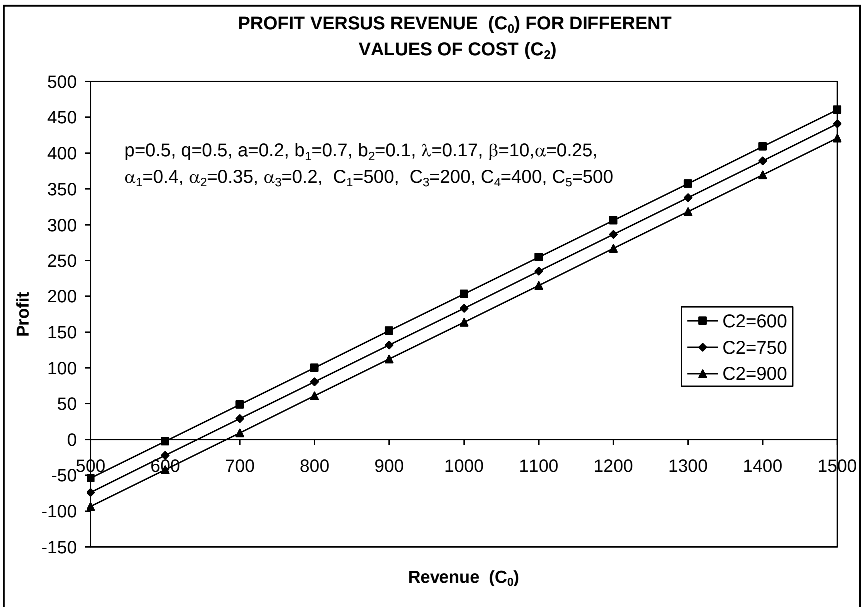

Fig 1.5

Fig. 1.6 depicts the pattern of profit with respect to cost (C_5) for different values of discussion rate (β). The profit decreases with increase in the values of (C_5) and is higher for higher values of discussion rate (β). Following observations can be made:

(i) For $\beta = 1$, $P_2 >$ or = or < 0 according as $C_5 <$ or = or $>$ 1200. So system is profitable if $C_5 < 1200$.
(ii) For $\beta = 2$, $P_2 >$ or = or < 0 according as $C_5 <$ or = or $>$ 1300. So system is profitable if $C_5 < 1300$.
(iii) For $\beta = 3$, $P_2 >$ or = or < 0 according as $C_5 <$ or = or $>$ 1380. So, for this particular case we should take $C_5 < 1380$.

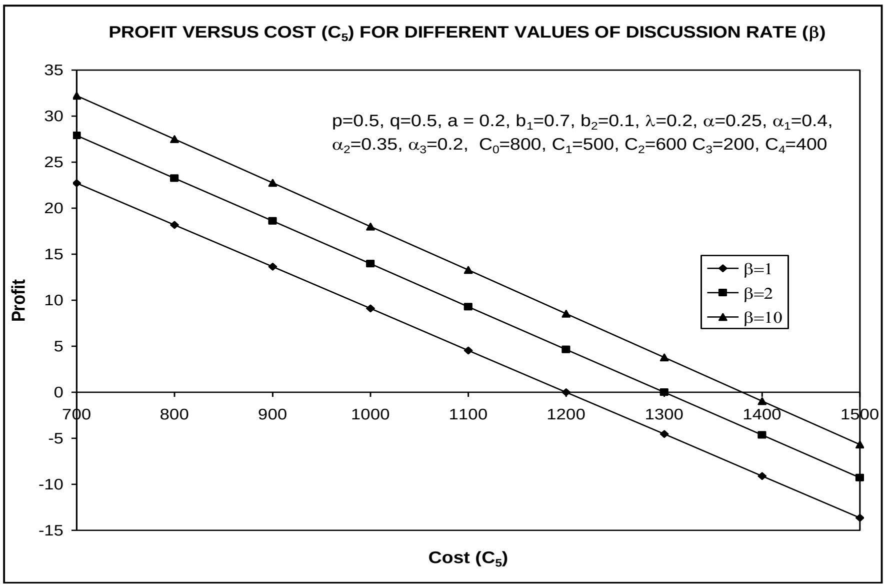

Fig 1.6

Fig. 1.7 shows the behaviour of profit with respect to probability (p) for different values of probability (a). Profit increases as p increases and becomes higher for higher values of repair rate (α). Following conclusions can be drawn:
(i) For $a = 0.1$, $P_2 >$ or = or < 0 according as $p >$ or = or $<$ 0.697. Therefore p should be greater than 0.697.
(ii) For $a = 0.4$, $P_2 >$ or = or < 0 according as $p >$ or = or $<$ 0.688. Therefore, system is profitable if $p > 0.688$.
(iii) For $a = 0.7$, $P_2 >$ or = or < 0 according as $p >$ or = or $<$ 0.678. Therefore, system is profitable if $p > 0.678$.

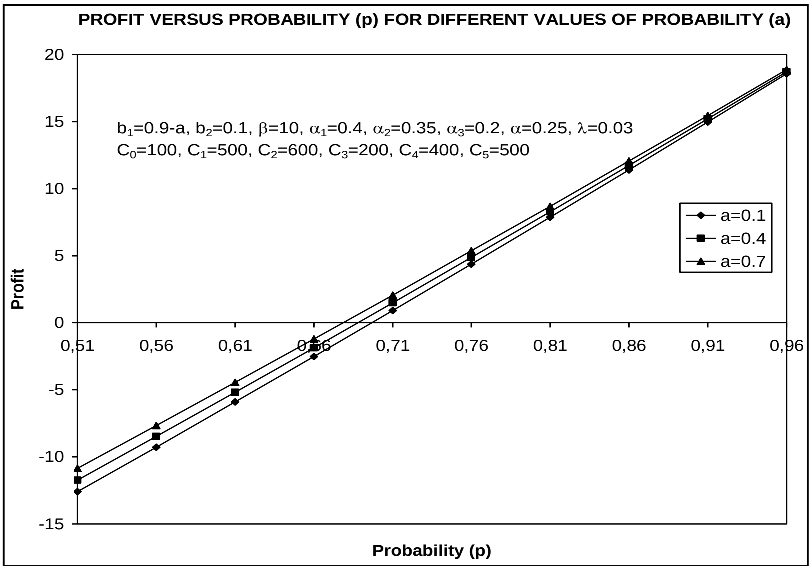

Fig 1.7

It is also observed that the three curves converge as $p \rightarrow 1$ which implies that profit comes out to be same as $p \rightarrow 1$ irrespective of the values of probability (a).

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