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Arpita Lakhre
School of mathematics, Statistics and Computational Science, Central University of Rajasthan, Ajmer, Rajasthan, India

Rajesh Tailor
S.S in statistics, Vikram University, Ujjain, Madhya Pradesh, India

Correspondence
Arpita Lakhre
School of mathematics, Statistics and Computational Science, Central University of Rajasthan, Ajmer, Rajasthan, India

Improved ratio-cum-product estimators of finite population mean using known parameters of two auxiliary variates in double sampling

Arpita Lakhre and Rajesh Tailor

Abstract
Use of auxiliary information has been in practice to improve the efficiency of the estimators of parameters. Ratio, product and regression methods are good examples of use of auxiliary information. Ratio, product and regression type estimators essentially require the knowledge of population mean of auxiliary variates. But many times, the information on population mean of the auxiliary variate is not available. In this type of situations, double sampling is used. Ajagaonkar (1975) and Sisodia and Dwivedi (1982) discussed problem of estimation using single auxiliary variate whereas Khan and Tripathi (1967), Rao (1975) and Singh and Namjoshi (1988) considered the use of multi auxiliary variates in double sampling.
Singh (1967) used information on two auxiliary variates and envisaged a ratio-cum-product estimator of finite population mean of the study variate assuming that the population mean of the auxiliary variates are known. Upadhyaya and Singh (1999) proposed some ratio type estimators using coefficient of variation and coefficient of kurtosis. Tailor *et al.* (2011) suggested ratio-cum-product estimators using coefficient variation and coefficient of kurtosis of two auxiliary variates in simple random sampling. In this paper, authors study the Tailor *et al.* (2011) ratio-cum-product estimators in double sampling.

Keywords: Ratio-cum-product estimator, double sampling, population mean, Bias, Mean squared error

Introduction
This paper considers the problem of estimation of finite population mean in double sampling. In this paper, two ratio-cum-product estimators of finite population mean, using known coefficient of variation and coefficient of kurtosis of two auxiliary variates, have been suggested. Suggested estimators have been compared with usual unbiased estimator, classical ratio and product estimators in double sampling and double sampling versions of Singh (1967) [3] and Upadhyaya and Singh (1999) estimators. To judge the performance of the suggested estimators over other considered estimators an empirical study also has been carried out.

Let us consider a finite population $U = \{U_1, U_2, ..., U_N\}$ of size N . Let y , x_1 and x_2 be the study variate and auxiliary variates taking values y_i x_{1i} and x_{2i} respectively on $U_i\,(i = 1,2,..., N)$ Let the auxiliary variates x_1 and x_2 be positively and negatively correlated with the study variate y respectively.

Let us define

$\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} y_i$: Population mean of the study variate y ,

$\bar{X}_1 = \frac{1}{N}\sum_{i=1}^{N} x_{1i}$: Population mean of the auxiliary variate x_1,

$\bar{X}_2 = \frac{1}{N}\sum_{i=1}^{N} x_{2i}$: Population mean of the auxiliary variate x_2.

Let $\bar{y} = \sum_{i=1}^{n} y_i / n$, $\bar{x}_1 = \sum_{i=1}^{n} x_{1i} / n$ and $\bar{x}_2 = \sum_{i=1}^{n} x_{2i} / n$ be the unbiased estimators of population mean $\bar{Y}$, $\bar{X}_1$ and $\bar{X}_2$ respectively.

$\bar{x}_1^{/} = \frac{1}{n}\sum_{i=1}^{n} x_{1i}$: First phase sample mean of the auxiliary variate x_1 based on sample size $n^{/}$,

$\bar{x}_2^{/} = \frac{1}{n}\sum_{i=1}^{n} x_{2i}$: First phase sample mean of the auxiliary variate x_2 based on sample size $n^{/}$,

$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$: Second phase sample mean of the study variate y based on sample size n,

$\bar{x}_1 = \frac{1}{n}\sum_{i=1}^{n} x_{1i}$: Second phase sample mean of the auxiliary variate x_1 based on sample size n,

$\bar{x}_2 = \frac{1}{n}\sum_{i=1}^{n} x_{2i}$: Second phase sample mean of the auxiliary variate x_2 based on sample size n,

$S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \bar{Y})^2$: Population mean square of the study variate y,

$S_{x_1}^2 = \frac{1}{N-1}\sum_{i=1}^{N}(x_{1i} - \bar{X}_1)^2$: Population mean square of the auxiliary variate x_1,

$S_{x_{21}}^2 = \frac{1}{N-1}\sum_{i=1}^{N}(x_{2i} - \bar{X}_2)^2$: Population mean square of the auxiliary variate x_2,

$S_{yx_1} = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \bar{Y})(x_{1i} - \bar{X}_1)$: Population covariance between the study variate y and auxiliary variate x_1

$S_{yx_2} = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \bar{Y})(x_{2i} - \bar{X}_2)$: Population covariance between the study variate y and auxiliary variate x_2,

$S_{x_1x_2} = \frac{1}{N-1}\sum_{i=1}^{N}(x_{1i} - \bar{X}_1)(x_{2i} - \bar{X}_2)$: Population covariance between the auxiliary variate x_1 and x_2,

$\rho_{yx_1} = \frac{S_{yx_1}}{\sqrt{S_y^2 S_{x_1}^2}}$: Population correlation coefficient between the study variate y and auxiliary variate x_1,

$\rho_{yx_2} = \frac{S_{yx_2}}{\sqrt{S_y^2 S_{x_2}^2}}$: Population correlation coefficient between the study variate y and auxiliary variate x_2,

$\rho_{x_1x_2} = \frac{S_{x_1x_2}}{\sqrt{S_{x_1}^2 S_{x_2}^2}}$: Population correlation coefficient between the auxiliary variate x_1 and auxiliary variate x_2,

$C_y = \frac{S_y}{\bar{Y}}$: Population coefficient of variation of the study variate y,

$C_{x_1} = \frac{S_{x_1}}{\bar{X}_1}$: Population coefficient of variation of the auxiliary variate x_1,

$C_{x_2} = \frac{S_{x_2}}{\bar{X}_2}$: Population coefficient of variation of the auxiliary variate x_2,

$\beta_2(x_1) = \dfrac{\sum (X_{1i} - \overline{X}_1)^4}{\sum (X_{1i} - \overline{X}_1)^2}$: Coefficient of kurtosis of the auxiliary variate x_1,

$\beta_2(x_2) = \dfrac{\sum (X_{2i} - \overline{X}_2)^4}{\sum (X_{2i} - \overline{X}_2)^2}$: Coefficient of kurtosis of the auxiliary variate x_2,

Cochran (1940) [2] envisaged classical ratio estimator for estimating the population mean $\overline{Y}$ when study variate y and auxiliary variate x_1 are positively correlated as

$$\hat{\overline{Y}}_R = \overline{y}\left(\frac{\overline{X}_1}{\overline{x}_1}\right) \tag{1.1}$$

In case of negative correlation between the study variate y and the auxiliary variate x_2, the classical product estimator was given by Robson (1957) [6] as

$$\hat{\overline{Y}}_P = \overline{y}\left(\frac{\overline{x}_2}{\overline{X}_2}\right) \tag{1.2}$$

Assuming that the population mean $\overline{X}_1$ and coefficient of variation C_{x_1} of the auxiliary variate x_1 are known, Sisodia and Dwivedi (1981) defined a ratio type estimator as

$$\hat{\overline{Y}}_{SDR} = \overline{y}\left(\frac{\overline{X}_1 + C_{x_1}}{\overline{x}_1 + C_{x_1}}\right), \tag{1.3}$$

When the correlation coefficient between the study variate y and auxiliary variate x_2 is negative product type estimator using coefficient of variation C_{x_2} is expressed as

$$\hat{\overline{Y}}_{SDP} = \overline{y}\left(\frac{\overline{x}_2 + C_{x_2}}{\overline{X}_2 + C_{x_2}}\right). \tag{1.4}$$

Singh *et al*. (2004) [10] defined ratio and product type estimators using coefficient of kurtosis $\beta_2(x_1)$ and $\beta_2(x_2)$ respectively as

$$\hat{\overline{Y}}_{SER} = \overline{y}\left(\frac{\overline{X}_1 + \beta_2(x_1)}{\overline{x}_1 + \beta_2(x_1)}\right), \tag{1.5}$$

and

$$\hat{\overline{Y}}_{SEP} = \overline{y}\left(\frac{\overline{x}_2 + \beta_2(x_2)}{\overline{X}_2 + \beta_2(x_2)}\right). \tag{1.6}$$

Upadhyaya and Singh (1999) utilized both coefficient of kurtosis as well as coefficient of variations of auxiliary variates and suggested two ratio and two product type estimators of population mean $\overline{Y}$ as

$$\hat{\overline{Y}}_{US1R} = \overline{y}\left(\frac{\overline{X}_1 C_{x_1} + \beta_2(x_1)}{\overline{x}_1 C_{x_1} + \beta_2(x_1)}\right), \tag{1.7}$$

$$\hat{\overline{Y}}_{US2R} = \overline{y}\left(\frac{\overline{X}_1 \beta_2(x_1) + C_{x_1}}{\overline{x}_1 \beta_2(x_1) + C_{x_1}}\right) \tag{1.8}$$

$$\hat{\overline{Y}}_{US1R} = \overline{y}\left(\frac{\overline{x}_2 C_{x_2} + \beta_2(x_2)}{\overline{X}_2 C_{x_2} + \beta_2(x_2)}\right). \tag{1.9}$$

$$\hat{\overline{Y}}_{US2P} = \overline{y}\left(\frac{\overline{x}_2 \beta_2(x_2) + C_{x_2}}{\overline{X}_2 \beta_2(x_2) + C_{x_2}}\right) \tag{1.10}$$

Singh (1967) [3] utilized information on known population means $\overline{X}_1$ and $\overline{X}_2$ of auxiliary variates x_1 and x_2 respectively and envisaged a ratio-cum-product estimator of population mean $\overline{Y}$ as

$$\hat{\hat{\bar{Y}}}_{SRP} = \bar{y}\left(\frac{\bar{X}_1}{\bar{x}_1}\right)\left(\frac{\bar{x}_2}{\bar{X}_2}\right) \tag{1.11}$$

The problem of estimating the population mean $\bar{Y}$ of y when the population means $\bar{X}_1$ and $\bar{X}_2$ of x_1 and x_2 respectively are known, has been discussed by many researchers including Singh and Tailor (2005) [9], Tailor and Tailor (2008) [12], Tailor *et al*. (2011a) [13], Tailor (2012) [11] and many others. When information is not available on $\bar{X}_1$ and $\bar{X}_2$ in advance, double sampling procedure is used. The standard double sampling procedure is described as

(i) a first phase sample S_1 of fixed size n' is drawn form U to observe only x_1 and x_2 to estimate $\bar{X}_1$ and $\bar{X}_2$ respectively then
(ii) a second phase sample S_2 of fixed size n is drawn either from S_1 from first phase sample or directly from the population.
These two cases may be designated as
Case I: As a sub sample from the first phase sample and
Case II: Drawn independently to the first phase sample.
In double sampling, the usual ratio and product estimators of population mean $\bar{Y}$ are respectively defined as

$$\hat{\hat{\bar{Y}}}_R^{(d)} = \bar{y}\left(\frac{\bar{x}_1'}{\bar{x}_1}\right), \tag{1.12}$$

and

$$\hat{\hat{\bar{Y}}}_P^{(d)} = \bar{y}\left(\frac{\bar{x}_2}{\bar{x}_2'}\right), \tag{1.13}$$

where $\bar{y}$, $\bar{x}_1$ and $\bar{x}_2$ are sample means based on second phase sample of size n whereas $\bar{x}_1' = \frac{1}{n'}\sum_{i=1}^{n'} x_{1i}$ and $\bar{x}_2' = \frac{1}{n'}\sum_{i=1}^{n'} x_{2i}$ are the first phase sample means of x_1 and x_2, which are unbiased estimates of population means $\bar{X}_1$ and $\bar{X}_2$ respectively.
In double sampling, Sisodia and Dwivedi (1981) ratio type and Pandey and Dubey (1988) [4] product type estimators of population mean $\bar{Y}$ are defined as

$$\hat{\hat{\bar{Y}}}_{SDR}^{(d)} = \bar{y}\left(\frac{\bar{x}_1' + C_{x_1}}{\bar{x}_1 + C_{x_1}}\right), \tag{1.14}$$

and

$$\hat{\hat{\bar{Y}}}_{SDP}^{(d)} = \bar{y}\left(\frac{\bar{x}_2 + C_{x_2}}{\bar{x}_2' + C_{x_2}}\right). \tag{1.15}$$

In double sampling, Singh *et al*. (2004) [10] ratio and product type estimators of population mean $\bar{Y}$ are defined as

$$\hat{\hat{\bar{Y}}}_{SER}^{(d)} = \bar{y}\left(\frac{\bar{x}_1' + \beta_2(x_1)}{\bar{x}_1 + \beta_2(x_1)}\right), \tag{1.16}$$

and

$$\hat{\hat{\bar{Y}}}_{SEP}^{(d)} = \bar{y}\left(\frac{\bar{x}_2 + \beta_2(x_2)}{\bar{x}_2' + \beta_2(x_2)}\right). \tag{1.17}$$

Double sampling versions of Upadhyaya and Singh (1999) ratio type estimators are

$$\hat{\hat{\bar{Y}}}_{US1R}^{(d)} = \bar{y}\left(\frac{\bar{x}_1' C_{x_1} + \beta_2(x_1)}{\bar{x}_1 C_{x_1} + \beta_2(x_1)}\right), \tag{1.18}$$

and

$$\hat{\hat{\bar{Y}}}_{US2R}^{(d)} = \bar{y}\left(\frac{\bar{x}_1' \beta_2(x_1) + C_{x_1}}{\bar{x}_1 \beta_2(x_1) + C_{x_1}}\right). \tag{1.19}$$

Double sampling version of Upadhyaya and Singh (1999) product type estimators are define as

$$\hat{\hat{\bar{Y}}}_{US1P}^{(d)} = \bar{y}\left(\frac{\bar{x}_2' C_{x_2} + \beta_2(x_2)}{\bar{x}_2 C_{x_2} + \beta_2(x_2)}\right), \tag{1.20}$$

and

$$\hat{\hat{\bar{Y}}}_{US2P}^{(d)} = \bar{y}\left(\frac{\bar{x}_2^{/}\beta_2(x_2)+C_{x_2}}{\bar{x}_2\beta_2(x_2)+C_{x_2}}\right). \quad (1.21)$$

In double sampling, Singh (1967) [3] ratio-cum-product estimator $\hat{\hat{\bar{Y}}}_{RP}$ of population mean $\bar{Y}$ is expressed as

$$\hat{\hat{\bar{Y}}}_{RP}^{(d)} = \bar{y}\left(\frac{\bar{x}_1^{'}}{\bar{x}_1}\right)\left(\frac{\bar{x}_2}{\bar{x}_2^{'}}\right). \quad (1.22)$$

The biases and mean squared errors $\hat{\hat{\bar{Y}}}_{R}^{(d)}$, $\hat{\hat{\bar{Y}}}_{P}^{(d)}$, $\hat{\hat{\bar{Y}}}_{SDR}^{(d)}$, $\hat{\hat{\bar{Y}}}_{SDP}^{(d)}$, $\hat{\hat{\bar{Y}}}_{SER}^{(d)}$, $\hat{\hat{\bar{Y}}}_{SEP}^{(d)}$, $\hat{\hat{\bar{Y}}}_{US1R}^{(d)}$, $\hat{\hat{\bar{Y}}}_{US2R}^{(d)}$, $\hat{\hat{\bar{Y}}}_{US1P}^{(d)}$, $\hat{\hat{\bar{Y}}}_{US2P}^{(d)}$ and $\hat{\hat{\bar{Y}}}_{RP}^{(d)}$ in both case I and case II are given below:

$$B(\hat{\hat{\bar{Y}}}_{R}^{(d)})_I = \bar{Y}f_3\, C_{x_1}^2\,(1-K_{01}), \quad (1.23)$$

$$B(\hat{\hat{\bar{Y}}}_{P}^{(d)})_I = \bar{Y}f_3\, C_{x_2}^2\,(1+K_{02}), \quad (1.24)$$

$$B(\hat{\hat{\bar{Y}}}_{SDR}^{(d)})_I = \bar{Y}f_3\, t_1C_{x_1}^2\,(t_1-K_{01}), \quad (1.25)$$

$$B(\hat{\hat{\bar{Y}}}_{SDP}^{(d)})_I = \bar{Y}f_3\, t_2C_{x_2}^2\,(t_2+K_{02}), \quad (1.26)$$

$$B(\hat{\hat{\bar{Y}}}_{SER}^{(d)})_I = \bar{Y}f_3\, t_3C_{x_1}^2\,(t_3-K_{01}), \quad (1.27)$$

$$B(\hat{\hat{\bar{Y}}}_{SEP}^{(d)})_I = \bar{Y}f_3\, t_4C_{x_2}^2\,(t_4+K_{02}), \quad (1.28)$$

$$B(\hat{\hat{\bar{Y}}}_{US1R}^{(d)})_I = \bar{Y}f_3\, t_5C_{x_1}^2\,(t_5-K_{01}), \quad (1.29)$$

$$B(\hat{\hat{\bar{Y}}}_{US1P}^{(d)})_I = \bar{Y}f_3\, t_6C_{x_2}^2\,(t_6+K_{02}), \quad (1.30)$$

$$B(\hat{\hat{\bar{Y}}}_{US2R}^{(d)})_I = \bar{Y}f_3\, t_7C_{x_1}^2\,(t_7-K_{01}), \quad (1.31)$$

$$B(\hat{\hat{\bar{Y}}}_{US2P}^{(d)})_I = \bar{Y}f_3\, t_8C_{x_2}^2\,(t_8+K_{02}), \quad (1.32)$$

$$B(\hat{\hat{\bar{Y}}}_{RP}^{(d)})_I = \bar{Y}f_3\left[C_{x_1}^2\,(1-K_{01})-C_{x_2}^2\,(K_{02}+K_{12})\right], \quad (1.33)$$

$$B(\hat{\hat{\bar{Y}}}_{R}^{(d)})_{II} = \bar{Y}f_1\, C_{x_1}^2\,(1-K_{01}), \quad (1.34)$$

$$B(\hat{\hat{\bar{Y}}}_{P}^{(d)})_{II} = \bar{Y}\, C_{x_2}^2\,(f_2+f_1K_{02}), \quad (1.35)$$

$$B(\hat{\hat{\bar{Y}}}_{SDR}^{(d)})_{II} = \bar{Y}f_1\, t_1C_{x_1}^2\,(t_1-K_{01}), \quad (1.36)$$

$$B(\hat{\hat{\bar{Y}}}_{SDP}^{(d)})_{II} = \bar{Y}t_2C_{x_2}^2\,(f_2t_2+f_1K_{02}), \quad (1.37)$$

$$B(\hat{\hat{\bar{Y}}}_{SER}^{(d)})_{II} = \bar{Y}f_1\, t_3C_{x_1}^2\,(t_3-K_{01}), \quad (1.38)$$

$$B(\hat{\hat{\bar{Y}}}_{SEP}^{(d)})_{II} = \bar{Y}t_4C_{x_2}^2\,(f_2t_4+f_1K_{02}), \quad (1.39)$$

$$B(\hat{\hat{\bar{Y}}}_{US1R}^{(d)})_{II} = \bar{Y}f_1\, t_5C_{x_1}^2\,(t_5-K_{01}), \quad (1.40)$$

$$B(\hat{\hat{\bar{Y}}}_{US1P}^{(d)})_{II} = \bar{Y}f_1\, t_6C_{x_2}^2\,(f_2t_6+f_1K_{02}), \quad (1.41)$$

$$B(\hat{\hat{\bar{Y}}}_{US2R}^{(d)})_{II} = \bar{Y}f_3\, t_7C_{x_1}^2\,(t_7-K_{01}), \quad (1.42)$$

$$B(\hat{\hat{\bar{Y}}}_{US2P}^{(d)})_{II} = \bar{Y}f_1\, t_8C_{x_2}^2\,(f_2t_8+f_1K_{02}), \quad (1.43)$$

$$B(\hat{\hat{\bar{Y}}}_{RP}^{(d)})_{II} = \bar{Y}\left[f_1C_{x_1}^2\,(1-K_{01})+C_{x_2}^2\,(f_2-K_{12}+f_1K_{02})\right], \quad (1.44)$$

$$MSE(\hat{\hat{\bar{Y}}}_{R}^{(d)})_I = \bar{Y}^2\left[f_1C_y^2+f_3\,C_{x_1}^2(1-2K_{01})\right], \quad (1.45)$$

$$MSE(\hat{\hat{\bar{Y}}}_{P}^{(d)})_I = \bar{Y}^2\left[f_1C_y^2+f_3\,C_{x_2}^2(1+2K_{02})\right], \quad (1.46)$$

$$MSE(\hat{\hat{\bar{Y}}}_{SDR}^{(d)})_I = \bar{Y}^2\left[f_1C_y^2+f_3\,t_1C_{x_1}^2(t_1-2K_{01})\right], \quad (1.47)$$

$$MSE(\hat{\bar{Y}}_{SDP}^{(d)})_I = \bar{Y}^2\left[f_1C_y^2 + f_3\, t_2C_{x_2}^2\,(t_2 + 2K_{02})\right], \tag{1.48}$$

$$MSE(\hat{\bar{Y}}_{SER}^{(d)})_I = \bar{Y}^2\left[f_1C_y^2 + f_3\, t_3C_{x_1}^2(t_3 - 2K_{01})\right], \tag{1.49}$$

$$MSE(\hat{\bar{Y}}_{SEP}^{(d)})_I = \bar{Y}^2\left[f_1C_y^2 + f_3\, t_4C_{x_2}^2\,(t_4 + 2K_{02})\right], \tag{1.50}$$

$$MSE(\hat{\bar{Y}}_{US1R}^{(d)})_I = \bar{Y}^2\left[f_1C_y^2 + f_3\, t_5C_{x_1}^2\,(t_5 - 2K_{01})\right], \tag{1.51}$$

$$MSE(\hat{\bar{Y}}_{US1P}^{(d)})_I = \bar{Y}^2\left[f_1C_y^2 + f_3\, t_5C_{x_2}^2\,(t_6 + 2K_{01})\right], \tag{1.52}$$

$$MSE(\hat{\bar{Y}}_{US2R}^{(d)})_I = \bar{Y}^2\left[f_1C_y^2 + f_3\, t_7C_{x_1}^2\,(t_7 - 2K_{01})\right], \tag{1.53}$$

$$MSE(\hat{\bar{Y}}_{US2P}^{(d)})_I = \bar{Y}^2\left[f_1C_y^2 + f_3\, t_8C_{x_2}^2\,(t_8 + 2K_{01})\right], \tag{1.54}$$

$$MSE(\hat{\bar{Y}}_{SRP}^{(d)})_I = \bar{Y}^2\left[f_1C_y^2 + f_3\, C_{x_1}^2(1 - 2K_{01}) + f_3\, C_{x_2}^2(1 + 2K_{02} - 2K_{12})\right], \tag{1.55}$$

$$MSE(\hat{\bar{Y}}_{R}^{(d)})_{II} = \bar{Y}^2\left[f_1C_y^2 + C_{x_1}^2\left\{(f_1 + f_2) - 2f_1K_{01}\right\}\right], \tag{1.56}$$

$$MSE(\hat{\bar{Y}}_{P}^{(d)})_{II} = \bar{Y}^2\left[f_1C_y^2 + C_{x_2}^2\left\{(f_1 + f_2) + 2f_1K_{02}\right\}\right], \tag{1.57}$$

$$MSE(\hat{\bar{Y}}_{SDR}^{(d)})_{II} = \bar{Y}^2\left[f_1C_y^2 + t_1C_{x_1}^2\left\{t_1(f_1 + f_2) - 2f_1K_{01}\right\}\right], \tag{1.58}$$

$$MSE(\hat{\bar{Y}}_{SDP}^{(d)})_{II} = \bar{Y}^2\left[f_1C_y^2 + t_2C_{x_2}^2\left\{t_2(f_1 + f_2) + 2f_1K_{02}\right\}\right], \tag{1.59}$$

$$MSE(\hat{\bar{Y}}_{SER}^{(d)})_{II} = \bar{Y}^2\left[f_1C_y^2 + t_3C_{x_1}^2\left\{t_3(f_1 + f_2) - 2f_1K_{01}\right\}\right], \tag{1.60}$$

$$MSE(\hat{\bar{Y}}_{SEP}^{(d)})_{II} = \bar{Y}^2\left[f_1C_y^2 + t_4C_{x_2}^2\left\{t_4(f_1 + f_2) + 2f_1K_{02}\right\}\right], \tag{1.61}$$

$$MSE(\hat{\bar{Y}}_{US1R}^{(d)})_{II} = \bar{Y}^2\left[f_1C_y^2 + t_5C_{x_1}^2\left\{t_5(f_1 + f_2) - 2f_1K_{01}\right\}\right], \tag{1.62}$$

$$MSE(\hat{\bar{Y}}_{US1P}^{(d)})_{II} = \bar{Y}^2\left[f_1C_y^2 + t_6C_{x_2}^2\left\{t_6(f_1 + f_2) + 2f_1K_{02}\right\}\right], \tag{1.63}$$

$$MSE(\hat{\bar{Y}}_{US2R}^{(d)})_{II} = \bar{Y}^2\left[f_1C_y^2 + t_7C_{x_1}^2\left\{t_7(f_1 + f_2) - 2f_1K_{01}\right\}\right], \tag{1.64}$$

$$MSE(\hat{\bar{Y}}_{US2P}^{(d)})_{II} = \bar{Y}^2\left[f_1C_y^2 + t_8C_{x_2}^2\left\{t_8(f_1 + f_2) + 2f_1K_{02}\right\}\right], \tag{1.65}$$

and

$$MSE(\hat{\bar{Y}}_{SRP}^{(d)})_{II} = \bar{Y}^2\left[f_1C_y^2 + C_{x_1}^2\left\{f_1 + f_2 - 2f_1K_{01})\right\} + C_{x_2}^2\left\{(f_1 + f_2 + 2f_1K_{02} - 2f_2K_{12})\right\}\right] \tag{1.66}$$

where $t_1 = \dfrac{\bar{X}_1}{(\bar{X}_1 + C_{x_1})}$, $t_2 = \dfrac{\bar{X}_2}{(\bar{X}_2 + C_{x_2})}$,

$t_3 = \dfrac{\bar{X}_1}{(\bar{X}_1 + \beta_2(x_1))}$, $t_4 = \dfrac{\bar{X}_2}{(\bar{X}_2 + \beta_2(x_2))}$,

$t_5 = \dfrac{\bar{X}_1C_{x_1}}{(\bar{X}_1C_{x_1} + \beta_2(x_1))}$, $t_6 = \dfrac{\bar{X}_2C_{x_2}}{(\bar{X}_2C_{x_2} + \beta_2(x_2))}$,

$t_7 = \dfrac{\bar{X}_1\beta_2(x_1)}{(\bar{X}_1\beta_2(x_1) + C_{x_1})}$, $t_8 = \dfrac{\bar{X}_2\beta_2(x_2)}{(\bar{X}_2\beta_2(x_2) + C_{x_2})}$.

$K_{01} = \rho_{yx}\dfrac{C_y}{C_x}$, $K_{02} = \rho_{yz}\dfrac{C_y}{C_z}$, $K_{12} = \rho_{xz}\dfrac{C_x}{C_z}$

$f_1 = \left(\dfrac{1}{n} - \dfrac{1}{N}\right)$, $f_2 = \left(\dfrac{1}{n'} - \dfrac{1}{N}\right)$ and $f_3 = f_1 - f_2$.

2. Suggested Ratio-Cum-Product Estimators

Tailor *et al*. (2011 a) proposed ratio-cum-product estimators of population mean $\bar{Y}$ using information on coefficient of variations and coefficient of kurtosis of auxiliary variates x_1 and x_2 as

$$\hat{\hat{\bar{Y}}}_{T1} = \bar{y}\left(\frac{\bar{X}_1 C_{x_1} + \beta_2(x_1)}{\bar{x}_1 C_{x_1} + \beta_2(x_1)}\right)\left(\frac{\bar{x}_2 C_{x_2} + \beta_2(x_2)}{\bar{X}_2 C_{x_2} + \beta_2(x_2)}\right), \quad (2.1)$$

and

$$\hat{\hat{\bar{Y}}}_{T2} = \bar{y}\left(\frac{\bar{X}_1 \beta_2(x_1) + C_{x_1}}{\bar{x}_1 \beta_2(x_1) + C_{x_1}}\right)\left(\frac{\bar{x}_2 \beta_2(x_2) + C_{x_2}}{\bar{X}_2 \beta_2(x_2) + C_{x_2}}\right). \quad (2.2)$$

The estimators $\hat{\hat{\bar{Y}}}_{T1}$ and $\hat{\hat{\bar{Y}}}_{T2}$ require the knowledge of $\bar{X}_1, \bar{X}_2, C_{x_1}, C_{x_2}\ \beta_2(x_1)$ and $\beta_2(x_2)$. When $\bar{X}_1$ and $\bar{X}_2$ are not known $\hat{\hat{\bar{Y}}}_{T1}$ and $\hat{\hat{\bar{Y}}}_{T2}$ cannot be used. Hence $\hat{\hat{\bar{Y}}}_{T1}$ and $\hat{\hat{\bar{Y}}}_{T2}$ in double sampling are studied as

$$\hat{\hat{\bar{Y}}}_{T1}^{(d)} = \bar{y}\left(\frac{\bar{x}'_1 C_{x_1} + \beta_2(x_1)}{\bar{x}_1 C_{x_1} + \beta_2(x_1)}\right)\left(\frac{\bar{x}_2 C_{x_2} + \beta_2(x_2)}{\bar{x}'_2 C_{x_2} + \beta_2(x_2)}\right), \quad (2.3)$$

and

$$\hat{\hat{\bar{Y}}}_{T2}^{(d)} = \bar{y}\left(\frac{\bar{X}_1 \beta_2(x_1) + C_{x_1}}{\bar{x}_1 \beta_2(x_1) + C_{x_1}}\right)\left(\frac{\bar{x}_2 \beta_2(x_2) + C_{x_2}}{\bar{X}_2 \beta_2(x_2) + C_{x_2}}\right) \quad (2.4)$$

To obtain the bias and mean squared error of $\hat{\hat{\bar{Y}}}_{T1}^{(d)}$ we write

$\bar{y} = \bar{Y}(1+e_0)$, $\bar{x} = \bar{X}(1+e_1)$, $\bar{x}'_1 = \bar{X}_1(1+e'_1)$ and $\bar{x}_2 = \bar{X}'_2(1+e'_2)$ such that

$\mathrm{E}(e_0) = \mathrm{E}(e_1) = \mathrm{E}(e_2) = \mathrm{E}(e_3) = \mathrm{E}(e_4) = 0$,

$\mathrm{E}(e_0^2) = f_1 C_y^2$,

$\mathrm{E}(e_1^2) = f_1 C_{x_1}^2$,

$\mathrm{E}(e_2^2) = f_1 C_{x_2}^2$,

$\mathrm{E}(e_1'^2) = f_2 C_{x_1}^2$,

$\mathrm{E}(e_2'^2) = f_2 C_{x_2}^2$,

$\mathrm{E}(e_0 e_1) = f_1 \rho_{yx_1} C_y C_{x_1}$,

$\mathrm{E}(e_0 e_2) = f_1 \rho_{yx_2} C_y C_{x_2}$,

$\mathrm{E}(e_0 e'_1) = f_2 \rho_{yx_1} C_y C_{x_1}$,

$\mathrm{E}(e_0 e'_2) = f_2 \rho_{yx_2} C_y C_{x_2}$,

$\mathrm{E}(e_1 e'_1) = f_2 C_{x_1}^2$,

$\mathrm{E}(e_2 e'_2) = f_2 C_{x_2}^2$,

$\mathrm{E}(e_1 e_2) = f_1 \rho_{x_1 x_2} C_{x_1} C_{x_2}$,

and $\mathrm{E}(e'_1 e_2) = \mathrm{E}(e'_1 e'_2) = \mathrm{E}(e_1 e'_2) = f_2 \rho_{x_1 x_2} C_{x_1} C_{x_2}$.

It is important to notice that $C_{x_1}, C_{x_2}\ \beta_2(x_1)$ and $\beta_2(x_2)$ also may not be available due to lack of information on population mean of auxiliary variates. In this type of situation, estimators of $C_{x_1}, C_{x_2}\ \beta_2(x_1)$ and $\beta_2(x_2)$ can be used. Expressing (2.2.2) in terms of e_i's we have

$$\hat{\hat{\bar{Y}}}_{T1}^{(d)} = \bar{Y}(1+e_0)\ \left[(1+t_5 e'_1)(1+t_5 e_1)^{-1}(1+t_6 e_2)(1+t_6 e'_2)^{-1}\right], \quad (2.5)$$

Now, using the standard technique, the bias and mean squared error of the suggested estimator $\hat{\hat{\bar{Y}}}_{T1}^{(d)}$ upto the first degree of approximation under cases I and II are respectively obtained as

$$\mathrm{B}(\hat{\hat{\bar{Y}}}_{T1}^{(d)})_I = \bar{Y}\Big[t_5^2\left(f_1 C_{x_1}^2 + f_2 C_{x_2}^2\right) - t_5 f_3 \rho_{yx_1} C_y C_{x_1} + t_6 f_3 \rho_{yx_2} C_y C_{x_2}$$
$$- t_5 t_6\left(f_2 \rho_{x_1 x_2} C_{x_1} C_{x_2} - f_2 C_{x_2}^2\right) + t_5^2 f_2 \rho_{yx_1} C_y C_{x_1}\Big],$$

or

$$\mathrm{B}(\hat{\hat{\bar{Y}}}_{T1}^{(d)})_I = \bar{Y}\left[t_5^2 C_{x_1}^2\left(t_5 f_1 - f_3 K_{01} + f_2 t_5 K_{01}\right) + C_{x_2}^2\left\{\left(f_2 t_5 (t_5 - t_6 K_{12}) + t_6 (f_2 t_5 + f_3 K_{02})\right\}\right] \quad (2.6)$$

$$\mathrm{B}(\hat{\hat{\bar{Y}}}_{T1}^{(d)})_{II} = \bar{Y}\Big[t_5^2\big(f_1 C_{x_1}^2 + f_2 C_{x_2}^2\big) - t_5 f_1 \rho_{yx_1} C_y C_{x_1} + t_6 f_2 \rho_{yx_2} C_y C_{x_2} - t_5 t_6 f_1 \rho_{x_1x_2} C_{x_1} C_{x_2} + t_5^2 f_2 C_{x_2}^2\Big],$$

or

$$\mathrm{B}(\hat{\hat{\bar{Y}}}_{T1}^{(d)})_{II} = \bar{Y}\Big[t_5^2 C_x^2\big(t_5 f_1 - t_5 f_1 K_{01}\big) + C_{x_2}^2\big(2t_5^2 f_2 + t_6 f_2 K_{02} - t_6 t_5 f_1 K_{12}\big)\Big], \quad (2.7)$$

$$MSE(\hat{\hat{\bar{Y}}}_{T1}^{(d)})_I = \bar{Y}^2\Big[f_1 C_y^2 + f_3 t_5^2 C_{x_1}^2 - 2t_5 f_3 \rho_{yx_1} C_y C_{x_1} + t_6^2 f_3 C_{x_2}^2 + 2t_6 f_3\big(\rho_{yx_2} C_y C_{x_2} - t_5 \rho_{x_1x_2} C_{x_1} C_{x_2}\big)\Big],$$

or

$$MSE(\hat{\hat{\bar{Y}}}_{T1}^{(d)})_I = \bar{Y}^2\Big[f_1 C_y^2 + f_3 C_{x_1}^2 t_5(t_5 - 2K_{01}) + f_3 C_{x_2}^2 t_6(t_6 + 2K_{02} - 2t_5 K_{12})\Big], \quad (2.8)$$

$$MSE(\hat{\hat{\bar{Y}}}_{T1}^{(d)})_{II} = \bar{Y}^2\Big[f_1 C_y^2 + \big(f_1 + f_2\big)\big(t_5^2 C_{x_1}^2 + t_6^2 C_{x_2}^2\big) - 2t_5 f_1 C_{x_1}^2\big(K_{01} + t_6 K_{12}\big) + 2t_6 f_1 \rho_{yx_2} C_y C_{x_2}\Big],$$

or

$$MSE(\hat{\hat{\bar{Y}}}_{T1}^{(d)})_{II} = \bar{Y}^2\Big[f_1 C_y^2 + C_{x_1}^2 t_5\big\{t_5(f_1 + f_2) - 2 f_1 K_{01}\big\} + C_{x_2}^2 t_6\big\{t_6(f_1 + f_2) + 2 f_1 K_{02}\big\} - 2t_5 f_1 K_{12}\Big]. \quad (2.9)$$

Similarly the biases and mean squared errors of the suggested estimator $(\hat{\hat{\bar{Y}}}_{T2}^{(d)})$ are obtained as

$$\mathrm{B}(\hat{\hat{\bar{Y}}}_{T2}^{(d)})_I = \bar{Y}\Big[t_7^2\big(f_1 C_{x_1}^2 + f_2 C_{x_2}^2\big) - t_7 f_3 \rho_{yx_1} C_y C_{x_1} + t_8 f_3 \rho_{yx_2} C_y C_{x_2} - t_7 t_8\big(f_2 \rho_{x_1x_2} C_{x_1} C_{x_2} - f_2 C_{x_2}^2\big) + t_7^2 f_2 \rho_{yx_1} C_y C_{x_1}\Big],$$

or

$$\mathrm{B}(\hat{\hat{\bar{Y}}}_{T2}^{(d)})_I = \bar{Y}\Big[t_7^2 C_{x_1}^2\big(t_7 f_1 - f_3 K_{01} + f_2 t_7 K_{01}\big) + C_{x_2}^2\big\{(f_2 t_7(t_7 - t_8 K_{12}) + t_8(f_2 t_7 + f_3 K_{02})\big\}\Big], \quad (2.10)$$

$$\mathrm{B}(\hat{\hat{\bar{Y}}}_{T2}^{(d)})_{II} = \bar{Y}\Big[t_7^2\big(f_1 C_{x_1}^2 + f_2 C_{x_2}^2\big) - t_7 f_1 \rho_{yx_1} C_y C_{x_1} + t_8 f_2 \rho_{yx_2} C_y C_{x_2} - t_7 t_8 f_1 \rho_{x_1x_2} C_{x_1} C_{x_2} + t_7^2 f_2 C_{x_2}^2\Big],$$

or

$$\mathrm{B}(\hat{\hat{\bar{Y}}}_{T2}^{(d)})_{II} = \bar{Y}\Big[t_7^2 C_{x_1}^2\big(t_7 f_1 - t_7 f_1 K_{01}\big) + C_{x_2}^2\big(2t_7^2 f_2 + t_8 f_2 K_{02} - t_8 t_7 f_1 K_{12}\big)\Big], \quad (2.11)$$

$$MSE(\hat{\hat{\bar{Y}}}_{T1}^{(d)})_I = \bar{Y}^2\Big[f_1 C_y^2 + f_3 t_7^2 C_{x_1}^2 - 2t_7 f_3 \rho_{yx_1} C_y C_{x_1} + t_8^2 f_3 C_{x_2}^2 + 2t_8 f_3\big(\rho_{yx_2} C_y C_{x_2} - t_7 \rho_{x_1x_2} C_{x_1} C_{x_2}\big)\Big],$$

or

$$MSE(\hat{\hat{\bar{Y}}}_{T1}^{(d)})_I = \bar{Y}^2\Big[f_1 C_y^2 + f_3 C_{x_1}^2 t_7(t_7 - 2K_{01}) + f_3 C_{x_2}^2 t_8(t_8 + 2K_{02} - 2t_7 K_{12})\Big], \quad (2.12)$$

$$MSE(\hat{\hat{\bar{Y}}}_{T1}^{(d)})_{II} = \bar{Y}^2\Big[f_1 C_y^2 + \big(f_1 + f_2\big)\big(t_7^2 C_{x_1}^2 + t_8^2 C_{x_2}^2\big) - 2t_7 f_1 C_{x_1}^2\big(K_{01} + t_8 K_{12}\big) + 2t_8 f_1 \rho_{yx_2} C_y C_{x_2}\Big],$$

or

$$MSE(\hat{\hat{\bar{Y}}}_{T1}^{(d)})_{II} = \bar{Y}^2\Big[f_1 C_y^2 + C_{x_1}^2 t_7\big\{t_7(f_1 + f_2) - 2 f_1 K_{01}\big\} + C_{x_2}^2 t_8\big\{t_8(f_1 + f_2) + 2 f_1 K_{02} - 2t_7 f_1 K_{12}\big\}\Big], \quad (2.13)$$

3. Efficiency Comparisons

3.1 Efficiency Comparisons of $\hat{\hat{\bar{Y}}}_{T1}^{(d)}$ in Case I

It is well known that under simple random sampling without replacement (SRSWOR) variance of unbiased estimator $\bar{y}$ is defined as

$$V(\bar{y}) = \bar{Y}^2 f_1 C_y^2. \quad (3.1.1)$$

It is observed from (1.6) and (3.3.1), that the suggested estimator $\hat{\hat{\bar{Y}}}_{T1}^{(d)}$ would be more efficient than the usual unbiased estimator $\bar{y}$ if

$$\frac{C_{x_2}^2}{C_{x_1}^2} < \frac{t_5(2K_{01}-t_5)}{t_6(t_6+2K_{02}-t_5K_{12})}. \quad (3.1.2)$$

It is noted from (1.45) and (2.6) that the suggested estimator $\hat{\bar{Y}}_{T1}^{(d)}$ is more efficient than usual double sampling ratio estimator $\hat{\bar{Y}}_{R}^{(d)}$ if

$$\frac{C_{x_2}^2}{C_{x_1}^2} < \frac{(1-t_5)(1+t_5-2K_{01})}{t_6(t_6+2K_{02}-t_5K_{12})}. \quad (3.1.3)$$

Comparison of (2.1.46) and (2.2.6) shows that the suggested estimator $\hat{\bar{Y}}_{T1}^{(d)}$ would be more efficient than usual double sampling product estimator $\hat{\bar{Y}}_{P}^{(d)}$ if

$$\frac{C_{x_2}^2}{C_{x_1}^2} < \frac{t_5(2K_{01}-t_5)}{(t_6-1)\{(1+t_6+2K_{02})-2t_5t_6K_{12}\}}. \quad (3.1.4)$$

Comparison of (1.66) and (2.6) shows that the suggested estimator $\hat{\bar{Y}}_{T1}^{(d)}$ would be more efficient than double sampling version of Sisodia and Diwedi (1981) ratio type estimator i.e. $\hat{\bar{Y}}_{SDR}^{(d)}$

$$\frac{C_{x_2}^2}{C_{x_1}^2} < \frac{\{t_1(t_1-2K_{01})-t_5(t_5-2K_{01})\}}{t_6(t_6+2K_{02}-2t_5K_{12})}. \quad (3.1.5)$$

Comparison of (1.48) and (2.6) shows that the suggested estimator $\hat{\bar{Y}}_{T1}^{(d)}$ would be more efficient than double sampling version of Sisodia and Diwedi (1981) product type estimator i.e. $\hat{\bar{Y}}_{SDP}^{(d)}$

$$\frac{C_{x_2}^2}{C_{x_1}^2} < -\frac{t_5(t_5-2K_{01})}{\{t_6(t_6+2K_{02}-2t_5K_{12})-t_2(t_2+2K_{02})\}}. \quad (3.1.6)$$

Comparing (1.49) and (2.6) reveals that the suggested estimator $\hat{\bar{Y}}_{T1}^{(d)}$ is more efficient than the double sampling version of Singh *et al*. (2004) [10] ratio type estimator $\hat{\bar{Y}}_{SER}^{(d)}$ if

$$\frac{C_{x_2}^2}{C_{x_1}^2} < \frac{\{t_3(t_3-2K_{01})-t_5(t_5-2K_{01})\}}{t_6(t_6+2K_{02}-2t_5K_{12})}. \quad (3.1.7)$$

From (1.50) and (2.6) it is observed that the suggested estimator $\hat{\bar{Y}}_{T1}^{(d)}$ is more efficient than the double sampling version of Singh *et al*. (2004) [10] product type estimator $\hat{\bar{Y}}_{SEP}^{(d)}$ if

$$\frac{C_{x_2}^2}{C_{x_1}^2} < -\frac{t_5(t_5-2K_{01})}{\{t_6(t_6+2K_{02}-2t_5K_{12})-t_4(t_4+2K_{02})\}}. \quad (3.1.8)$$

Comparison of (1.51) and (2.6) shows that the suggested estimator $\hat{\bar{Y}}_{T1}^{(d)}$ would be more efficient than double sampling version of Upadhyaya and Singh (1999) ratio estimator $\hat{\bar{Y}}_{US1R}^{(d)}$ if

$$t_6+2K_{02}-2t_5K_{12}<0. \quad (3.1.9)$$

Comparison of (1.52) and (2.6) shows that the suggested estimator $\hat{\bar{Y}}_{T1}^{(d)}$ would be more efficient than double sampling version Upadhyaya and Singh (1999) product estimator $\hat{\bar{Y}}_{US1P}^{(d)}$ if

$$\frac{C_{x_2}^2}{C_{x_1}^2} < -\frac{(t_5-2K_{01})}{2t_6K_{12}}. \quad (3.1.10)$$

Comparison of (1.55) and (3.2.6) that the suggested estimator $\hat{\bar{Y}}_{T1}^{(d)}$ is more efficient than the double sampling version of Singh (1967) [3] ratio-cum-product type estimator $\hat{\bar{Y}}_{RP}^{(d)}$ if

$$\frac{C_{x_2}^2}{C_{x_1}^2} < \frac{(1-t_5)(1+t_5-2K_{01})}{t_6(t_6+2K_{02}-t_5K_{12})-(1+2K_{02}-2K_{12})}. \quad (3.1.11)$$

Expressions (2.3.2) to (2.3.11) are conditions for case I under which the suggested estimator $\hat{\hat{\bar{Y}}}_{T1}^{(d)}$ has less mean squared error than usual unbiased estimator $\bar{y}$, $\hat{\hat{\bar{Y}}}_{R}^{(d)}$, $\hat{\hat{\bar{Y}}}_{P}^{(d)}$, $\hat{\hat{\bar{Y}}}_{SDR}^{(d)}$, $\hat{\hat{\bar{Y}}}_{SDP}^{(d)}$, $\hat{\hat{\bar{Y}}}_{SER}^{(d)}$, $\hat{\hat{\bar{Y}}}_{SEP}^{(d)}$, $\hat{\hat{\bar{Y}}}_{US1R}^{(d)}$, $\hat{\hat{\bar{Y}}}_{US1P}^{(d)}$, and $\hat{\hat{\bar{Y}}}_{RP}^{(d)}$.

3.2 Efficiency Comparisons of $\hat{\hat{\bar{Y}}}_{T1}^{(d)}$ in Case II

From (1.56), (1.57), (1.58), (1.59), (1.60), (1.61), (1.62) and (2.6) it is observed that the suggested estimator $\hat{\hat{\bar{Y}}}_{T}^{(d)}$ under case II ($(\hat{\hat{\bar{Y}}}_{T1}^{(d)})_{II}$), would be more efficient than

(i) Usual unbiased estimator $\bar{y}$ if

$$\frac{C_{x_2}^2}{C_{x_1}^2} < \frac{t_5\{2f_1K_{01} - t_5(f_1+f_2)\}}{t_6\{t_6(f_1+f_2) + 2f_1K_{02} - 2f_1t_5K_{12}\}}, \tag{3.2.1}$$

(ii) Double sampling ratio estimator $\hat{\hat{\bar{Y}}}_{R}^{(d)}$ if

$$\frac{C_{x_2}^2}{C_{x_1}^2} < \frac{(1-t_5)\{(1+t_5)(f_1+f_2) - 2f_1K_{01}\}}{t_6\{t_6(f_1+f_2) + 2f_1K_{02} - 2f_1t_5K_{12}\}}, \tag{3.2.2}$$

(iii) Double sampling product estimator $\hat{\hat{\bar{Y}}}_{P}^{(d)}$ if

$$\frac{C_{x_2}^2}{C_{x_1}^2} < \frac{t_5\{2f_1K_{01} - t_5(f_1+f_2)\}}{[(t_6-1)\{(f_1+f_2)(t_6+1) + 2f_1K_{02}\}] - 2f_1t_5t_6K_{12}}, \tag{3.2.3}$$

(iv) Sisodia and Dwivedi (1981) double sampling ratio type estimator $\hat{\hat{\bar{Y}}}_{SDR}^{(d)}$ if

$$\frac{C_{x_2}^2}{C_{x_1}^2} < \frac{(t_5-t_1)\{2f_1K_{01} - (f_1+f_2)(t_5+t_1)\}}{t_6\{(f_1+f_2) + 2f_1K_{02} - 2t_5f_1K_{12}\}}, \tag{3.2.4}$$

(v) Pandey and Dubey (1988) [4] product estimator in double sampling $\hat{\hat{\bar{Y}}}_{SDP}^{(d)}$ if

$$\frac{C_{x_2}^2}{C_{x_1}^2} < \frac{t_5\{-t_5(f_1+f_2) + 2f_1K_{01}\}}{[(t_6-t_2)\{(t_6+t_2)(f_1+f_2) + 2f_1K_{01} - 2t_5f_1K_{12}\}]}, \tag{3.2.5}$$

(vi) Singh *et al*. (2004) [10] ratio type estimator in double sampling $\hat{\hat{\bar{Y}}}_{SER}^{(d)}$ if

$$\frac{C_{x_2}^2}{C_{x_1}^2} < \frac{(t_5-t_3)\{2f_1K_{01} - (f_1+f_2)(t_5+t_3)\}}{t_6\{(f_1+f_2)t_6 + 2f_1K_{02} - 2t_5f_1K_{12}\}}, \tag{3.2.6}$$

(vii) Singh *et al* (2004) [10] product type estimator in double sampling $\hat{\hat{\bar{Y}}}_{SEP}^{(d)}$ if

$$\frac{C_{x_2}^2}{C_{x_1}^2} < \frac{t_5\{-t_5(f_1+f_2) + 2f_1K_{01}\}}{[(t_6-t_4)\{(t_6+t_4)(f_1+f_2) + 2f_1K_{01} - 2t_5f_1K_{12}\}]}, \tag{3.2.7}$$

(viii) Upadhyaya and Singh (1999) double sampling ratio type estimator $\hat{\hat{\bar{Y}}}_{US1R}^{(d)}$ if

$$t_6(f_1+f_2) + 2f_1K_{02} - 2t_5f_1K_{12} < 0, \tag{3.2.8}$$

(ix) Upadhyaya and Singh (1999) double sampling ratio type estimator $\hat{\hat{\bar{Y}}}_{US1R}^{(d)}$

$$\frac{C_{x_2}^2}{C_{x_1}^2} > \frac{t_5\{t_5(f_1+f_2) - 2f_1K_{01}\}}{2t_5t_6f_1K_{12}}, \tag{3.2.9}$$

(x) Singh (1967) [3] double sampling ratio-cum-product estimator $\hat{\bar{Y}}_{SRP}^{(d)}$

$$\frac{C_{x_2}^2}{C_{x_1}^2} < \frac{(t_5-1)\{2f_1K_{01}-(f_1+f_2)(t_5+1)\}}{(t_6-1)\{(f_1+f_2)(t_6+1)+2f_1K_{02}\}-(t_5-1)2f_1K_{12}}. \qquad (3.2.10)$$

Expression (3.2.1) and (3.2.10) are the conditions under which the suggested estimators $\hat{\bar{Y}}_{T1}^{(d)}$ would be more efficient than $\bar{y}$, $\hat{\bar{Y}}_{R}^{(d)}$, $\hat{\bar{Y}}_{P}^{(d)}$, $\hat{\bar{Y}}_{SDR}^{(d)}$, $\hat{\bar{Y}}_{SDP}^{(d)}$, $\hat{\bar{Y}}_{SER}^{(d)}$, $\hat{\bar{Y}}_{SEP}^{(d)}$, $\hat{\bar{Y}}_{US1R}^{(d)}$, $\hat{\bar{Y}}_{US1P}^{(d)}$, and $\hat{\bar{Y}}_{RP}^{(d)}$ in case II.

3.3 Efficiency Comparisons of $\hat{\bar{Y}}_{T2}^{(d)}$ in Case I

Comparison of (1.45), (1.46), (1.47), (1.48), (1.49), (1.50), (1.53), (1.54), (1.55), (2.12) and (3.3.1) shows that the suggested estimator $\hat{\bar{Y}}_{T2}^{(d)}$ would more efficient than

(i) $\bar{y}$ if

$$\frac{C_{x_2}^2}{C_{x_1}^2} < \frac{t_7(2K_{01}-t_7)}{t_8(t_8+2K_{02}-t_7K_{12})}, \qquad (3.3.1)$$

(ii) $\hat{\bar{Y}}_{R}^{(d)}$ if

$$\frac{C_{x_2}^2}{C_{x_1}^2} < \frac{(1-t_7)(1+t_7-2K_{01})}{t_8(t_8+2K_{02}-t_7K_{12})}, \qquad (3.3.2)$$

(iii) $\hat{\bar{Y}}_{P}^{(d)}$ if

$$\frac{C_{x_2}^2}{C_{x_1}^2} < \frac{t_7(2K_{01}-t_7)}{(t_8-1)\{(1+t_8+2K_{02})-2t_7t_8K_{12}\}}, \qquad (3.3.3)$$

(iv) $\hat{\bar{Y}}_{SDR}^{(d)}$ if

$$\frac{C_{x_2}^2}{C_{x_1}^2} < \frac{\{t_1(t_1-2K_{01})-t_7(t_7-2K_{01})\}}{t_8(t_8+2K_{02}-2t_7K_{12})}, \qquad (3.3.4)$$

(v) $\hat{\bar{Y}}_{SDP}^{(d)}$

$$\frac{C_{x_2}^2}{C_{x_1}^2} < -\frac{t_7(t_7-2K_{01})}{\{t_8(t_8+2K_{02}-2t_7K_{12})-t_2(t_2+2K_{02})\}}, \qquad (3.3.5)$$

(vi) $\hat{\bar{Y}}_{SER}^{(d)}$ if

$$\frac{C_{x_2}^2}{C_{x_1}^2} < \frac{\{t_3(t_3-2K_{01})-t_7(t_7-2K_{01})\}}{t_8(t_8+2K_{02}-2t_7K_{12})}, \qquad (3.3.6)$$

(vii) $\hat{\bar{Y}}_{SEP}^{(d)}$ if

$$\frac{C_{x_2}^2}{C_{x_1}^2} < -\frac{t_7(t_7-2K_{01})}{\{t_8(t_8+2K_{02}-2t_7K_{12})-t_4(t_4+2K_{02})\}}, \qquad (3.3.7)$$

(viii) $\hat{\bar{Y}}_{US2R}^{(d)}$ if

$$t_8+2K_{02}-2t_7K_{12}<0, \qquad (3.3.8)$$

(ix) $\hat{\bar{Y}}_{US2P}^{(d)}$ if

$$\frac{C_{x_2}^2}{C_{x_1}^2} < -\frac{(t_7 - 2K_{01})}{2t_8 K_{12}}, \tag{3.3.9}$$

(x) $\hat{\hat{\bar{Y}}}_{RP}^{(d)}$ if

$$\frac{C_{x_2}^2}{C_{x_1}^2} < \frac{(1-t_7)(1+t_7-2K_{01})}{t_8(t_8+2K_{02}-t_7K_{12})-(1+2K_{02}-2K_{12})}, \tag{3.3.10}$$

Expressions (3.3.1) and (3.3.10) are the conditions under which suggested estimator $\hat{\hat{\bar{Y}}}_{T2}^{(d)}$ would be more efficient than $\bar{y}$, $\hat{\hat{\bar{Y}}}_{R}^{(d)}$, $\hat{\hat{\bar{Y}}}_{P}^{(d)}$, $\hat{\hat{\bar{Y}}}_{SDR}^{(d)}$, $\hat{\hat{\bar{Y}}}_{SDP}^{(d)}$, $\hat{\hat{\bar{Y}}}_{SER}^{(d)}$, $\hat{\hat{\bar{Y}}}_{SEP}^{(d)}$, $\hat{\hat{\bar{Y}}}_{US2R}^{(d)}$, $\hat{\hat{\bar{Y}}}_{US2P}^{(d)}$, and $\hat{\hat{\bar{Y}}}_{RP}^{(d)}$ in case I

3.4 Efficiency Comparisons of $\hat{\hat{\bar{Y}}}_{T2}^{(d)}$ in Case

From (1.56), (1.57), (1.58), (1.59), (1.60), (1.61), (1.64), (1.65), (2.13) and (3.1.1), it is observed that the suggested estimator $\hat{\hat{\bar{Y}}}_{T2}^{(d)}$ under case II would be more efficient than

(i) usual unbiased estimator $\bar{y}$ if

$$\frac{C_{x_2}^2}{C_{x_1}^2} < \frac{t_7\{2f_1K_{01} - t_7(f_1+f_2)\}}{t_8\{t_8(f_1+f_2)+2f_1K_{02}-2f_1t_7K_{12}\}}, \tag{3.4.1}$$

(ii) double sampling ratio estimator $\hat{\hat{\bar{Y}}}_{R}^{(d)}$ if

$$\frac{C_{x_2}^2}{C_{x_1}^2} < \frac{(1-t_7)\{(1+t_7)(f_1+f_2)-2f_1K_{01}\}}{t_8\{t_8(f_1+f_2)+2f_1K_{02}-2f_1t_7K_{12}\}}, \tag{3.4.2}$$

(iii) double sampling product estimator $\hat{\hat{\bar{Y}}}_{P}^{(d)}$ if

$$\frac{C_{x_2}^2}{C_{x_1}^2} < \frac{t_7\{2f_1K_{01} - t_7(f_1+f_2)\}}{[(t_8-1)\{(f_1+f_2)(t_8+1)+2f_1K_{02}\}]-2f_1t_7t_8K_{12}}, \tag{3.4.3}$$

(iv)) Sisodia and Dwivedi (1981) ratio type estimator in double sampling $\hat{\hat{\bar{Y}}}_{SDR}^{(d)}$ if

$$\frac{C_{x_2}^2}{C_{x_1}^2} < \frac{(t_7-t_1)\{2f_1K_{01}-(f_1+f_2)(t_7+t_1)\}}{t_8\{(f_1+f_2)+2f_1K_{02}-2t_7f_1K_{12}\}}, \tag{3.4.4}$$

(v) Pandey and Dubey (1988) [4] product type estimator in double sampling $\hat{\hat{\bar{Y}}}_{SDP}^{(d)}$ if

$$\frac{C_{x_2}^2}{C_{x_1}^2} < \frac{t_7\{-t_7(f_1+f_2)+2f_1K_{01}\}}{[(t_8-t_2)\{(t_8+t_2)(f_1+f_2)+2f_1K_{01}-2t_7f_1K_{12}\}]}, \tag{3.4.5}$$

(vi) Singh *et al* (2004) [10] ratio type estimator in double sampling $\hat{\hat{\bar{Y}}}_{SER}^{(d)}$

$$\frac{C_{x_2}^2}{C_{x_1}^2} < \frac{(t_7-t_3)\{2f_1K_{01}-(f_1+f_2)(t_7+t_3)\}}{t_8\{(f_1+f_2)t_8+2f_1K_{02}-2t_7f_1K_{12}\}}, \tag{3.4.6}$$

(vii) Singh *et al* (2004) [10] product type estimator in double sampling $\hat{\hat{\bar{Y}}}_{SEP}^{(d)}$

$$\frac{C_{x_2}^2}{C_{x_1}^2} < \frac{t_7\{-t_7(f_1+f_2)+2f_1K_{01}\}}{[(t_8-t_4)\{(t_8+t_4)(f_1+f_2)+2f_1K_{01}-2t_7f_1K_{12}\}]}, \tag{3.4.7}$$

(viii) Upadhyaya and Singh (1999) double sampling ratio type estimator $\hat{\hat{\bar{Y}}}_{US1R}^{(d)}$

$$t_8(f_1+f_2)+2f_1K_{02}-2t_7f_1K_{12}<0, \quad (3.4.8)$$

(xi) Upadhyaya and Singh (1999) double sampling ratio type estimator $\hat{\bar{Y}}_{US1R}^{(d)}$

$$\frac{C_{x_2}^2}{C_{x_1}^2}>\frac{t_7\{t_7(f_1+f_2)-2f_1K_{01}\}}{2t_7t_8f_1K_{12}}, \quad (3.4.9)$$

(x) Singh (1967) [3] double sampling ratio-cum-product estimator $\hat{\bar{Y}}_{SRP}^{(d)}$

$$\frac{C_{x_2}^2}{C_{x_1}^2}<\frac{(t_7-1)\{2f_1K_{01}-(f_1+f_2)(t_7+1)\}}{(t_8-1)\{(f_1+f_2)(t_8+1)+2f_1K_{02}\}-(t_7-1)2f_1K_{12}} \quad (3.4.10)$$

Expressions (3.4.1) to (3.4.10) are the conditions under which the suggested estimator $\hat{\bar{Y}}_{T2}^{(d)}$ in case II would be more efficient than simple mean estimator $\bar{y}$, usual double sampling ratio and product estimators $\bar{X}_h=\frac{1}{N_h}\sum_{i=1}^{N_h}x_{hi}$ and $\hat{\bar{Y}}_P^{(d)}$, double sampling versions of estimators suggested by Sisodia and Dwivedi (1981), Pandey and Dubey (1988) [4], Singh *et al.* (2004) [10], Upadhyaya and Singh (1999) and Singh (1967) [3].

4. Empirical Study

To analyze the performance of the suggested estimator $\hat{\bar{Y}}_{T1}^{(d)}$ of population mean $\bar{Y}$ in double sampling in comparison to other considered estimators, a natural population data set is being considered. We have computed Percent relative efficiencies (PREs) of $\hat{\bar{Y}}_R^{(d)}$, $\hat{\bar{Y}}_P^{(d)}$, $\hat{\bar{Y}}_{SDR}^{(d)}$, $\hat{\bar{Y}}_{SDP}^{(d)}$, $\hat{\bar{Y}}_{SER}^{(d)}$, $\hat{\bar{Y}}_{SEP}^{(d)}$, $\hat{\bar{Y}}_{US1R}^{(d)}$, $\hat{\bar{Y}}_{US1P}^{(d)}$ $\hat{\bar{Y}}_{US2R}^{(d)}$, $\hat{\bar{Y}}_{US2P}^{(d)}$, $\hat{\bar{Y}}_{RP}^{(d)}$, h^{th} and $\hat{\bar{Y}}_{T2}^{(d)}$ with respect to $\bar{y}$. The description of the population is given below.

Population: [Source: Steel and Torrie (1960, p.282)]
y = Log of leaf burn in seconds,
x_1 = Potassium percentage,
h^{th} Chlorine percentage,
N=30, n=6, n'=14,
$\bar{Y}$ = 0.6860, C_y = 0.4803, $\beta_2(x_1)$ =1.56, ρ_{01} = 0.1794,
$\bar{X}_1$ = 4.6537, C_{x_1} = 0.2295, $\beta_2(x_2)$ =1.40, ρ_{02} = -0.4996,
$\bar{X}_2$ = 0.8077, C_{x_2} = 0.7493, and $\hat{\bar{Y}}_{RC}=\bar{y}_{st}\left(\frac{\bar{X}}{\bar{x}_{st}}\right)$ = 0.4074.

Table 4.1: Percent relative efficiencies of $\hat{\bar{Y}}_R^{(d)}$, $\hat{\bar{Y}}_P^{(d)}$, $\hat{\bar{Y}}_{SDR}^{(d)}$, $\hat{\bar{Y}}_{SDP}^{(d)}$, $\hat{\bar{Y}}_{SER}^{(d)}$, $\hat{\bar{Y}}_{SEP}^{(d)}$, $\hat{\bar{Y}}_{US1R}^{(d)}$, $\hat{\bar{Y}}_{US1P}^{(d)}$ $\hat{\bar{Y}}_{US2R}^{(d)}$, $\hat{\bar{Y}}_{US2P}^{(d)}$, $\hat{\bar{Y}}_{RP}^{(d)}$, z and $\hat{\bar{Y}}_{T2}^{(d)}$ (Under case-I) with respect to $\bar{y}$

Estimator	PRE
$\bar{y}$	100.00
$\hat{\bar{Y}}_R^{(d)}$	96.10
$\hat{\bar{Y}}_P^{(d)}$	61.54
$\hat{\bar{Y}}_{SDR}^{(d)}$	96.95
$\hat{\bar{Y}}_{SDP}^{(d)}$	112.33
$\hat{\bar{Y}}_{SER}^{(d)}$	100.02
$\hat{\bar{Y}}_{SEP}^{(d)}$	121.16
$\hat{\bar{Y}}_{US1R}^{(d)}$	102.34

$\hat{\bar{Y}}_{US1P}^{(d)}$	121.61
$\hat{\bar{Y}}_{US2R}^{(d)}$	96.66
$\hat{\bar{Y}}_{US2P}^{(d)}$	104.25
$\hat{\bar{Y}}_{RP}^{(d)}$	81.18
$\hat{\bar{Y}}_{T1}^{(d)}$	134.00
$\hat{\bar{Y}}_{T2}^{(d)}$	134.99

Table 4.2: Percent relative efficiencies of $\hat{\bar{Y}}_{R}^{(d)}$, $\hat{\bar{Y}}_{P}^{(d)}$, $\hat{\bar{Y}}_{SDR}^{(d)}$, $\hat{\bar{Y}}_{SDP}^{(d)}$, $\hat{\bar{Y}}_{SER}^{(d)}$, $\hat{\bar{Y}}_{SEP}^{(d)}$, $\hat{\bar{Y}}_{US1R}^{(d)}$, $\hat{\bar{Y}}_{US1P}^{(d)}$ $\hat{\bar{Y}}_{US2R}^{(d)}$, $\hat{\bar{Y}}_{US2P}^{(d)}$, $\hat{\bar{Y}}_{RP}^{(d)}$, $\hat{\bar{Y}}_{T1}^{(d)}$ and $\hat{\bar{Y}}_{T2}^{(d)}$ or (Under case-II) with respect to $\bar{y}$

Estimator	PRE
$\bar{y}$	100
$\hat{\bar{Y}}_{R}^{(d)}$	89.12
$\hat{\bar{Y}}_{P}^{(d)}$	38.91
$\hat{\bar{Y}}_{SDR}^{(d)}$	90.64
$\hat{\bar{Y}}_{SDP}^{(d)}$	96.76
$\hat{\bar{Y}}_{SER}^{(d)}$	96.50
$\hat{\bar{Y}}_{SEP}^{(d)}$	117.85
$\hat{\bar{Y}}_{US1R}^{(d)}$	102.17
$\hat{\bar{Y}}_{US1P}^{(d)}$	122.76
$\hat{\bar{Y}}_{US2R}^{(d)}$	90.12
$\hat{\bar{Y}}_{US2P}^{(d)}$	83.72
$\hat{\bar{Y}}_{RP}^{(d)}$	39.70
Z	139.11
$\hat{\bar{Y}}_{T2}^{(d)}$	105.27

5. Conclusion

Sections 3.1,3.2, 3.3, and 3.4 provide the conditions under which suggested estimators $\hat{\bar{Y}}_{T1}^{(d)}$ and $\hat{\bar{Y}}_{T2}^{(d)}$ would have less mean squared error as compared to $\hat{\bar{Y}}_{R}^{(d)}$, $\hat{\bar{Y}}_{P}^{(d)}$, $\hat{\bar{Y}}_{SDR}^{(d)}$, $\hat{\bar{Y}}_{SDP}^{(d)}$, $\hat{\bar{Y}}_{SER}^{(d)}$, $\hat{\bar{Y}}_{SEP}^{(d)}$, $\hat{\bar{Y}}_{US1R}^{(d)}$, $\hat{\bar{Y}}_{US1P}^{(d)}$ $\hat{\bar{Y}}_{US2R}^{(d)}$, $\hat{\bar{Y}}_{US2P}^{(d)}$, $\hat{\bar{Y}}_{RP}^{(d)}$. Tables 2.7.1 clearly shows that suggested estimator $\hat{\bar{Y}}_{T1}^{(d)}$ has highest percent relative efficiency as compared to other considered estimators. Thus larger gain in efficiency is observed by using suggested estimator $\hat{\bar{Y}}_{T1}^{(d)}$ over other estimators.

Table 4.2 exhibits that in case II i.e. when sample is taken as a sub sample from the first sample, $\hat{\bar{Y}}_{T1}^{(d)}$ has highest percent relative efficiency but $\hat{\bar{Y}}_{T2}^{(d)}$ has lower percent relative efficiency as compared to other estimators. It may be due to not satisfying conditions obtained in section 2.6. Hence there is need to explore another data set where conditions obtained in section 2.6 are satisfied. Finally, it can be concluded that suggested estimators can be used for the estimation of population mean when conditions obtained in sections 3.1, 3.2, 3.4 and 3.5 are satisfied.

6. References

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