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Mamta Kumari
Research Scholar, Maharshi Dayanand University, Rohtak, Haryana, India

Correspondence
Mamta Kumari
Research Scholar, Maharshi Dayanand University, Rohtak, Haryana, India

New special homogeneous Bianchi type I Massive string cosmological models with anisotropic dark energy

Mamta Kumari

Abstract
A special law of variation for Hubble's parameter is presented in a spatially homogeneous Bianchi type-I massive string cosmological models with space-time that yields a constant value of deceleration parameter. Using the law of variation for Hubble's parameter, exact solutions of Einstein's field equations are obtained for Bianchi-I space-time filled with perfect fluid in two different cases where the universe exhibits power-law and exponential expansion. It is found that the solutions are consistent with the recent observations of type Ia supernovae. A detailed study of physical and kinematical properties of the models is carried out.

Keywords: Hubble's parameter, cosmological models, anisotropic dark energy

Introduction
The study of cosmic strings has been a subject of much interest for cosmologists. It is believed that these strings give rise to density perturbations leading to the formation of galaxies (Zel'dovich [1980]). String theory is a hypothetical framework in which the point-like particles are replaced by one-dimensional objects called strings. On distance scales larger than the string scale, a string in believed to work just like an ordinary particle. In the early universe (strings dominated era), the strings produce fluctuations in the density of particles. We may speculate that as strings vanish and particles become important, then the fluctuations grow in such a way that finally we shall end up with galaxies. The presence of string in the early universe can be explained using grand unified theories (GUT) as already discussed by Kibble (1976), Everet (1981), Vilenkin (1982). These strings have stress energy and are classified as geometric and massive strings.

Letelier (1979, 1981) studied a gauge onvariant model of a cloud formed by geometric strings and used the model as a source of gravitational field. Stachel (1980) has developed the classical theory of geometric strings as a theory of simple surface forming time-like bivector field in an arbitrary background space-time. Letelier (1979) has formulated the energy-momentum tensor for massive strings and explained that the massive strings are formed by geometric strings with particles attached its extension. Further, Letelier (1983) used this idea for deriving cosmological models of Bianchi type- I and Kantowski-Sachs space-times in the presence of massive strings. Matraverse (1988) has presented a class of exact solutions of Einstein's field equations with a two-parameter family of classical strings. Krori *et al*. (1990) have obtained some exact solutions in string cosmology for homogenous spaces for Bianchi type II, VI, VIII and IX. Banerjee *et al*. (1990) have studied Bianchi type- I string cosmological models with and without the source-free magnetic field. Tikekar and Patel (1992) have obtained some Bianchi type III cosmological solutions of massive strings in the presence of a magnetic field. Shri Ram and Singh (1995) have presented some spatially homogenous type I Bianchi massive string models with and without source free magnetic field. Chakraborty (1991) has discussed string cosmology in Bianchi type VI_o space-time. Bali *et al*. (2003, 2008) have studied spatially homogenous string cosmological models in different physical contexts. Wang (2006) has investigated a Bianchi type III massive string cosmological model with magnetic field. Pradhan (2011) has discussed anisotropic Bianchi type I magnetized string cosmological models with decaying vacuum energy density $\wedge$ (t).

Saha and Visinescu (2010) and Saha *et al*. (2010) have studied Bianchi Type I models with cosmic strings in the presence of magnetic flux.

In general relativity, cosmological models are usually constructed under the assumption that the matter content is an idealized prefect fluid. This assumption may be a good approximation to the actual matter content of the universe at present time. The evolution of isotropic cosmological models filled with perfect fluids has been extensively studied by many workers. It is certainly of considerable interest to study cosmologies with richer structure both geometrically and physically than the standard prefect fluid FRW models. It is of interest to take into account dissipative process such as viscosity and heat conduction in cosmological models. Misner (1968) has studied the effect of viscosity on the evolution of cosmological models.

Several authors viz. Belinski and Khalatnikov (1976), Banerjee and Santos (1984), Padmanabhan and Chitre (1987), Beesham (1993), Bali *et al*. (2008) Shri Ram *et al*. (2009) have obtained exact solutions of Einstein's field equations by taking viscous effects in isotropic as well as anisotropic space-times. The recent observations of Riess *et al*. (1998). Perlmutter (1998) have led to the belief that a cosmological constant in a kind of repulsive pressure, dubbed as dark energy, and is most suitable candidate to explain the recent observations that the universe is not only expanding but also accelerating. The cosmological term $\wedge$ is also interpreted as the vacuum energy density. Cosmological models with time-dependent $\wedge$ term have been investigated so far by many authors. Recently, Bali and Swati (2016) have investigated a Bianchi type-I massive string cosmological model with bulk viscosity and vacuum energy density.

In this paper, we obtain a new spatially homogenous Bianchi type-I massive string cosmological models with bulk viscosity and vacuum energy density. The plan of the paper is as follows. In Sect. 2, we present the metric and field equations by assuming a special law of variation of Hubble parameter. We discuss the physical and kinematical properties of the model is Sect. 4. In Sect. 5 we outline some concluding remarks.

Metric and field equations

We consider the spatially homogenous Bianchi type-I line element given by

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + C^2dz^2 \qquad (1)$$

where the metric potentials A, B and C are functions of cosmic time t.

The energy-momentum tensor for a cloud of massive string dust with a bulk viscous fluid of string in given by Letelier (1983) and Landau and Lifshitz (1963) as

$$T_\mu^\upsilon = \rho\upsilon_\mu\, \upsilon^\nu - \lambda x_\mu x^\nu - \xi\theta\left(\delta_\mu^\upsilon + v_\mu v^\upsilon\right) \qquad (2)$$

where υ_μ and x_μ satisfy conditions

$$\upsilon_\mu\upsilon^\mu = -x_\mu x^\mu = 1,\ \upsilon_\mu x^\mu = 0. \qquad (3)$$

Here ρ is the proper energy density for a could of strings with particles attached to them, λ is the string tension density, υ^μ is the four-velocity of the particles, x^μ is a unit space-like vector representing the direction of string, ξ is the coefficient of bulk viscosity and θ is the expansion scalar. If the particle density of the configuration is denoted by ρ_p, then we have

$$\rho = \rho_p + \lambda \qquad (4)$$

Einstein's field equations for a system of strings with vacuum energy density are given by

$$R_\mu^\upsilon - \frac{1}{2}R\delta_\mu^\upsilon = -T_\mu^\upsilon + \wedge\delta_\mu^\upsilon \qquad (5)$$

where $R_{\mu\nu}$ is the Ricci tensor and R is the scalar curvature in the gravitational unit $8\pi G = 1$, c=1

We assume that the string's direction is along the x-axis, so that $x^\mu = \left(\frac{1}{A},0,0,0\right)$. In comoving coordinate system $v^\mu = (0,0,0,1)$, the field equations (5) together with (2) and (3) lead to the following system of equations.

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = \lambda + \xi\theta + \wedge, \qquad (6)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = \xi\theta + \wedge, \qquad (7)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = \xi\theta + \wedge, \qquad (8)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} = \rho + \wedge. \qquad (9)$$

where an overdot denotes ordinary derivative with respect to t.

For the model (1), the spatial volume (V) and average scale factor a are given by

$$V = a^3 = ABC. \qquad (10)$$

The expansion scalar (θ), shear scalar (σ), the anisotropy parameter (Am) and, Hubble parameter H are given by

$$\theta = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}, \qquad (11)$$

$$\sigma = \frac{1}{2}\left(\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B} + \frac{\dot{C}^2}{C}\right) - \frac{1}{6}\theta^2, \qquad (12)$$

$$Am = \frac{1}{3}\sum_{\mu=1}^{3}\left(\frac{H_\mu - H}{H}\right)^2, \qquad (13)$$

$$H = \frac{1}{3}(H_1 + H_2 + H_3). \qquad (14)$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$ and $H_3 = \frac{\dot{C}}{C}$ are the directional Hubble parameters in the direction of x, y and z-axes respectively.

An important observational quantity in cosmology is the deceleration parameter (q) defined by

$$q = -\frac{a\ddot{a}}{\dot{a}^2}. \qquad (15)$$

The sign of q indicates whether the model inflates or not. The positive value of q corresponds to the standard decelerating universe and the negative value indicates inflation.

Solutions of the field equations

We now obtain exact solutions of (6)-(9) which are four equations in seven unknowns A, B, C, ρ, λ, ξ and $\wedge$. Therefore to obtain a deterministic model, we need extra conditions depending upon the physical nature of the problem.

Subtracting (8) from (7), we get

$$\frac{\ddot{B}}{B}-\frac{\ddot{C}}{C}+\frac{\dot{A}}{A}\left(\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right)=0. \quad (16)$$

Equation (16), on integration, provides

$$\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=\frac{k}{a^3} \quad (17)$$

where k is an arbitrary constant. To treat (17) we consider the following relation

$$A^n = BC,\ n\neq 0 \ \text{(constant)} \quad (18)$$

as suggested by Goswami *et al*. (2016). Future, we set

$$B = A^{n/2}D,\ C = A^{n/2}D^{-1} \quad (19)$$

where D(t) is an arbitrary function. Substituting (19) in (17) and integrating of the resulting equation we get

$$\frac{\dot{D}}{D}=\frac{K}{a^3}, \quad (20)$$

where $K=\frac{k}{2}$. From (10) and (18), we find that

$$V = a^3 = A^{n+1} \quad (21)$$

We can determine the function D(t) from (20) if the average scale factor a is a known function of time. Hence to obtain an expression for the average scale factor a, we can apply the special law of variation of Hubble parameter proposed by Berman (1983) that yields constant value of the deceleration parameter. It may be noted that here most- of the well known models in general relativity and alternative theories including inflationary models are model with constant deceleration parameter. For an accelerating models of the universe, we take the constant negative. Then (15) gives the solution

$$a=(ct+d)^{\frac{1}{1+q}} \quad (22)$$

where $c\neq 0$ and d are integration constants. This equation implies that the condition of expansion is $1+q>0$.

Substituting (22) in (20) and integrating, we obtain

$$D = N\exp\left\{-\frac{K(1+q)}{c(2-q)}(ct+d)^{\frac{q-2}{1+q}}\right\} \quad (23)$$

where N is integration constant. Without loss of generality we can take N=1. Therefore from (19), (21), (22) and (23), the solutions for the scale factor A, B and C are obtained as

$$A=(ct+d)^{\frac{3}{(n+1)(1+q)}} \quad (24)$$

$$B=(ct+d)^{\frac{3n}{2(n+1)(1+q)}}\exp\left\{-\frac{K(1+q)}{c(2-q)}(ct+d)^{\frac{q-2}{1+q}}\right\} \quad (25)$$

$$C=(ct+d)^{\frac{3n}{2(n+1)(1+q)}}\exp\left\{\frac{K(1+q)}{c(2-q)}(ct+d)^{\frac{q-2}{1+q}}\right\} \quad (26)$$

To get a deterministic model of universe, we assume that

$$\wedge=\frac{\alpha}{a^3} \quad (27)$$

where α is a constant. Then, from (22) and (27), the vacuum energy density $\wedge$ has the value given by

$$\wedge=\frac{\alpha}{(ct+d)^{\frac{3}{1+q}}} \quad (28)$$

Physical feature of cosmological model

For the present model, we obtain

$$H_1=\frac{3c}{(1+q)(n+1)(ct+d)}, \quad (29)$$

$$H_2=\frac{3nc}{2(1+q)(n+1)(ct+d)}+\frac{K}{(ct+d)^{\frac{3}{1+q}}}, \quad (30)$$

$$H_3=\frac{3nc}{2(1+q)(n+1)(ct+d)}-\frac{K}{(ct+d)^{\frac{3}{1+q}}}, \quad (31)$$

$$H=\frac{c}{(1+q)(ct+d)}. \quad (32)$$

The expansion scalar, shear scalar and anisotropy parameter are obtained as

$$\theta=\frac{3c}{(1+q)(ct+d)}, \quad (33)$$

$$\sigma^2=\frac{9c^2(2n^2-2n+5)}{2(1+q)^2(n+1)^2(ct+d)^2}+\frac{2K^2}{(ct+d)^{6/1+q}} \quad (34)$$

$$\Delta=\frac{1}{2}\left(\frac{n-2}{n+1}\right)^2+\frac{2}{3}\frac{K^2c^2(1+q)^2}{(ct+d)^{4-2q/1+q}} \quad (35)$$

The energy density, string tensor density, particle density and bulk viscosity coefficient are given by

$$\rho=\frac{9n(n+4)c^2}{4(1+q)^2(n+1)^2(ct+d)^2}-\frac{K^2}{(ct+d)^{\frac{6}{1+q}}}-\frac{\alpha}{(ct+d)^{\frac{3}{1+q}}}, \quad (36)$$

$$\lambda=\frac{9c^2(n^2-n-2)}{2(1+q)^2(n+1)^2(ct+d)^2}+\frac{3c^2(2-n)}{2(1+q)\ (n+1)\ (ct+d)^2}-\frac{\alpha}{(ct+d)^{\frac{3}{1+q}}}, \quad (37)$$

$$\rho_p=\frac{9c^2(4+6n-n^2)}{4(1+q)^2(n+1)^2(ct+d)^2}+\frac{3c^2(n-2)c^2}{2(1+q)\ (n+1)\ (ct+d)^2}-\frac{K^2}{(ct+d)^{\frac{6}{1+q}}}, \quad (38)$$

$$\xi=\frac{3c\ (n^2+2n+4)}{4(1+q)\ (n+1)^2(ct+d)}-\frac{c\ (n+2)}{2(n+1)\ (ct+d)}+\frac{K^2(1+q)}{3c(ct+d)^{\frac{5-q}{1+q}}}-\frac{\alpha(1+q)}{(ct+d)^{\frac{2-q}{1+q}}} \quad (39)$$

We observe that the spatial volume is zero at t= - b/a$\equiv$$t_o$. As t $\rightarrow t_o$. the Hubble parameter, the scalar expansion and shear scalar assume infinitely large values whereas with the growth of cosmic time they decrease to null values as t$\rightarrow\infty$. At the instant $t=t_o$, p, λ, ρ_p are all infinite. Thus, the model starts evolving with a big-bang singularity at $t=t_o$. As t tends to infinity, the spatial becomes infinite and the physical and kinematical parameter all tend to zero. Therefore, the model essentially gives an empty space-time for large time. The anisotropy parameters Δ, being infinite initially, tends to a constant as $t\rightarrow\infty$. This means that the anisotropy of the model is maintained throughout the passage of time. Since the deceleration parameter is negative, the present model represents an accelerating phase of the universe. The strings eventually disappear from the universe for sufficiently large time. The model is compatible with the results of recent observations on present-day universe from type Ia supernova (Riess, *et al*. 1998; Perlmutter *et al*. 1998.)

Conclusions

In this paper, a spatially homogenous Bianchi type I model representing massive strings with bulk viscosity and vacuum energy density has been studied. The exact solutions of the field equations have been obtained by applying a special law of variation of Hubble parameter which yields a constant value of the deceleration parameter. By assuming the negative constant deceleration parameter and a time-decaying form vacuum energy density, an accelerating model of the universe has been presented. The model starts evolving from a finite big-bang singularity. The anisotropy in the model is maintained throughout its evolution. The strings dominate in the early universe and eventually disappear from the universe for sufficiently large time. For sufficiently large time, the model would essentially gives an empty space-time as ρ, λ and ρ_p all tend to zero as $t\rightarrow\infty$. The model presented here is physically meaningful as the associated parameters behave reasonably.

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