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Comparative study of convergence and solution of algebraic and transcendental equations using zero-finding methods

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Abstract
The paper is about Newton-Raphson Method which is used to solve non-square and non-linear problems. The study also aims to comparing the rate of performance, rate of convergence, root finding. It also represents a new approach calculation using non-linear terms and this will be similar to Newton Raphson simple method. An inverse Jacobian matrix will be used for the iteration process and this will be further used for distributed power load flow calculation and will also be helpful in some of the application.

Keywords: Rate of convergence, Jacobian matrix, non-linear equations

Introduction
Here we consider the nonlinear equation and try to find the solution of it. We have to found the solution with the help of Newton -Raphson method. This method is very familiar for its fast rate of convergence and for improving the convergence property. Newton's method, named after Isaac- Newton and Joseph -Raphson.

Any zero-finding method (Bisection Method, False Position Method, Newton-Raphson, etc.) can also be used to find a minimum or maximum of such a function, by finding a zero in the function's first derivative, see Newton's method has an optimization algorithm. Root finding is also one of the problem in practical applications. Newton method is very fast and efficient as compared to the other methods. Newton method requires only one iteration and the derivative evaluation per iteration. The result of comparing the rate of convergence of Bisection, Newton and Secant methods came as Bisection method < Newton method < Secant method which in terms of number is that the Newton method is 7.678622465 times better than the Bisection method whereas Secant method is 1.389482397 times better than the Newton method. Finding roots of the nonlinear equation with the help of Newton Raphson method, provides good result with fast convergence speed and Mat lab also adopted this method for finding the roots and tool used for such calculations is scientific calculator.

Formulation
In numerical analysis, Newton's method (also known as the Newton–Raphson method), named after Isaac Newton and Joseph Raphson, is a method for finding successively better approximations to the roots (or zeroes) of a real valued function.

$$x: f(x)=0.$$

The Newton–Raphson method in one variable is implemented as follows:

Given a function f defined over the reals x, and its derivative f', we begin with a first guess x_0 for a root of the function f. Provided the function satisfies all the assumptions made in the derivation of the formula, a better approximation x_1 is

$$x_1=x_0-\frac{f(x_0)}{f'(x_0)}$$

Geometrically, $(x_1, 0)$ is the intersection with the x-axis of the tangent to the graph of f at $(x_0, f(x_0))$.
The process is repeated as

$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$$

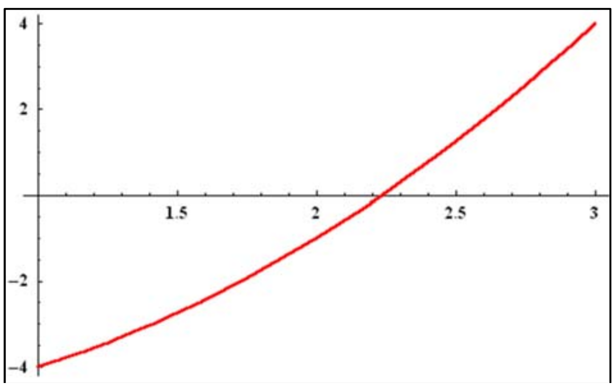
Until a sufficiently accurate value is reached.

Applications

Square root of a number

Consider the problem of finding the square root of a number. Newton's method is one of many methods of computing square roots.

Example. Let us find an approximation to $\sqrt{5}$ to ten decimal places.
Note that $\sqrt{5}$ is an irrational number. Therefore the sequence of decimals which defines $\sqrt{5}$ will not stop. Clearly r = $\sqrt{5}$ is the only zero of $f(x) = x^2 - 5$ on the interval [1, 3].



Let $\{x_n\}$ be the successive approximations obtained through Newton's method. We have

$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}=x_n-\frac{x_n^2-5}{2x_n}\,.$$

Let us start this process by taking $x_1 = 2$.

$$\begin{aligned} x_1 &= 2 \\ x_2 &= 2.25 \\ x_3 &= 2.2361111111111111111111111111111111 \\ x_4 &= 2.2360679779158040027605244996549344 \\ x_5 &= 2.2360679774997896964478728283271100 \\ x_6 &= 2.2360679774997896964091736687312762 \end{aligned}$$

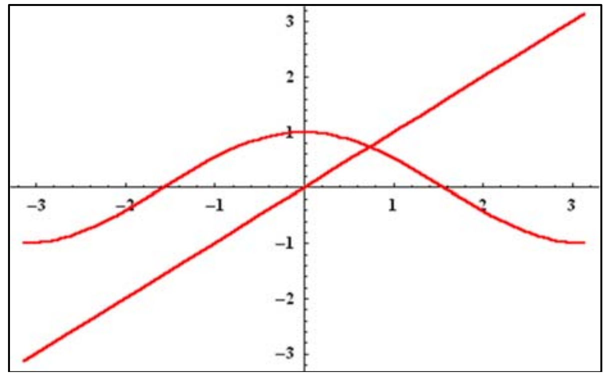
It is quite remarkable that the results stabilize for more than ten decimal places after only 5 iterations!

Trigonometry function

Let us approximate the only solution to the equation

$$x=\cos(x)\,.$$

In fact, looking at the graphs we can see that this equation has one solution.



This solution is also the only zero of the function f(x) = x-cos(x). So now we see how Newton's method may be used to approximate *r*. Since *r* lies between 0 and $\pi/2$, we set $x_1 = 1$. The rest of the sequence is generated through the formula

$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}=x_n-\frac{x_n-\cos(x_n)}{1+\sin(x_n)}\,.$$

We have

$$\begin{aligned} x_1 &= 1. \\ x_2 &= 0.750363867840243893034942306682177 \\ x_3 &= 0.739112890911361670360585290904890 \\ x_4 &= 0.739085133385283969760125120856804 \\ x_5 &= 0.739085133215160641661702625685026 \\ x_6 &= 0.739085133215160641655312087673873 \\ x_7 &= 0.739085133215160641655312087673873 \\ x_8 &= 0.739085133215160641655312087673873 \end{aligned}$$

Convergence of Newton-Raphson Method

Suppose x_r is a root of f(x)=0 and x_n is an estimate of x_r s.t. $|x_r - x_n| = \delta << 1$. Then by Taylor's series expansion we have,

$0=f(x_r)=f(x_n+\delta)=f(x_n)+f'(x_n)(x_r-x_n)+(f''(x_n)/2)(x_r-x_n)^2$,

for some between x_n and x_r. By Newton-Raphson method, we know that

$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$$

i.e.

$f(x_n)=f'(x_n)(x_n-x_{n+1})$

$\Rightarrow 0= f'(x_n)(x_n-x_{n+1})+(f''(x_n)/2)(x_r-x_n)^2$

Say

$e_n=(x_r - x_n)$, $e_{n+1}=(x_r-x_{n+1})$

Where e_n, e_{n+1} denote the error in the solution at n and (n+1) iterations.

Therefore,

$e_{n+1}= -(f''(x_n)/2 f'(x_n) \approx e_n^2$

$\Rightarrow e_{n+1} \propto e_n^2$

Therefore, Newton Raphson Method is said to have quadratic convergence.

Conclusion

From this paper we have concluded that, the convergence rate of Newton method is fast as compared to other methods. Secant method is the most effective method as it has a convergence rate close to that of the Newton Raphson method but it requires only a single function evaluating per iteration. We have also cocluded that the convergence rate of bisection method is very slow and it is difficult to extend such kind of systems equations. So in comparison, Newton's method have a fast converging rate. The effectiveness of using scientific calculator in solving non-linear equations using Newton-Raphson method also reduces the time complexity for solving nonlinear equations. In this work a sequence of iterative methods for solving nonlinear equation f(x) = 0 with higher-order convergence is developed. The method can be continuously applied to generate an iterative scheme with arbitrarily specified order of convergence. We also concluded that the Newton Raphson method can be used very effectively to determine the intrinsic value based on its measured permittivity.

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