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Stochastic models for reservoir inflow simulation

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Abstract
Forecasting of inflow to a new reservoir and modelling of the historical behaviour of an existing reservoir can be accomplished through simulation models. First order stationary Markov model assuming stationary mean, standard deviation and lag1 correlation for the historical and generated data would result in a perfect simulation for annual stream flows and reservoir inflows. Seasonal models can be introduced in which the seasons can be either months, or any intra year period variation and here the inherent periodicity would introduce non stationarity in the data. First order Markov model addressing the non stationarity is a better alternative recommended in this situation. Simulations for Karappara (10^0 27' 25"N 76^0 38' 51" E) Stream flow and Idamalayar (10^0 13' 18" N 76^0 42' 21" E) Reservoir Inflow belonging to Kerala state have been realised with respect to both stationary and non-stationary data.

Keywords: Stream flow, reservoir inflow, simulation, markov model

1. Introduction

The main impacts of climate change have been discussed globally over last two decades. It would affect agriculture in several ways. As far as water resources are concerned a warmer climate would accelerate the hydrological cycle and it can alter the intensity and timing of rainfall. Global warming and the resulted decline in rainfall may reduce net recharge and can affect ground water levels. Decrease in winter precipitation would reduce the total seasonal precipitation and can impose greater water stress. At the same time, soil erosion would be at a greater rate with increased rainfall amounts [1].

In this climate change scenario, distribution of water for agricultural purposes is a complicated issue and its management is generally affected by social, environmental and political factors [2]. Surplus water during rainy seasons can be stored in reservoirs and can be utilised for irrigation in drought periods. Prevention of floods can also be ensured. So a well organised operation of reservoir system is important for getting maximum net benefit from the available water resource. It will regulate inflows and provide outflows in a regular rate according to demand. Generally water management involves the supply and distribution of the right amount of water at the right time to the right place so that the agricultural, industrial and commercial activities and other day to day domestic needs of the society are satisfied. A serious constraint in this regard is the shortage of reservoir water. For a precise estimate of reservoir yield, simulation models need to be developed.

Hence a study was carried out to develop stochastic models to simulate stream flow as well as reservoir inflow using the long term observed data and to compare the generated flow with the observed values and consequently to anticipate future flows with reliability.

In general reservoir simulation models may be used directly to forecast the performance of a new reservoir or to model the historical behaviour of an existing reservoir [3]. Forecasts are made for a variety of operating conditions through the history matched model. Economic models are combined with these models to make decisions concerning the regular operations of the reservoir. The expected performance of the system can be evaluated for a set of given design and policy parameters. Based on given operating rules, the performance of the system for the next 50 or 100 years can be simulated. Determining the sequence of annual operations of irrigation and hydropower, the benefits derived out of it can be estimated.

The sequential nature of the reservoir management decisions, together with the inherent randomness of natural water inflows direct us to use Markov decision processes for modelling reservoir management problems and their optimisation through stochastic dynamic programming [4]. Markov chain model was found to be useful as a predictive device for studying reservoir elevation of Shiroro Dam [5].

Synthetically generated sequences of the inflow are made use of for simulation of the reservoir operations. This branch of synthetic hydrology is well appreciated by Engineers and hydrologists because it can accommodate complex inflow models. The important elements involved in the modelling are the time unit of operation of the reservoir, the persistence of flows, the seasonality, the unit of volume, the release and finally the finiteness or otherwise of the reservoir [6]. A monthly time series model incorporating both seasonality and autocorrelation would be resulted if the time unit is a month. Markov processes with seasonality varying transition probabilities can be effectively used to model the persistence and the seasonality. The time unit can be selected as 10 days, 15 days or months according to the situation.

1.1 Location of the Study

The study was conducted to simulate reservoir inflow of Idamalyar Dam (10^0 13' 18" N 76^0 42' 21" E). It is a multipurpose concrete gravity Dam built across Idamalayar river, a tributary of Periyar river in Kerala. It originates in the Anamala hills at about 2500m above sea level. It has an annual rainfall of 6000mm and inflow of $5539Mm^3$. In addition, the storage is supplemented with water let from Nirar under interstate agreement and a portion of the excess water from the Poringalkuth reservoir of Chalakudy river. The water stored in the reservoir is used for generating electricity at the 75MW power station and the tail water is led to the Idamalayar river itself and collected at Bhoothathankettu barrage of Periyar Valley Irrigation Project. The irrigation benefits covers an area of 14394 hectares of agricultural land. The cultivable command area is 13209 hectares.

The next location of study was the Karappara river near Nelliampathy in Palakkad District of Kerala. It is a tributary of Chalakudy river having a catchment area of $47.695km^2$ for the gauging weir. Karappara- Kuriarkutty multi-purpose project for power generation and irrigation in the Chittur taluk, in Palakkad district which experiences severe shortage of water and power was proposed years ago.

2. Data and Methodology

Daily data on Idamalayar Reservoir **i**nflow for a period from 1989 june -2015 May and the daily stream flow at Karappara gauging weir for a period from June '76 to May 2002 were used for the study.

The main principle behind data generation in hydrology is that there is a statistical regularity of the hydrologic processes unless major changes occur. Information collected from the historical periods are used to make an assessment of how the process is likely to behave in the future and using this principle, data for the future is generated. The essential information content, in terms of probability, general stochastic behaviour of the process and statistical parameters are captured. Using these informations of the past data, future sequence of observations are generated. It is always better to base the future decisions upon several sequences rather than basing on a single sequence to avoid the risk of occurrence of extreme events like floods, droughts etc. All the parameters of the historical data would be extended or preserved so that the several sequences generated would follow the same distribution as that of the past.

As far as hydrologic processes are concerned, there is a persistence in nature. That means there is a tendency of flows to follow the trend of immediate past. The generating models reproduce the statistical distribution and persistence of historical flows and it will have the same mean, S.D and lag1 correlation as that of the historical data.

2.1 Data generation –uncorrelated data

If from the correlogram (plot of auto correlations ρ_k for different lags, against lags k), all the autocorrelations are found to be statistically insignificant (purely random stochastic process), then if the distribution of the observed sequence is known or can be estimated, the data can be generated for the future by the specific distribution.

In hydrology, most of the data are serially correlated. The value realised during a particular period may be related to the value of the previous period. In such cases, a method based on lag one correlation can be adopted. The first order Markov process can be effectively used in this case. If X_t is correlated with $X_{t-\tau}$ and if ρ_k (auto correlation) is exponentially decaying, then correlation at any lag can be obtained using correlation at lag1. This also indicates that the memory of the process is short. According to different situations, one time step memory, two time step memory etc. can be used. The first order Markov process can be defined as

$P[X_t \mid X_{t-1}, X_{t-2, \ldots\ldots}, X_0] = P[X_t \mid X_{t-1}]$. The entire information contained in the history of the process given by X_{t-1}, $X_{t-2, \ldots\ldots,}$ X_0 can be expressed by X_{t-1}.

2.2 Data generation through first order Markov process – stationary Data

If large time steps like annual time steps are taken neglecting periodicities, then stationary Markov process can be employed as

$X_{t+1} = \mu_x + \rho_1 (X_t - \mu_x) + €_{t+1}$

X_{t+1} is the generated value for the time period t+1, μ_x is the long term observed mean of the process, ρ_1 is the lag1 correlation of X_{t+1} with $X_{t,}$ $€ \approx N(0, \neg_e^2)$. This model is stationary with respect to mean and variance.

A sequence is to be generated with the same mean μ_x and variance $\neg_x^2$ € will have a mean 0. But to maintain the same variance $\neg_x^2$ for the process, properties of $€_{t+1}$ are important.

$$E[X_{t+1}] = E[\mu_x + \rho_1(X_t - \mu_x) + €_{t+1}] = \mu_x$$

$$\neg_x^2 = E[X^2] - (E[X])^2 = E[(\mu_x + \rho_1(X_t - \mu_x) + €_{t+1})^2] - (E[X_{t+1}])^2 = \rho_1^2 \neg_x^2 + \neg_e^2$$

Therefore $\neg_e^2 = \neg_x^2 (1- \rho_1^2)$ ie the variance of $€_{t+1}$ would be $\neg_e^2 = \neg_x^2 (1- \rho_1^2)$

If X_t follows normal distribution with mean μ_x and variance $\neg_x^2$ and € should also be normally distributed with mean 0 and variance $\neg_e^2$. Let a new variable U_t be introduced which follows N (0, 1), then $U_t \neg_{e\ ie}$ $U_t \neg_x \sqrt{}(1- \rho_1^2)$ is N $(0, \neg_e^2)$. Thus it can be ensured that the random component has 0 mean and variance $\neg_e^2$. Introducing a standard normal deviate U_{t+1}, the first order Markov model can be written as

$X_{t+1} = \mu_x + \rho_1 (X_t - \mu_x) + U_{t+1} \neg_x \sqrt{}(1- \rho_1^2)$. U_{t+1} follows standard normal distribution with mean 0 and variance 1. By using such a random number it can be ensured that the error component will have 0 mean and constant variance $\neg_e^2$. The model would be stationary because the same value of mean and S.D is used to generate different values.

The assumptions made to apply this model is that (1) the process X_t follows normal distribution with mean μ_x and variance $\neg_x^2$ and (2) the process is stationary in mean, S.D and lag1 correlation.

To start the generation procedure, first the moments viz; mean, S.D and lag1 correlation of the historical data are found out. To generate x_2 from x_1 using the model, x_1 is initially assumed to be μ_x so that the second term in the model will immediately be zero. The standard normal deviates U_{t+1} are straight away taken from the statistical tables or inbuilt programs. Once x_2 is generated it is used to generate x_3. Each time X_t and U_{t+1} are changed in the model but all other parameters remain constant. Since the process was started by assuming X_t as μ_x, the first 100 or 150 values are to be discarded to do away with this effect. By changing the set of random numbers of standard normal deviate, several number of sequences can be generated. In all these sequences first few values are discarded to make it free from the effect of the initial value assumed to start the process. Since standard normal deviates ranges from -3 to +3, practically, the generated values may contain negative numbers. In such cases retain the negative number as such to generate the next number but in real applications it is taken as zero as the variables in hydrology such as stream flow, rainfall etc. will not assume negative values.

2.3 Data Generation through first order Markov Model with Non-Stationarity

In the first order stationary Markov model it is assumed that the mean, S.D and lag1 correlation should be the same for the historical and generated data. But when hydrological time series is considered they exhibit non stationarity especially when monthly data such as stream flow are considered. In this case the mean, S.D and lag1 correlation will be significantly different from one month to another. So it is essential to incorporate these type of variations or non stationarity of the moments in the model. Relaxing the requirements of the first order stationary Markov model usually used for annual flows, seasonal models can be introduced in which the seasons can be either months, or any intra period or intra year variation. There may be periodicities existing in the data. The stream flow during a period may be correlated with a value in the previous periods and so on. In situations where there is a periodicity, it introduces non stationarity in the data. The periodicities would affect not only the mean and S.D but lag1 correlation also all of which appear in the Markov model.

In the stationary Markov model, nonstationarity is introduced in the mean, S.D and lag1 correlation by taking one new index $x_{i,\,j+1}$ where i is the year and j is the month. First order Markov model with non-stationarity, for data generation is given by

$$X_{i,j+1} = \mu_{j+1} + \rho_j \frac{\sigma_{j+1}}{\sigma_j} (X_{ij} - \mu_j) + t_{i,j+1}\sigma_{j+1}\sqrt{1-\rho_j^2}$$

where ρ_j is the serial correlation between flows of jth month and j+1 th month, $t_{i,\,j+1} \sim N(0,1)$.

Instead of the stationary mean μ_x, mean of a particular season for which data is generated is expressed as $\mu_{j+1,\ \rho j}$ is the lag1 correlation between month j and j+1, ⌐ j indicating S.D for the particular month, $t_{i,\,j+1}$ is drawn from N(0,1) ie random variables following standard normal distribution. To generate the values, first, $\mu_{x,}$ ⌐ j and ρj are estimated, then start with an assumed initial value and a value for the (j+1) th month is generated and so on.

3. Results and Discussion

The first order stationary Markov model as well as the model addressing the non stationarity are demonstrated using the data pertaining to Karappara stream flow and Idamalayar Reservoir Inflow. They were found to be very useful in modelling annual stream flows and reservoir inflows. These types of models can be effectively used in hydrological designing to fix the capacity of a reservoir, simulation of the performance of a reservoir etc. by using several sequences of generated data.

3.1 Simulation of Karappara Stream flow

The mean annual flow at Karappara weir was 79.42Mm^3 with a S.D of 26.81Mm^3 and lag1 correlation of 0.26. The yearly series of stream flow was found to follow normal distribution.

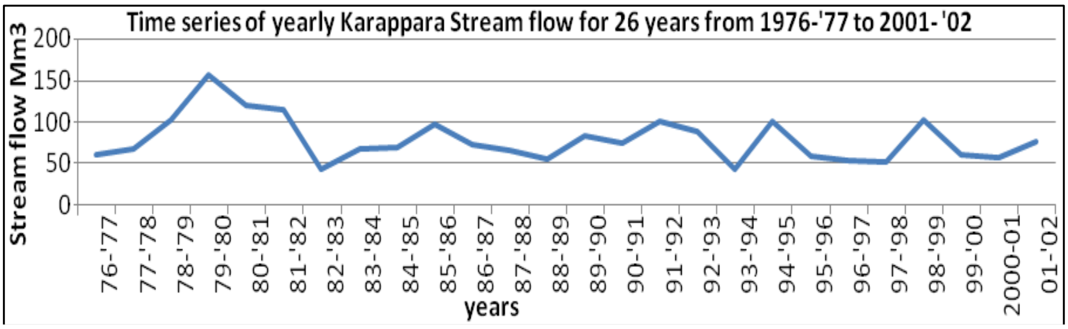

Fig 1: Time series of yearly Karappara stream flow for 26 years from 1976-'77 to 2001 – '02

Simulation of the data using the first order stationary Markov model has been tried using the moments of historical annual stream flow data. First 100 values of the generated data were discarded and the next 50 values were taken. The mean, S.D and lag1correlation of the generated values were very close to that of the existing data.

Table 1: Moments of the historical and generated series of annual flow at Karappara weir

Annual flow	Mean (Mm^3)	S.D (Mm^3)	Lag1 correlation
Historical data	79.42	26.81	0.26
Generated data	80.37	25.20	0.35

The seasonal first order Markov model can accommodate shorter period flows such as 10 day period, weekly etc. to simulate data. But when the model is applied to very short periods like less than 10, the assumption of normality of the data may be violated. When 10 day period is used, the year is divided into 36 time intervals. The time series of the 10 day period flow at Karappara weir is depicted in Fig. (2). Each month was divided into three viz; juneI, juneII, juneIII etc.. For each of these periods, the moments viz; mean, S.D and lag1 correlation were found out. The simulated values were then derived using First order Markov model. First hundred values generated were discarded and the characteristics of the next 50 values generated are summarised in Table2.

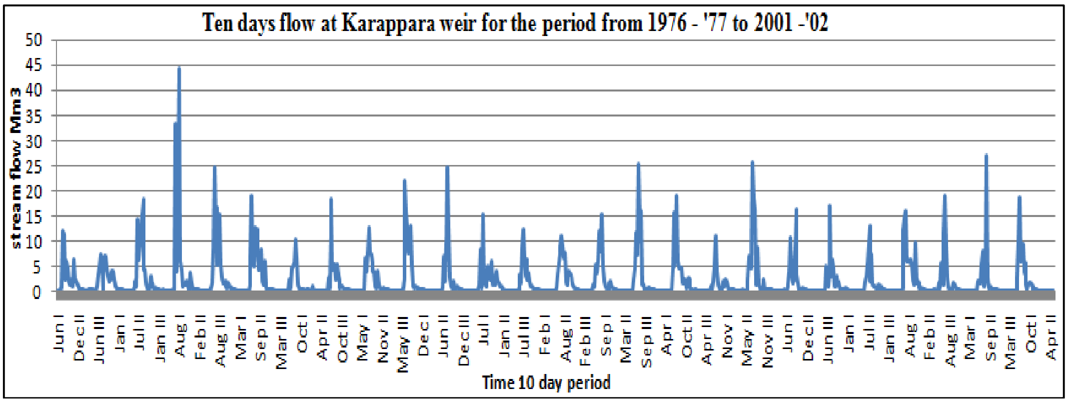

Fig 2: Ten days flow at Karappara weir for the period from 1976–77 to2001 –'02

Table 2: Moments of the historical and generated 10 day's flows (Mm^3) at Karappara weir

Period	mean		S.D		lag1 correlation	
	Original	generated	original	generated	original	Generated
June I	0.59	0.58	0.73	0.56	0.49	0.36
June II	2.84	2.87	2.52	2.32	-0.02	-0.08
June III	6.66	6.48	7.94	5.72	0.61	0.59
July-I	7.95	8.14	7.48	6.55	0.02	0.13
July II	8.74	8.73	5.09	5.28	0.15	0.08
July-III	11.06	12.08	6.51	6.71	0.40	0.38
Aug-I	9.72	10.48	8.89	8.79	0.18	0.01
Aug.-II	6.67	6.74	4.65	3.64	0.21	0.30
Aug.III	7.37	6.86	5.79	5.79	0.10	-0.001
Sep.-I	3.46	3.72	3.41	2.99	0.47	0.53
Sep-II	2.06	2.03	1.69	1.65	0.62	0.81
Sep.-III	1.90	2.28	1.84	1.92	0.27	0.19
Oct.-I	1.42	1.36	0.94	1.01	0.25	0.44
Oct.-II	1.62	1.79	1.88	1.47	0.22	0.08
Oct.-III	1.31	1.64	1.17	1.11	0.15	0.06
Nov.-I	1.43	1.80	1.40	1.32	0.48	0.40
Nov.-II	1.19	1.66	1.16	1.07	0.87	0.88
Nov-III	0.81	1.05	0.69	0.64	0.74	0.62
Dec.I	0.49	0.60	0.35	0.35	0.88	0.88
Dec.II	0.39	0.46	0.23	0.22	0.86	0.85
Dec.III	0.32	0.36	0.20	0.19	0.91	0.90
Jan.I	0.23	0.23	0.13	0.12	0.92	0.94
Jan.II	0.18	0.19	0.09	0.09	0.20	-0.03
Jan.III	0.18	0.27	0.18	0.16	0.27	0.07
Feb.I	0.09	0.11	0.06	0.06	0.90	0.91
Feb.II	0.07	0.08	0.05	0.04	0.85	0.86
Feb.III	0.05	0.05	0.04	0.03	0.82	0.79
Mar.I	0.04	0.05	0.04	0.04	0.92	0.93
Mar.II	0.03	0.03	0.03	0.03	0.52	0.53
Mar.III	0.03	0.03	0.03	0.03	0.57	0.53
Apr.I	0.04	0.06	0.06	0.05	0.50	0.44
Apr.II	0.07	0.08	0.08	0.07	0.51	0.36
Apr.III	0.05	0.07	0.06	0.05	0.70	0.82
MayI	0.07	0.08	0.06	0.06	0.67	0.68
MayII	0.07	0.09	0.07	0.06	0.23	0.02
MayIII	0.18	0.27	0.39	0.30	0.12	0.09

The moments of the historical and generated series compared well. The adequacy of the fitted model is shown in Fig.3, Fig.4 and Fig.5.

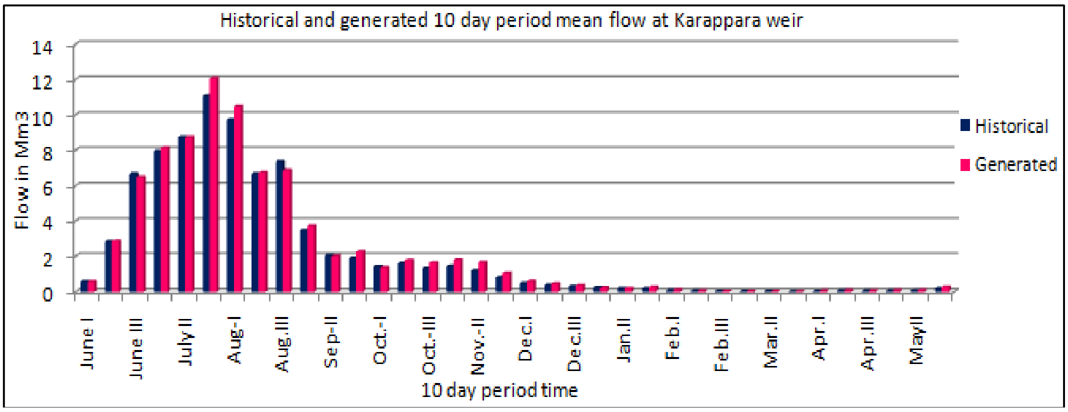

Fig 3: Historical and generated 10 day period mean flow at Karappara weir

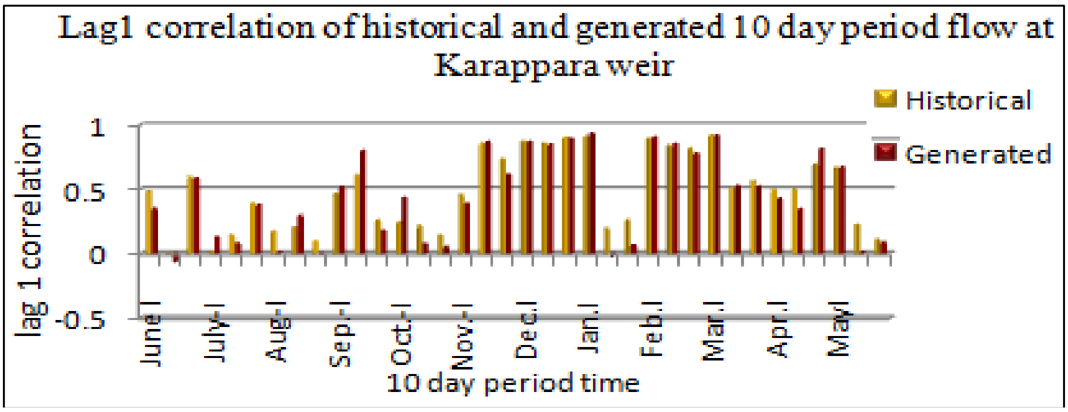

Fig 4: S.D of the historical and generated10 day period flow at Karappara weir

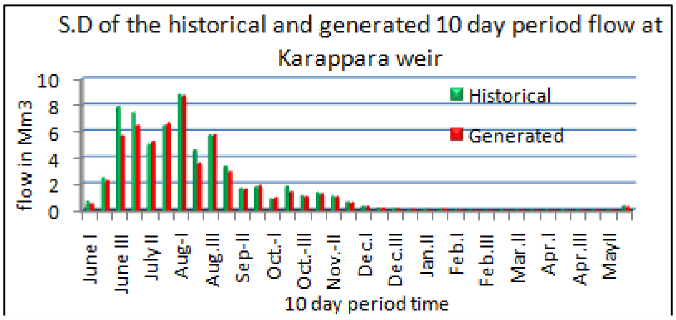

Fig 5: Lag1 correlation of historical and generated 10 day period flow at Karappara weir

3.2 Simulation of monthly inflow to Idamalayar Reservoir

Augmented Dickey-Fuller test showed that the series of monthly inflow to Idamalayar Reservoir was non stationary. Also the series was not normal. Therefore the series of inflow was transformed to logarithmic values and the first order Markov model addressing the non stationarity was fitted. The improved simulated series compared perfectly well with the historical data. The prescribed model was fairly acceptable in terms of mean, S.D and lag1 correlation. The results in Table (3) shows the adequacy of the fitted model to account for serial dependence in the data.

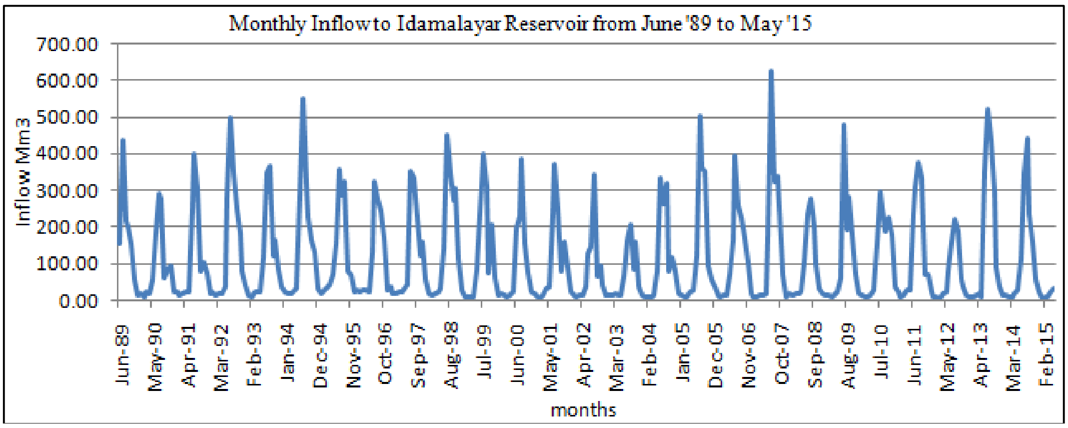

Fig 6: Monthly Inflow to Idamalayar Reservoir from june '89 to May '15

Table 3: Moments of the historical and generated logarithmic series of monthly Inflow (Mm^3) to Idamalayar Reservoir

period	mean		S.D		lag1 correlation	
	original	generated	original	generated	original	Generated
June	5.02	4.96	0.55	0.57	0.34	0.38
July	5.83	5.89	0.38	0.39	0.23	0.40
August	5.71	5.67	0.22	0.21	0.18	-0.02
September	5.15	5.10	0.58	0.60	0.03	0.22
October	4.86	4.80	0.41	0.48	0.45	0.50
November	4.17	4.13	0.52	0.57	0.45	0.36
December	3.10	3.06	0.41	0.45	0.28	0.18
January	2.58	2.59	0.44	0.43	0.70	0.52
February	2.31	2.36	0.41	0.38	0.67	0.74
March	2.61	2.60	0.42	0.34	0.60	0.53
April	2.93	2.92	0.35	0.33	-0.05	-0.17
May	3.41	3.46	0.50	0.48	0.23	0.16

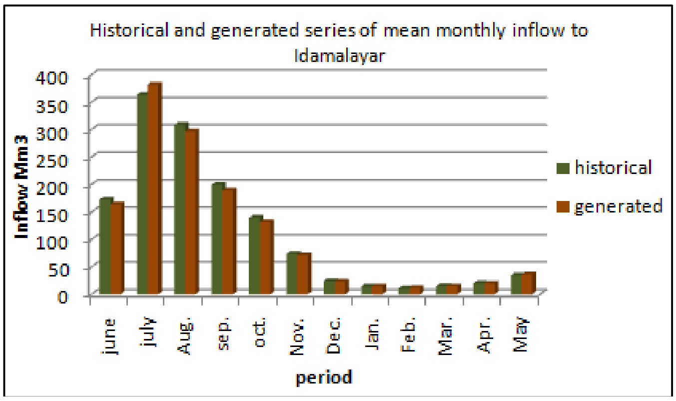

Fig 7: Historical and generated series of mean monthly inflow to Idamalayar

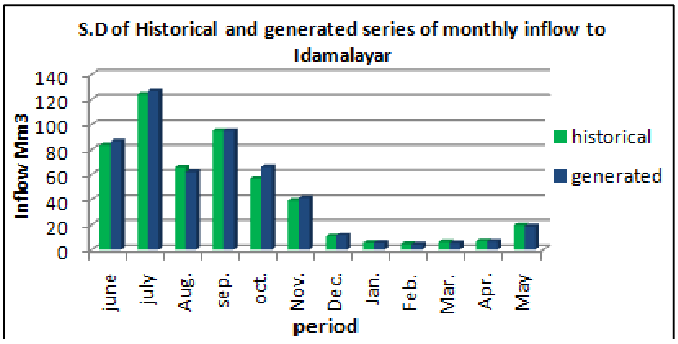

Fig 8: S.D of the historical and generated series of monthly inflow to Idamalayar

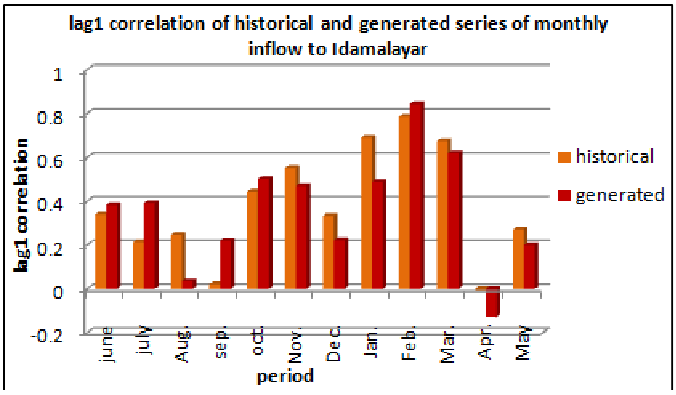

Fig 9: Lag1 correlation of historical and generated series of monthly inflow to Idamalayar

Several sequences of reservoir inflow can be generated using different sequences of random numbers and this information can be made use of for planning the routine operations of the Reservoir and to discuss about how the system would perform in future according to this generated data.

The model described above is meant only for simulating sequences preserving the historical mean, S.D and lag1 correlation. The peak flows may not be reproduced by the model. When a stochastic model is selected, the purpose for which it is used is very important. So these models are usually recommended, addressing the issues such as whether it is needed to model peak flows, is it important to include the time and volume of peak flows, duration of flow etc.

4. Conclusion

Reservoirs are probably one among the largest human interventions on earth. Most of them are mainly intended for irrigation purpose. Other major purposes are Hydro- power generation, water supply, flood control, etc. Some of them function as Multi-purpose with combination of the above. As water is inevitable for our existence and the requirement for power generation and irrigation is increasing, storage reservoirs have become indispensable for the development of a country. Planning of new efficient and effective reservoirs necessitate proper assessment of current and future requirement of water and its availability. The inflow to the reservoir is seasonal and depends on the variation in the rainfall pattern, due to climate change. Specific tools and techniques from different disciplines are required to face such challenges and making decisions and real time reservoir operation rules for existing or proposed projects. For a precise estimate of reservoir yield, simulation models need to be developed. The sequential nature of the reservoir management decisions, together with the inherent randomness of natural water inflows lead us to use Markov decision processes for modelling reservoir management problems and their optimisation through stochastic dynamic programming.Thus first order stationary Markov model can be effectively used to simulate annual stream flows and annual reservoir inflows whereas Markov model addressing the nonstationarity is to be used for nonstationary data like monthly inflow data. In this study, competent simulation models are demonstrated pertaining to Karappara stream flow and Idamalayar reservoir inflow pertaining to Kerala state.

5. Acknowledgement

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