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Dr. Manju Sharma
Department of Mathematics,
Agra College Agra U.P, India

Rajvir Singh
Department of Mathematics,
Agra College Agra U.P, India

Correspondence
Dr. Manju Sharma
Department of Mathematics,
Agra College Agra U.P, India

Hydromagnetic heat and mass transfer flow with time dependent suction and porous medium in rotating system

Dr. Manju Sharma and Rajvir Singh

Abstract
In the present paper free convection and mass transfer effects in the unsteady flow of an incompressible, electrically conducting, viscous liquid in a porous medium past an infinite porous vertical plate with time dependent suction in a rotating system in the presence of uniform magnetic field applied perpendicular to the flow region is studied. The effects of magnetic parameter, rotation parameter, Grashof number, heat source parameter, Prandtl number, permeability parameter, modified Grashof number and Schmidt number or primary velocity and secondary velocity for cooling case and the effects of the above stated parameters on primary velocity and secondary velocity for cooling case and the effects of the above stated parameters on primary velocity and secondary velocity for heating case are discussed with the help of graphs while the effects of these parameters on skin-friction are discussed with the help of graphs while the effects of these parameters on skin-friction are discussed with the help of tables. The rate of heat transfer and the rate of mass transfer are also discussed with the help of tables.

Keywords: Hydro magnetic heat, porous medium, mass transfer flow, velocity, magnetic parameter

1. Introduction

Free convection problem have attracted a considerable amount of interest because of its importance in atmospheric and oceanic circulations, nuclear reactors, powder transformers etc. Extensive review of the free convection flows in rotating system has been done by Gebhart (1973). Jana & Dutta (1977), Place & Daguenet (1987) have studied problems on flow of rotating convection fluids in absence of magnetic field. Singh (1994) has presented MHD free convection and mass transfer flow through porous medium bounded by a vertical porous plate in a rotating system. Singh (1995) has studied asymptotic behaviour of the unsteady laminar incompressible boundary layer flows over a roatating disc in the presence of uniformly applied magnetic field. Seth & Ghosh (1995) have discussed MHD flow in a rotating channel in the presence of applied magnetic field. Seth and Banerjee (1996) have studied combined free and forced convection flow of a viscous fluid in rotating channel in the presence of a uniform transverse magnetic field applied parallel the axis of rotation. Kumar & Sharma (1998) have investigated unsteady multiple boundary layers viscous flow though a porous medium bounded by a porous plate with suction or injection in a rotating system. Gupta & Pathak (1999) have analysed elasticplastic transition in non-homogeneous thick walled rotating cylinders. More recently, Singh *et al*. (2000) have discussed free convection flow of a rotating channel in the presence of a uniform transverse magnetic field applied parallel to the axis of rotation. Kumar & Sharma (1998) have investigated unsteady multiple boundary layers viscous flow though a porous medium bounded by a porous plate with suction 'or injection in a rotating system. Gupta & Pathak (1999) have; analysied elasticplastic transition ion non-homogenous thick walled rotating cylinders. More recently, Singh *et al*. (2000) have discussed free convection flow of a rotating dusty viscous liquid though a porous medium past a porous vertical plate. In addition, Singh *et al*. (2000) have studied effect of Hall current on unsteady hydromagnetic boundary layer flow in a roating dusty viscous liquid. More recently Singh & Singh (2000) have studied MHD effects ion heat and mass transfer in stratified flow of a viscous fluid through parallel permeable beds.

Singh & Singh (2001) have discussed MHD flow of a viscous fluid in a jotating channel taking induced magnetic field into account. Singh *et al*. (2002) have studied free convection and mass transfer in MHD flow of a dusty viscoelastic (water is liquid Model B) liquid between vertical porous parallel plate. Shaoo, Dutta and B Iswal (2003) were investigated MHD unsteady free convection flow past an infinite vertical plate 'with constant heat source.

In the present chapter, free convection and mass transfer effects in the unsteady flow of an incompressible, electrically conducting, viscous liquid in a porous medium past an infinite porous vertical plate with time dependent suction in a rotating system in the presence of uniform magnetic field applied perpendicular to the flow region is studied. The effects of magnetic parameter, rotation parameter, Grashof number, heat source parameter, Prandtl number, permeability parameter, modified Grashof number and Schmidt number on primary velocity and secondary velocity for cooling case and the effects of the above stated parameters on primary velocity an secondary velocity for heating case are discussed with the help of graphs while the effects of these parameters on skin-friction are a discussed with the help of tables. Te rate of heat transfer and the rate of mass transfer are also discussed with the help of tables.

2. Formulation of the Problem

Consider an oscillatory flow of an incompressible, electrically conducting, viscous liquid through a porous medium past an infinite, hot, vertical, porous plate with constant heat source, in Cartesian coordinates system *x, y, z* in the presence of uniform magnetic field. We assume *x-axis* and *y-axis* in the plane of porous plate an z-axis normal to the plate with velocity components *(u*, v, w) in these directions respectively. Both the liquid and the plate are considered in a state of rigid body rotation about z-axis with uniform angular velocity Ω. Further we assume that the uniform magnetic field $\vec{B}_0 = \mu_e \vec{H},$, where $\vec{H} = (0,0,H_0)$ is applied in the z-direction and the magnetic Reynolds number is assumed small. The constant heat source Q is assumed at z = 0 and the suction velocity at the plate is $w = -w_0\left(1 + \in e^{\text{int}}\right)$ where w_0 is a positive real number. Initially, when $t \le 0$ the plate and the fluid are assumed to be at the same temperature T_0 and the foreign mass is assumed to be uniformly distributed in the flow region such that it is everywhere C_0. *At* $t > 0$, the temperature of the plate is instantaneously raised to T_w and the concentration of species is raised to C_w and thereafter maintained constant. In this analysis buoyancy force, Hall effect, effect due to perturbation of the field, induced magnetic field and polarization effect are ignore. Since the plate in infinite in extent, all physical variables depend on *z* and *t* only.

In addition to above, we take the heat source of absorption type following Vajravelu & Sastri (1978) and the foreign mass is present at low level. In the present configuration, the equation of motion and energy equation are:

$$\frac{\partial u}{\partial t} - w_0\left(1 + \in e^{\text{int}}\right)\frac{\partial u}{\partial z} - 2\Omega v = \frac{\partial U}{\partial t} + \vartheta\frac{\partial^2 u}{\partial z^2} + g\beta_0\left(T - T_\infty\right)$$

$$+ g\beta\left(C - C_\infty\right) - \frac{\vartheta}{k}\left(u - U\right) - \frac{\sigma}{\rho}\mu_e^2 H_0^2\left(u - U\right) \quad \text{.... (1)}$$

$$\frac{\partial v}{\partial t} - w_0\left(1 + \in e^{\text{int}}\right)\frac{\partial v}{\partial z} - 2\Omega\left(u - U\right) = \vartheta\frac{\partial^2 v}{\partial z^2} - \frac{\vartheta}{k}v - \mu_e^2 H_0^2 v \quad \text{.... (2)}$$

$$\frac{\partial T}{\partial t} - w_0\left(1 + \in e^{\text{int}}\right)\frac{\partial T}{\partial z} = \frac{K}{\mu C_p}\frac{\partial^2 T}{\partial z^2} - \frac{Q_0\left(T - T_\infty\right)}{\rho C_p} \quad \text{.... (3)}$$

$$\frac{\partial C}{\partial t} w_0\left(1 + \in e^{\text{int}}\right)\frac{\partial C}{\partial z} = D\frac{\partial^2 C}{\partial z^2} \quad \text{.... (4)}$$

where g is the acceleration due to gravity, β_0 is due volumetric coefficient of thermal expansion, σ is electrical conductivity of the liquid, ρ is the density of the liquid, μ_e is the magnetic permeability, H_0 is the constant magnetic field, k_0 is the constant permeability of the medium, μ is the coefficient of viscosity, K is the thermal conductivity, C_P is the specific heat at constant pressure, T is the temperature, T_w is the plate temperature and T_α is the temperature far away from the plate, C is the concentration of species, C_w is the concentration of species at the plate and the other symbols have their usual meaning.

The boundary conditions are:

$$u = U_0\left(1 + \in e^{\text{int}}\right),\ v = 0, \qquad T = T_w + \in\left(T_w - T_\infty\right)e^{\text{int}},$$

$$C = C_w + \in\left(C_w - C_\infty\right)e^{\text{int}} \qquad at \quad z = 0$$

$$u \to U(t) = U_0\left(1 + \in e^{\text{int}}\right),\ v \to 0, \qquad T \to T_\infty,$$

$$C \to C_\infty \qquad as \qquad z \to 0 \quad \text{.... (5)}$$

We introduced the following non-dimensional quantities:

$$z^* = \frac{w_0 z}{\upsilon}, \quad t^* = \frac{w_0^2 t}{\upsilon}, \quad u^* = \frac{u}{U_0}, \quad U^* = \frac{U}{U_0}, \quad n^* = \frac{\upsilon n}{w_0^2},$$

$$U^* = \frac{U}{U_0}, \quad k_0^* = \frac{w_0^2 k_0}{\vartheta^2}, \quad C^* = \frac{C - C_\infty}{C_w - C_\infty}, \; and \; T^* = \frac{T - T_\infty}{T_w - T_\infty}$$

Using these non-dimensional quantities, the equations (1), (2), (3) and (4) after ignoring the starts, reduce to:

$$\frac{\partial u}{\partial t} - \left(1 + \in e^{\text{int}}\right)\frac{\partial u}{\partial z} - 2Ev = \frac{\partial U}{\partial t} + \frac{\partial^2 u}{\partial z^2} + G_r T + G_m C - \left[M^2 + \frac{1}{k}\right](u - U) \quad \text{…. (6)}$$

$$\frac{\partial v}{\partial t} - \left(1 + \in e^{\text{int}}\right)\frac{\partial v}{\partial z} - 2E(u - U) = \frac{\partial^2 v}{\partial z^2} - \left[M^2 + \frac{1}{k}\right]v \quad \text{…. (7)}$$

$$\frac{\partial T}{\partial t} - \left(1 + \in e^{\text{int}}\right)\frac{\partial T}{\partial z} = \frac{1}{P_r}\frac{\partial^2 T}{\partial z^2} - \alpha_0 T \quad \text{…. (8)}$$

$$\frac{\partial C}{\partial t} - \left(1 + \in e^{\text{int}}\right)\frac{\partial C}{\partial z} = \frac{1}{S_c}\frac{\partial^2 C}{\partial z^2} \quad \text{…. (9)}$$

where $P_r = \frac{\vartheta \rho C_p}{K}$ (Prandtl number),

$S_c = \frac{\upsilon}{D}$ (Schmidt number)

$G_r = \frac{g\beta_0 \vartheta (T_w - T_\infty)}{U_0 w_0^2}$ (Grash of number),

$G_m = \frac{g\beta_0 \vartheta (C_w - C_\infty)}{U_0 w_0^2}$ (Modified Grash of number),

$E = \frac{\Omega \vartheta}{w_0^2}$ (Rotation parameter),

$M^2 = \frac{\sigma \mu_e^2 H_0^2 \vartheta}{\rho U_0 w_0^2}$ (Magnetic parameter) and $\alpha_0 = \frac{Q_0 \vartheta^2}{K w_0^2}$ (Heat source parameter)

Using $q = u + iv$ in (6) and (7), we obtain

$$\frac{\partial q}{\partial t} - \left(1 + \in e^{\text{int}}\right)\frac{\partial q}{\partial z} + \left[M^2 + \frac{1}{k} + 2iE\right](q - U) = \frac{\partial U}{\partial t} + \frac{\partial^2 q}{\partial z^2} + G_r T + G_m C \quad \text{…. (10)}$$

The equations (2.8) and (2.9) can be written as:

$$\frac{\partial^2 T}{\partial z^2} + P_r\left(1 + \in e^{\text{int}}\right)\frac{\partial T}{\partial z} - P_r\frac{\partial T}{\partial t}\alpha_0 T = 0 \quad \text{…. (11)}$$

$$\frac{\partial^2 C}{\partial z^2} + S_c\left(1 + \in e^{\text{int}}\right)\frac{\partial C}{\partial z} - S_c\frac{\partial C}{\partial t} = 0 \quad \text{…. (12)}$$

The boundary condition (2.5) are transformed to :

$$q = 1 + \in e^{\text{int}}, \quad T = 1 + \in e^{\text{int}} \quad at \quad z = 0$$

$$q \to \left(1 + \in e^{\text{int}}\right), T \to 0, \quad C \to 0 \quad as\; z \to \infty \quad \text{…. (13)}$$

3. Solution of the Problem

In order to solve the equations (10), (11) and (12), we assume the velocity, temperature and concentration of the liquid in the neighbourhood of the plates as:

$$q(z,t) = (1 - q_0) + \in (1 - q_1)e^{\text{int}} \quad \text{…. (14)}$$

$$U(t) = 1 + \in e^{\text{int}} \quad \text{…. (15)}$$

$T(z,t) = T_0 + \in T_1 e^{\text{int}}$ …. (16)

and $C(z,t) = C_0 + \in C_1 e^{\text{int}}$ …. (17)

where $(1 - q_0)e^{\text{int}} = (M_r + iM_i)(\cos nt + i \sin nt)$

$T_1 e^{\text{int}} = (T_r + iT_i)(\cos nt + i \sin nt) \quad and \quad \in << 1$

The transient velocity profiles can be deduced from the equation (14) for $nt = \frac{\pi}{2}$. Hence, cos*nt* =0 and sin*nt* =1. So we have:

$u(z,t) = u_0(z) - \in M_i \quad and \quad v(z,t) = v_0(z) + \in M_r$ …. (18)

Where $u_0(z) + iv_0(z) = (1 - q_0)$

Also transient temperature and transient concentration can be deduced from the equations (16) and (17) respectively, for $nt = \frac{\pi}{2}$.

Hence, so we have,

$T(z,t) = T_0(z) - \in T_i$ …. (19)

$C(z,t) = C_0(z) - \in C_i$ …. (20)

Using (14) – (17) in (10) – (11) and comparing the similar terms, we have the following equations:

$q_0''(z) + q_0'(z) - (M_1 + 2_i E)(z) = G_r T_0(z) + G_m C_0(z)$ …. (21)

$q_1''(z) + q_1'(z) - [M_1 + i(2E + n)]q_1(z) = G_r T_1(z) + G_m C_1(z) - q_0'(z)$ …. (22)

$T_0''(z) + P_r T_0'(z) - \alpha_0 T_0(z) = 0$ …. (23)

$T_1''(z) + P_r T_0'(z) - (inP_r + \alpha_0)T_1(z) = -P_r T_0'$ …. (24)

$C_0''(z) + S_C C_0(z) = 0$ …. (25)

$C_1''(z) + S_C C_1'(z) - inS_C C_1(z) = -S_C C_0'$ …. (26)

The corresponding boundary conditions (2.13) are reduced to:

$q_0 = 0, \; q_1 = 0, \; T_0 = 1, \; C_0 = 1, \; C_1 = 0, \quad at \quad z = 0$

$q_0 \to 0, \; q_1 \to 0, T_0 \to 0, T_1 \to 0, C_0 \to 0, C_1 \to 0, \; as \quad z \to \infty$ …. (27)

The solutions of equations (3.5) to (3.10), under the boundary condition (3.14) are :

$T_0(z) = e^{-A_0^z}$ …. (28)

$T_0(z) = L_1 e^{-A_0^z} + (1 - L_1)e^{-K_{1z}}$ …. (29)

$C_0(z) = e^{-S_{cz}}$ …. (30)

$C_1(z) = R_2 e^{-S_{cz}} + (1 - R_2)e^{-R1z}$ …. (31)

$q_0(z) = L_2\left(e^{-A_0 z} - e^{-K_2 z}\right) + R_3\left(e^{-S_0 z} - e^{-K_2 z}\right)$ …. (32)

and $q_1(z) = L_3 e^{-A_0 z} - L_4 e^{-K_1 z} + L_4 e^{-A_0 z} - L_6 e^{-K_2 z}$

$+ R_4 e^{-S_c z} - R_5 e^{-R_1 z} + R_6 e^{-S_c z} - R_7 e^{-K_2 z}$

$+ (L_6 - L_5 - L_4 - L_3 + R_7 - R_6 - R_5 - R_4)e^{-K_3 z}$ …. (33)

Substituting the value 0 and in the equation (14), the values of and in the equation (15) and the values of and in the equation (16), we obtain:

$q_1(z,t) = \left[1 - L_2 e^{-A_0 z} + L_2 e^{-K_2 z} - R_3 e^{-S_c z} + R_3 e^{-K_2 z}\right] + \left[1 - (L_6 - L_5 - L_4 - L_3 + R_7 - R_6 - R_5 - R_4)e^{-K_3 z}\right.$

$\left. + R_4 e^{-S_c z} - R_5 e^{-R_1 z} + R_6 e^{-S_c z} - R_7 e^{-K_2 z} \; L_3 e^{-A_0 z} - L_4 e^{-K_1 z} - L_5 e^{-A_0 z} + L_6 e^{-K_2 z}\right]e^{\text{int}}$ …. (34)

$T(z,t) = e^{-A_0 z} + \in \left[L_1 e^{-A_0 z} + (1 - L_1)e^{-K_1 z}\right]e^{\text{int}}$ …. (35)

$C(z,t) = e^{-S_c z} + \in \left[R_2 e^{-S_c z} + (1 - R_2)e^{-R_1 z}\right]e^{\text{int}}$ …. (36)

Hence, Primary velocity (u) and secondary velocity (v)

$$u(z,t)=u_0+\in\left[u_1\cos nt-u_1\sin nt\right] \quad \text{…. (37)}$$

$$u(z,t)=v_0+\in\left[v_1\cos nt-v_1\sin nt\right] \quad \text{…. (38)}$$

where $1-q_0(z)=u_0(z)+iu_0(z),$

$1-q_1(z)=u_1(z)+iu_1(z)$

$$u_0(z)=1-A_5e^{-A_0z}+\left(A_5\cos B_2z+B_5\sin B_2z\right)e^{-A_2z}$$
$$-P_5e^{-S_cz}+\left(P_3\cos B_2z+Q_3\sin B_2z\right)e^{-A_2z}$$

$$v_0(z)=\left(B_5\cos B_2z+B_5\sin B_2z\right)e^{-A_2z}-B_5e^{-A_2z}$$
$$-Q_3e^{-S_cz}+\left(Q_3\cos B_2z+P_3\sin B_2z\right)e^{-A_2z}$$

$$u_1(z)=1-\{A_{10}\cos B_3z+B_{10}\sin B_3z\}e^{-A_3z}$$
$$+\left\{A_9\cos B_zz+B_9\sin B_2z\right\}e^{-A_3z}$$
$$-\left(A_7\cos B_1z+B_7\sin B_1z\right)e^{-A_3z}-\left(A_6+A_8\right)e^{-A_0z}$$
$$-\left(P_7\cos Q_1z+Q_5\sin Q_1z\right)e^{-P_1z}-\left(P_4+P_6\right)e^{-S_cz}$$
$$+\left(P_7\cos B_2z+Q_7\sin B_2z\right)e^{-A_2z}$$

$$v_1(z)=\{B_{10}\sin B_3z+A_{10}\cos B_3z\}e^{-A_3z}$$
$$+\left\{B_9\cos B_zz+A_9\sin B_2z\right\}e^{-A_2z}$$
$$-\left(B_7\cos B_1z+A_7\sin B_1z\right)e^{-A_1z}-\left(B_6+B_8\right)e^{-A_0z}$$
$$-\left(Q_5\cos Q_1z+P_5\sin Q_1z\right)e^{-P_1z}-\left(Q_4+Q_6\right)e^{-S_cz}$$
$$+\left(Q_7\cos B_2z+P_7\sin B_2z\right)e^{-A_2z}$$

From (28), the transient primary velocity (u) and transient secondary velocity (v) for $nt=\frac{\pi}{2}$ using (16) and (17) are as follows:

$$u\left(z,\frac{\pi}{2n}\right)=u_0(z)-\in M_i \quad \text{….. (39)}$$

$$v\left(z,\frac{\pi}{2n}\right)=v_0(z)-\in M_r \quad \text{….. (40)}$$

Where

$$M_r=1-\{A_{10}\cos B_3z+B_{10}\sin B_3z\}e^{-A_3z}$$
$$+\left\{A_9\cos B_2z+B_9\sin B_2z\right\}e^{-A_2z}$$
$$-\left(A_7\cos B_1z+B_7\sin B_1z\right)e^{-A_1z}-\left(A_6+A_8\right)e^{-A_0z}$$
$$-\left(P_5\cos Q_1z+Q_5\sin Q_1z\right)e^{-P_1z}-\left(P_4+P_6\right)e^{-S_cz}$$
$$+\left(P_7\cos B_2z+Q_7\sin B_2z\right)e^{-A_2z}$$

$$M_r=1-\{A_{10}\cos B_3z+B_{10}\sin B_3z\}e^{-A_3z}$$
$$+\left\{A_9\cos B_2z+B_9\sin B_2z\right\}e^{-A_2z}$$
$$-\left(A_7\cos B_1z+B_7\sin B_1z\right)e^{-A_1z}-\left(A_6+A_8\right)e^{-A_0z}$$
$$-\left(P_5\cos Q_1z+Q_5\sin Q_1z\right)e^{-P_1z}-\left(P_4+P_6\right)e^{-S_cz}$$
$$+\left(P_7\cos B_2z+Q_7\sin B_2z\right)e^{-A_2z}$$

$$M_i = \{B_{10}\sin B_3 z - A_{10}\cos B_3 z\}e^{-A_3 z}$$
$$+\{B_9\cos B_2 z + A_9\sin B_2 z\}e^{-A_2 z}$$
$$-(B_7\cos B_1 z + A_7\sin B_1 z)e^{-A_1 z} - (B_6 + B_8)e^{-A_0 z}$$
$$-(Q_5\cos Q_1 z + P_5\sin Q_1 z)e^{-P_1 z} - (Q_4 + Q_6)e^{-S_c z}$$
$$+(Q_7\cos B_2 z + P_7\sin B_2 z)e^{-A_2 z}$$

where $M_1 = M^2 + \frac{1}{k}$, $\quad A_0 = \frac{1}{2}\left[P_r + \sqrt{P_r^2 + 4\alpha_0}\right]$,

$$K_1 = A_1 + iB_1 = \frac{1}{2}\left[P_r + \sqrt{(P_r^2 + 4\alpha_0) + i4nP_r}\right],$$

$$K_2 = A_2 + iB_2 = \frac{1}{2}\left[1 + \sqrt{(1 + 4M_1) + i8E}\right],$$

$$K_3 = A_3 + iB_3 = \frac{1}{2}\left[1 + \sqrt{(1 + 4M_1) + i4(2E + n)}\right],$$

$$L_1 = A_4 + iB_4 = \frac{A_0 P_r}{(A_0^2 - A_0 P_r - \alpha_0) - inP_r},$$

$$L_2 = A_5 + iB_5 = \frac{G_r}{(A_0^2 - A_0 - M_1) - 2iE},$$

$$L_3 = A_6 + iB_6 = \frac{L_1 G_r}{(A_0^2 - A_0 - M_1) - i(2E + n)},$$

$$L_4 = A_7 + iB_7 = \frac{G_r(1 - L_2)}{(K_1^2 - K_1 - M_1) - i(2E + n)},$$

$$L_5 = A_8 + iB_8 = \frac{A_0 L_2}{(A_0^2 - A_0 - M_1) - i(2E + n)},$$

$$L_6 = A_9 + iB_9 = \frac{K_2 L_2}{(K_2^2 - K_2 - M_1) - i(2E + n)},$$

$$R_1 = P_1 + iQ_1 = \frac{1}{2}\left[S_c + \sqrt{S_c^2 + 4inS_c}\right]^{1/2}, \qquad R_2 = P_2 + iQ_2 = \frac{iS_c}{n},$$

$$R_3 = P_3 + iQ_3 = \frac{G_m}{S_c^2 - S_c - M_1 - 2iE},$$

$$R_4 = P_4 + iQ_4 = \frac{G_m R_2}{S_c^2 - S_c - M_1 - i(2E + n)},$$

$$R_5 = P_5 + iQ_5 = \frac{G_m(1 - R_2)}{S_c^2 - R_1 - M_1 - i(2E + n)},$$

$$R_6 = P_6 + iQ_6 = \frac{S_c R_3}{S_c^2 - S_c - M_1 - i(2E + n)},$$

$$R_7 = P_7 + iQ_7 = \frac{R_3 K_2}{K_2^2 - K_2 - M_1 - i(2E + n)},$$

$$A_1 = P_r + \frac{1}{2\sqrt{2}}\left[\left(P_r^2 + 4\alpha_0\right) + \sqrt{\left(P_r^2 + 4\alpha_0\right)^2 + \left(4nP_r\right)^2}\right]^{1/2},$$

$$B_1 = \frac{1}{2\sqrt{2}}\left[\sqrt{\left(P_r^2 + 4\alpha_0\right)^2 + \left(4nP_r\right)^2} - \left(P_r^2 + 4\alpha_0\right)\right]^{1/2},$$

$$A_2 = \frac{1}{2} + \frac{1}{2\sqrt{2}}\left[1 + 4M_1 + \sqrt{\{1 + 4M_1\}^2 + \left(8E\right)^2}\right]^{1/2},$$

$$B_2 = \frac{1}{2\sqrt{2}}\left[\sqrt{\{1 + 4M_1\}^2 + \left(8E\right)^2} - \left(1 + 4M_1\right)\right]^{1/2},$$

$$A_3 = \frac{1}{2} + \frac{1}{2\sqrt{2}}\left[1 + 4M_1 + \sqrt{\{1 + 4M_1\}^2 + \left(8E + 4n\right)^2}\right]^{1/2},$$

$$B_3 = \frac{1}{2\sqrt{2}}\left[\sqrt{\{1 + 4M_1\}^2 + \left(8E + 4n\right)^2} - \{1 + 4M_1\}^2\right]^{1/2},$$

$$A_4 = \frac{A_0 d_1 P_r}{d_1^2 + n^2 P_r^2}, \qquad B_4 = \frac{nA_0 d_1 P_r^2}{d_1^2 + n^2 P_r^2},$$

$$A_5 = \frac{G_r d_2}{d_2^2 + \left(2E + n\right)^2}, \qquad B_5 = \frac{G_r\left(2E + n\right)}{d_2^2 + \left(2E + n\right)^2},$$

$$A_6 = \frac{d_3}{d_2^2 + \left(2E + n\right)^2}, \qquad B_6 = \frac{d_4}{d_2^2 + \left(2E + n\right)^2},$$

$$A_7 = \frac{d_9}{d_7^2 + d_8^2}, \qquad B_7 = \frac{-d_{10}}{d_7^2 + d_8^2},$$

$$A_8 = \frac{d_{11}}{d_2^2 + \left(2E + n\right)^2}, \qquad B_8 = \frac{d_{12}}{d_2^2 + \left(2E + n\right)^2},$$

$$A_9 = \frac{d_{17}}{d_{15}^2 + d_{16}^2}, \qquad B_9 = \frac{d_{18}}{d_{15}^2 + d_{16}^2},$$

$$A_{10} = A_9 - A_8 - A_7 - A_6 + P_7 - P_6 - P_5 - P_4,$$

$$B_{10} = B_9 - B_8 - B_7 - B_6 + Q_7 - Q_6 - Q_5 - Q_4,$$

$$P_1 = \frac{S_c}{2} + \frac{1}{2\sqrt{2}}\left[S_c + \sqrt{\left(S_c^2 + 16n^2\right)^2} + S_c^2\right]^{1/2}, P_2 = 0$$

$$Q_1 = \frac{1}{2\sqrt{2}}\left[S_c + \sqrt{\left(S_c^2 + 16n^2\right)^2} - S_c^2\right]^{1/2} Q_2 \frac{S_c}{n},$$

$$P_3 = \frac{G_m G_1}{G_1^2 + 4E^2}, \qquad Q_3 = \frac{2G_m E}{G_1^2 + 4E^2},$$

$$P_4 = \frac{G_4}{G_1^2 + \left(2E + n\right)^2}, \qquad Q_4 = \frac{G_5}{G_1^2 + \left(2E + n\right)^2}$$

$$P_5 = \frac{G_{10}}{G_8^2 + G_9^2}, \qquad Q_5 = \frac{-G_{11}}{G_8^2 + G_9^2},$$

$$P_6 = \frac{G_{12}}{G_1^2 + \left(2E + n\right)^2}, \qquad Q_6 = \frac{G_{13}}{G_1^2 + \left(2E + n\right)^2},$$

$$P_7 = \frac{G_{18}}{G_{16}^2 + G_{17}^2}, \qquad Q_7 = \frac{G_{19}}{G_{16}^2 + G_{17}^2}$$

$$d_1 = A_0^2 - A_0 P_r - \alpha_0, \qquad d_2 = A_0^2 - A_0 - M_1$$

$$d_3 = G_r A_4 d_2 - G_r B_4\left(2E + n\right), \qquad d_4 = G_r B_4 d_2 - G_r A_4\left(2E + n\right),$$

$$d_5 = G_r\left(1 - A_4\right), \qquad d_6 = G_r B_4,$$

$$d_7 = A_1^2 - B_1^2 - A_1 - M_1, \qquad d_8 = 2A_1 B_1 - B_1 - 2E - n,$$

$$d_9 = d_5 d_7 - d_6 d_8, \qquad d_{10} = d_6 d_7 - d_5 d_8,$$

$$d_{11} = A_0 A_5 d_5 - A_0 B_5\left(2E + n\right), \qquad d_{12} = A_0 B_5 d_2 - A_0 B_5\left(2E + n\right),$$

$d_{13} = A_2 A_5 - B_2 B_5$ $\quad d_{14} = B_2 A_5 - A_2 B_5$

$d_{15} = A_2^2 - B_2^2 - A_2 - M_1,$ $\quad d_{16} = 2A_2 B_2 - B_2 - 2E - n,$

$d_{17} = d_{13} d_{15} + d_{14} d_{16},$ $\quad d_{18} = d_{14} d_{15} + d_{13} d_{16},$

$G_1 = S_c^2 - S_c - M_1,$ $\quad G_2 = G_m P_2,$

$G_3 = G_m Q_2$ $\quad G_4 = G_1 G_2 - G_3 (2E + n),$

$G_5 = G_1 G_3 - G_2 (2E + n),$ $\quad G_6 = G_m (1 - P_2),$

$G_7 = G_m Q_2$ $\quad G_8 = P_1^2 - Q_1^2 - P_1 - M_1,$

$G_9 = 2P_1 Q_1 - Q_1 - 2E - n$ $\quad G_{10} = G_8 G_6 - G_7 G_9,$

$G_{11} = G_8 G_7 - G_6 G_9,$ $\quad G_{12} = S_c G_1 P_3 - S_c Q_3 (2E + n),$

$G_{13} = S_c G_1 Q_3 - S_c P_3 (2E + n),$ $\quad G_{14} = P_3 A_2 - Q_3 B_2,$

$G_{15} = Q_3 A_2 - P_3 B_2,$ $\quad G_{16} = A_2^2 - B_2^2 - A_2 - M_1,$

$G_{17} = 2A_2 B_2 - B_2 - 2E - n,$ $\quad G_{18} = G_{14} d_{16} + d_{15} d_{17},$

$and G_{19} = G_{15} d_{16} + d_{14} d_{17},$

4. Skin Friction:-

The skin friction for fluid at the plate is:

$$\tau = \left(\frac{\partial q}{\partial z}\right)_{z=0} = \left(\frac{\partial q_0}{\partial z}\right)_{z=0} + \in e^{\text{int}} \left(\frac{\partial q_1}{\partial z}\right)_{z=0} = \tau_p + \tau_s \quad \text{….. (41)}$$

Hence, primary skin-friction and secondary skin-friction are:

$$\tau_p = A_{11} + \in (A_{12} \cos nt - B_{12} \cos nt) \quad \text{….. (42)}$$

$$\tau_S = B_{11} + \in (B_{12} \cos nt - A_{12} \cos nt) \quad \text{….. (43)}$$

where

$$A_{11} = A_0 A_5 - A_2 A_5 + B_2 B_5 + S_C P_3 - A_2 P_3 + B_2 Q_3$$

$$B_{11} = A_0 B_5 - B_2 A_5 + A_2 B_5 + S_C Q_3 - B_2 P_3 + A_2 Q_3,$$

$$A_{12} = (A_3 A_{10} - B_3 B_{10}) - (A_2 A_9 - B_2 B_9) + (A_1 A_7 - B_1 B_7) + A_0 (A_6 - A_8) + S_c (P_4 - P_6) + (P_1 P_6 - Q_1 Q_6) - (A_2 P_7 - B_2 Q_7)$$

$$and\ B_{12} = (B_3 A_{10} - A_3 B_{10}) - (B_2 A_9 - A_2 B_9) + (B_1 A_7 - A_1 B_7) + A_0 (B_6 - B_8) + S_c (Q_4 - Q_6) + (Q_1 P_6 - P_1 Q_6) - (B_2 P_7 - A_2 Q_7)$$

5. Rate of Heat and Mass Transfer

The rate of heat transfer in terms of Nusselt number (N_u) at the plate is:

$$N_u = \left(\frac{\partial T}{\partial z}\right)_{z=0} = \left(\frac{\partial T_0}{\partial z}\right)_{z=0} + \in e^{\text{int}} \left(\frac{\partial T_1}{\partial z}\right)_{z=0} e^{\text{int}} \quad \text{….. (44)}$$

$$N_u = -A_0 + \in (A_{13} \cos nt - B_{13} \sin nt) + i \in (B_{13} \cos nt - A_{13} \sin nt) \quad \text{….. (45)}$$

where $A_{13} = -A_0 A_4 + A_1 A_4 - B_1 B_4 - A_1$ and $B_{13} = -A_0 B_4 + B_1 A_4 - A_1 B_4 - B_1$

The rate of mass transfer in terms of Sherwood number (S_h) at the plate is:

$$S_h = \left(\frac{\partial C}{\partial z}\right)_{z=0} = \left(\frac{\partial C_0}{\partial z}\right)_{z=0} + \in \left(\frac{\partial C_1}{\partial z}\right)_{z=0} e^{\text{int}} \quad \text{….. (46)}$$

$$S_h = -S_h + \in (A_{14} \cos nt - B_{14} \sin nt) + i \in (B_{14} \cos nt - A_{14} \sin nt) \quad \text{….. (47)}$$

where $A_{14} = -P_1 - Q_1 Q_2$

and $B_{14} = -P_1 Q_2 - Q_1 - S_c Q_2$

Table 1: Skin friction due to primary velocity
(Cooling case at n = 5.0, t = 1.0 and ∈ = 0.002)

P_r	S_C	M	k_0	α_0	G_r	G_m	E	ι_p
0.71	0.30	1.5	5.0	2.0	8.00	10.0	1.0	6.42479
11.4	0.30	1.5	5.0	2.0	8.00	10.0	1.0	6.33248
0.71	0.66	1.5	5.0	2.0	8.00	10.0	1.0	5.45810
0.71	0.30	2.5	5.0	2.0	8.00	10.0	1.0	5.04565
0.71	0.30	1.5	25.0	2.0	8.00	10.0	1.0	6.50994
0.71	0.30	1.5	5.0	3.0	8.00	10.0	1.0	6.37206
0.71	0.30	1.5	5.0	2.0	10.00	10.0	1.0	6.60367
0.71	0.30	1.5	5.0	2.0	8.00	12.0	1.0	7.56665
0.71	0.30	1.5	5.0	2.0	8.00	10.0	2.0	5.23188

Table 2: Skin friction due to secondary velocity
(Cooling case at n = 5.0, t = 1.0 and ∈ = 0.002)

P_r	S_C	M	k_0	α_0	G_r	G_m	E	ι_p
0.71	0.30	1.5	5.0	2.0	8.00	10.0	1.0	5.57355
11.4	0.30	1.5	5.0	2.0	8.00	10.0	1.0	5.96123
0.71	0.66	1.5	5.0	2.0	8.00	10.0	1.0	6.18484
0.71	0.30	2.5	5.0	2.0	8.00	10.0	1.0	6.15907
0.71	0.30	1.5	25.0	2.0	8.00	10.0	1.0	5.94892
0.71	0.30	1.5	5.0	3.0	8.00	10.0	1.0	5.83473
0.71	0.30	1.5	5.0	2.0	10.00	10.0	1.0	5.47619
0.71	0.30	1.5	5.0	2.0	8.00	12.0	1.0	6.76615
0.71	0.30	1.5	5.0	2.0	8.00	10.0	2.0	5.48001

Table 3: Skin friction due to secondary velocity
(Heating case at n = 5.0, t = 1.0 and ∈ = 0.002)

P_r	S_C	M	k_0	α_0	G_r	G_m	E	ι_p
0.71	0.30	1.5	5.0	2.0	- 8.00	- 10.0	1.0	- 6.42479
11.4	0.30	1.5	5.0	2.0	- 8.00	- 10.0	1.0	- 6.33248
0.71	0.66	1.5	5.0	2.0	- 8.00	- 10.0	1.0	- 5.45810
0.71	0.30	2.5	5.0	2.0	- 8.00	- 10.0	1.0	- 5.04565
0.71	0.30	1.5	25.0	2.0	- 8.00	- 10.0	1.0	- 6.50994
0.71	0.30	1.5	5.0	3.0	- 8.00	- 10.0	1.0	- 6.37206
0.71	0.30	1.5	5.0	2.0	- 10.00	- 10.0	1.0	- 6.60367
0.71	0.30	1.5	5.0	2.0	- 8.00	- 12.0	1.0	- 7.56665
0.71	0.30	1.5	5.0	2.0	- 8.00	- 10.0	2.0	-5.23188

Table 4: Skin friction due to secondary velocity
(Heating case at n = 5.0, t = 1.0 and ∈ = 0.002)

P_r	S_C	M	k_0	α_0	G_r	G_m	E	ι_p
0.71	0.30	1.5	5.0	2.0	- 8.00	- 10.0	1.0	- 5.57355
11.4	0.30	1.5	5.0	2.0	- 8.00	- 10.0	1.0	- 5.96123
0.71	0.66	1.5	5.0	2.0	- 8.00	- 10.0	1.0	- 6.18484
0.71	0.30	2.5	5.0	2.0	- 8.00	- 10.0	1.0	- 6.15907
0.71	0.30	1.5	25.0	2.0	- 8.00	- 10.0	1.0	- 5.94892
0.71	0.30	1.5	5.0	3.0	- 8.00	- 10.0	1.0	- 5.83473
0.71	0.30	1.5	5.0	2.0	- 10.00	- 10.0	1.0	- 5.47619
0.71	0.30	1.5	5.0	2.0	- 8.00	- 12.0	1.0	- 6.76615
0.71	0.30	1.5	5.0	2.0	- 8.00	- 10.0	2.0	- 5.48001

Table 5: Rate of heat transfer in terms of Nusslt Number, (∈ = 0.002)

S. No.	Pr	*n*	*α*	*t*	Nu
1.	0.71	5.00	2.0	1.0	-1.81789
2.	0.025	5.00	2.0	1.0	-1.42963
3.	7.00	5.00	2.0	1.0	-7.3174
4.	11.4	5.00	2.0	1.0	-11.6166
5.	0.71	10.00	2.0	1.0	-1.81815
6.	0.71	5.00	3.0	1.0	-2.12833
7.	0.71	5.00	2.0	2.0	-1.81768

Table 6: Rate of mass transfer in terms of Sherwood number, ($\in$ = 0.002)

S. No.	Pr	*n*	*t*	Nu
1.	0.30	5.00	1.0	-0.30158
2.	0.22	5.00	1.0	-0.22127
3.	0.60	5.00	1.0	-0.60264
4.	0.66	5.00	1.0	-0.66366
5.	0.78	5.00	1.0	-0.78323
6.	0.30	10.00	1.0	-030146
7.	0.30	5.00	2.0	-0.30106

6. Discussion and Conclusion

To get physical insight into the problem, we have observed the effects of modified Grashof number (G_m), constant permeability parameter (k_0), magnetic parameter (M), rotation parameter (E), Grashof number (G_r), heat source parameter (α_0), Schmidt number (S_c), and Prandtl number (P_r) on primary velocity (for cooling case $G_r > 0$). These effects are shown in Fig. –1 and Fig.2. The effects of modified Grashof number (G_m), constant permeability parameter (k_0), magnetic parameter (M), rotation parameter (E), Grashof number (G_r), heat source parameter(α_0), Schmidt number (S_c), and prandtl number (P_r) on secondary velocity (for cooling case $G_r > 0$) are shown. The effects of above stated parameter on primary velocity we reach on following conclusions:

1. An increase in G_m, G_r or k_0 increases primary velocity while an increase in M, E, α_0 , S_c or P_r decreases primary velocity in cooling case ($G_r > 0$).
2. An increase in G_m, G_r or k_0 decreases secondary velocity while an increase in M, E, α_0 , S_c or P_r increases secondary velocity in cooling case ($G_r >0$).
3. An increase in M, E, α_0 , S_c or P_r increases primary velocity while an increase in G_m, G_r or k_0 decreases primary velocity in heating case (Gr>0).
4. An increase in M, E, α_0 , S_c or P_r decreases velocity while an increase in G_m, G_r or k_0 increases secondary velocity in heating case ($G_r < 0$).

The effects of the parameter namely prandtl number (P_r), Schmidt number (S_c), magnetic parameter (M), permeability parameter (k_0), heat source parameter (α_0), Grashof number (G_r), modified Grashof number (G_m) and rotation parameter (E), at n = 5.0, t = 1.0, and $\in = 0.002,$ on skin-friction (τ_p) due to primary velocity for cooling case ($G_r > 0$) and skin-friction (τ_S) due to secondary velocity for cooling case ($G_r > 0$) are shown in Table-1 and Table-2. The effects of these parameters on skin-friction at n = 5.0, t = 1.0, and $\in = 0.002,$ for heating case ($G_r <0$) are respectively represented in Table-3 and Table-4, Table-5 represents the rate of heat transfer in terms of Nusselt number (N_u) at $\in = 0.002$. While the table-6 represents the rate of mass transfer in terms of Sherwood number (S_h) at ε = 0.002. The value of Prandtl number (P_r) is chosen as P_r = 0.71 which corresponds to air, P_r = 11.4 which corresponds to water. The numerical values of the remaining parameters are chosen arbitrarily. From these tables we reach on the following conclusions:

1. An increase in P_r, M, α_0 , or E deceases skin-friction due to primary velocity while an increase in S_c, k_0, G_r, or G_m increases skin-friction due to primary velocity in cooling case (Gr > 0).
2. An increase in P_r, S_c, k_0, α_0 or G_m increases skin-friction due to secondary velocity an increase in M, G_r or E decreases skin-friction due to secondary velocity in cooling case ($G_r > 0$).
3. An increase in S_c, k_0, G_r, or G_m decreases skin-friction due to primary velocity while an increase in P_r, M, α_0 or E increases skin-friction due to primary velocity in heating case ($G_r < 0$).
4. An increase in M, G_r or E increases skin-friction due to secondary velocity an increase in P_r, S_c, k_0, α_0 or G_m decreases skin-friction due to secondary velocity in heating case ($G_r < 0$).
5. An increase in P_r, n or α_0 decreases the rate of heat transfer while an increase in t increases the rate of heat transfer.
6. An increase in S_c decreases the rate of mass transfer while an increase in n or t increases the rate of mass transfer.

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