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**Rashmi Gupta**
Dept. of Mathematics,
Vaish College of Engg,
Rohtak, Haryana, India

**Correspondence**
**Rashmi Gupta**
Dept. of Mathematics,
Vaish College of Engg,
Rohtak, Haryana, India

# Reliability analysis of a model with regard to undertaking the failed unit by ordinary or expert repairman with the concept of instruction and replacement time

**Rashmi Gupta**

**Abstract**
The present paper introduces the instruction time and the possibility that ordinary repairman may damage the unit to the extent that: (i) it rather goes to more degraded stage but repairable (ii) it may become irreparable and hence replaced. Two-unit cold standby system is examined with the assumptions that every failed unit first goes under the repair of ordinary repairman who starts repair after getting instructions from expert. The model has been analyzed by making use of Semi-Markov Processes and regenerative point technique. Various measures of system effectiveness including profit incurred have been evaluated. Various conclusions have been drawn through graphical study for a particular case.

**Keywords:** instruction and replacement Time, expert repairman, undertaking the failed unit

## Introduction

In order to increase the reliability, concept of redundancy is used by the users of various systems. As a result, two-unit standby systems have widely been studied in the field of reliability. Concept of two types of repairman has been considered in some of these studies including [3-7] wherein one of the repairman had been taken as an ordinary and the other as an expert. The ordinary repairman may not be able to do some complex repairs and then an expert comes. Long stay of the expert with the system may be costly and hence idea of instruction time was introduced by Kumar *et al* [8].

There may also be situations when the ordinary repairman even after getting the instruction may damage the failed unit during his try for repair. This leads to the unit in more degraded stage and sometimes to a stage where we are left with no other option but to replace it by a new one.

The purpose of the present study is.

1. to introduce redundancy
2. to introduce a new type of repair policy which is defined as : “when the ordinary repairman makes the unit damaged and leads it to more degraded stage due to mishandling, it is undertaken by the expert at much earlier stage than the stage at which its repair has been started by the ordinary repairman”
3. to make the replacement when the failed unit is made no more repairable by the ordinary repairman
4. To reduce the stay of the expert.

The present paper, therefore, investigates two-unit cold standby system introducing the aforesaid repair policy together with instruction and replacement. The additional assumptions taken for the model is that every failed unit first goes under the repair of ordinary repairman who starts repair after getting instructions from expert. Failure times are assumed to follow exponential distributions whereas other time distributions are arbitrary. Other assumptions are as usual.

The system has been analysed by making use of Semi-Markov Processes and regenerative point technique. Various measures of system effectiveness including profit incurred have been evaluated. Various conclusions have been drawn through graphical study for a particular case.

**Notations**

$\lambda$ : constant failure rate of a unit

$p_1$ : probability that the ordinary repairman is able to complete the repair

$q_1$ : probability that the ordinary repairman is unable to complete the repair

a : probability that resume repair policy is adopted

$b_2$ : probability that unit is damaged but repairable

$b_3$ : probability that the unit is damaged but irreparable

g(t), G(t) : p.d.f. and c.d.f. of the repair time of the ordinary repairman

$g_1(t)$ $G_1(t)$ : p.d.f. and c.d.f. of repair time of the expert repairman when resume repair policy is adopted

$g_2(t)$, $G_2(t)$ : p.d.f. and c.d.f. of repair time of the expert repairman when repeat repair policy (type-I) is adopted

$g_3(t)$, $G_3(t)$ : p.d.f. and c.d.f. of repair time of the expert repairman when repeat repair policy (type-II) is adopted

$g_4(t)$, $G_4(t)$ : p.d.f. and c.d.f. of replacement time

i(t), I(t) : p.d.f. and c.d.f. of time when expert gives instruction to ordinary repairman

**Symbols for the state of system are**

o: operative unit

cs: cold standby unit

$F_r$: failed unit under repair of ordinary repairman

$F_R$: repair of the failed unit by the ordinary repairman is continuing from previous state

$F_{re_1}$ : Failed unit under repair of the expert repairman when resume repair policy is adopted

$F_{Re_1}$ : Repair of the failed unit by the expert repairman is continuing from the previous state under resume repair policy

$F_{re_2}$ : Failed unit under repair of the expert repairman when repeat repair policy (type-I) is adopted

$F_{Re_2}$ : Repair of the failed unit by the expert repairman is continuing from the previous state under repeat repair policy (type-I)

$F_{re_3}$ : Failed unit under repair of the expert repairman when repeat repair policy (type-II) is adopted

$F_{Re_3}$ : Repair of the failed unit by the expert repairman is continuing from the previous state under repeat repair policy (type-II)

$F_{rep}$ : failed unit under replacement

$F_{Rep}$ : replacement of the failed unit continuing from the previous state

$F_{ei}$ : failed unit under instructions given by expert repairman to the ordinary repairman

**Transition Probabilities and Mean Sojourn Times**

The transition diagram showing the various states of the system is shown as in Fig. 1.1 the epochs of entry into states 0, 1, 2, 3, 4, 5, 10, 11, 12 are regeneration points and thus these states are called regenerative states. States 6, 7, 8, 9, 10, 11 and 12 are failed states.

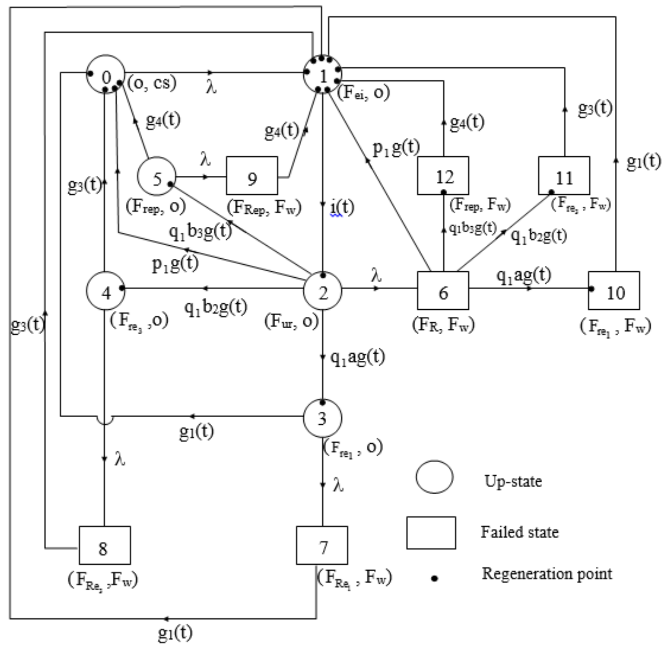


**Fig 1.1**

The non–zero elements $p_{ij}$ are:

$p_{01} = p_{12} = 1$, $p_{20} = p_1\, g^*(\lambda)$, $p_{23} = q_1\, a\, g^*(\lambda)$

$p_{24} = q_1\, b_2\, g^*(\lambda)$, $p_{25} = q_1\, b_3\, g^*(\lambda)$, $p_{26} = 1 - g^*(\lambda)$

$p_{21}^{(6)} = p_1\,(1-g^*(\lambda))$, $p_{2,10}^{(6)} = q_1\, a\,(1 - g^*(\lambda))$

$p_{2,11}^{(6)} = q_1\, b_2\,(1 - g^*(\lambda))$, $p_{2,12}^{(6)} = q_1\, b_3\,(1 - g^*(\lambda))$

$p_{30} = g_1^*(\lambda)$, $p_{37} = p_{31}^{(7)} = 1 - g_1^*(\lambda)$

$p_{40} = g_3^*(\lambda)$, $p_{48} = p_{41}^{(8)} = 1 - g_3^*(\lambda)$

$p_{50} = g_4^*(\lambda)$, $p_{59} = p_{51}^{(9)} = 1 - g_4^*(\lambda)$

By these transition probabilities it can be verified that

$p_{01} = p_{12} = 1$

$p_{20} + p_{23} + p_{24} + p_{25} + p_{26} = 1$

$p_{20} + p_{23} + p_{24} + p_{25} + p_{21}^{(6)} + p_{2,10}^{(6)} + p_{2,11}^{(6)} + p_{2,12}^{(6)} = 1$

$p_{30} + p_{37} = 1$, $p_{30} + p_{31}^{(7)} = 1$

$p_{40} + p_{48} = 1$, $p_{40} + p_{41}^{(8)} = 1$

$p_{50} + p_{59} = 1$, $p_{50} + p_{51}^{(9)} = 1$

The mean sojourn time ($\mu_i$) in state i are

$\mu_0 = \dfrac{1}{\lambda}$, $\mu_1 = -i^{*\prime}(0)$, $\mu_2 = \dfrac{1-g^*(\lambda)}{\lambda}$, $\mu_3 = \dfrac{1-g_1^*(\lambda)}{\lambda}$,

$\mu_4 = \dfrac{1-g_3^*(\lambda)}{\lambda}$, $\mu_5 = \dfrac{1-g_4^*(\lambda)}{\lambda}$, $\mu_{10} = -g_1^{*\prime}(0)$, $\mu_{11} = -g_3^{*\prime}(0)$, $\mu_{12} = -g_4^{*\prime}(0)$

The unconditional mean time taken by the system to transit for any state j when it is counted from epoch of entrance into state i is mathematically stated as

$$m_{ij} = \int_0^\infty t\, q_{ij}(t)\, dt = -\, q_{ij}^{*\prime}(0)$$

Thus,

$m_{01} = \mu_0$; $m_{12} = \mu_1$

$m_{20} + m_{23} + m_{24} + m_{25} + m_{26} = \mu_2$

$m_{20} + m_{23} + m_{24} + m_{25} + m_{21}^{(6)} + m_{2,10}^{(6)} + m_{2,11}^{(6)} + m_{2,12}^{(6)} = k_1$ (say)

$m_{30} + m_{37} = \mu_3$ ; $m_{30} + m_{31}^{(7)} = \mu_{10}$

$m_{40} + m_{48} = \mu_4$ ; $m_{40} + m_{41}^{(8)} = \mu_{11}$

$m_{50} + m_{59} = \mu_5$ ; $m_{50} + m_{51}^{(9)} = \mu_{12}$

**Mean Time to System Failure (MTSF) =** $T_0 = N/D$
**Availability Analysis =** $A_0 = N_1/D_1$
**Busy Period Analysis of the Ordinary Repairman =** $B_0 = N_2/D_1$
**Busy Period Analysis of the Expert Repairman (Repair Time Only)**
**Expected Instruction Time =** $IT_0 = N_4/D_1$
**Expected Number of Visits by the Ordinary Repairman =** $V_0 = N_5/D_1$
**Expected Number of Visits by the Expert Repairman =** $V_0^e = N_6/D_1$
**Busy Period Analysis of Repairman (Replacement Time Only) =** $B_0^R = N_7/D_1$
**Expected Number of Replacements =** $RP_0 = N_8/D_1$

where
$N = \mu_0 + \mu_1 + \mu_2 + \mu_3\, p_{23} + \mu_4 p_{24} + \mu_5 p_{25}$
$D = 1 - p_{20} - p_{23}\, p_{30} - p_{24}\, p_{40} - p_{25}\, p_{50}$
$N_1 = \mu_0[p_{20} + p_{23}\, p_{30} + p_{24}\, p_{40} + p_{25}\, p_{50}] + \mu_1 + \mu_2 + \mu_3\, p_{23} + \mu_4\, p_{24} + \mu_5\, p_{25}$
$D_1 = \mu_0\, [\, p_{20} + p_{23}\, p_{30} + p_{24}\, p_{40} + p_{25}\, p_{50}] + \mu_1 + k_1 + \mu_{10}\, [1 + p_{23}] + \mu_{11}[1 + p_{24}] + \mu_{12}[1 + p_{25}]$ $N_2 = k_1$
$B_0^e = N_3/D_1$
$N_3 = \mu_{10}(p_{23} + p_{2,10}^{(6)}) + \mu_{11}(p_{24} + p_{2,11}^{(6)})$
$N_4 = \mu_1$
$N_5 = 1$
$N_6 = 1 + [p_{23}\, p_{30} + p_{24} p_{40} + p_{25} p_{50} - p_{25} - p_{2,12}^{(6)}]$
$N_7 = \mu_{12}(p_{25} + p_{2,12}^{(6)})$
$N_8 = p_{25} + p_{2,12}^{(6)}$

**Profit Analysis**
The expected total profit incurred to the system in steady–state is given by
$P_{62} = C_0A_0 - C_1B_0 - C_2\, B_0^e - C_4V_0 - C_5V_0^e - C_6\, B_0^R - C_7RP_0 - C_8IT_0$

Where
$C_0$ = revenue per unit up time of the system
$C_1$ = cost per unit time for which the ordinary repairman is busy for repairing the failed unit
$C_2$ = cost per unit time for which the expert repairman is busy for repairing the unit
$C_4$ = cost per visit of the ordinary repairman
$C_5$ = cost per visit of the expert repairman.
$C_6$ = cost per unit time for which the repairman is busy for replacing the unit
$C_7$ = cost per replacement
$C_8$ = cost per unit time for which the expert repairman is busy in giving the instructions to the ordinary repairman

**Particular Case**
For graphical interpretation, the following particular case is considered
$g(t) = \alpha e^{-\alpha t}$ ; $g_1(t) = \alpha_1\, e^{-\alpha_1 t}$
$g_3(t) = \alpha_3\, e^{-\alpha_3 t}$ ; $g_4(t) = \alpha_4\, e^{-\alpha_4 t}$
$i(t) = \gamma\, e^{-\gamma t}$

Therefore, we have

$p_{01} = p_{12} = 1$, $p_{20} = \frac{p_1\alpha}{\alpha + \lambda}$, $p_{23} = \frac{aq_1\alpha}{\alpha + \lambda}$, $p_{24} = \frac{b_2q_1\alpha}{\alpha + \lambda}$,

$p_{25} = \frac{b_3q_1\alpha}{\alpha + \lambda}$, $p_{26} = \frac{\lambda}{\alpha + \lambda}$

$p_{21}^{(6)} = \frac{p_1\lambda}{\alpha + \lambda}$, $p_{2,10}^{(6)} = \frac{aq_1\lambda}{\alpha + \lambda}$, $p_{2,11}^{(6)} = \frac{b_2q_1\lambda}{\alpha + \lambda}$,

$p_{2,12}^{(6)} = \frac{b_3q_1\lambda}{\alpha + \lambda}$, $p_{30} = \frac{\alpha_1}{\alpha_1 + \lambda}$,

$p_{37} = \frac{\lambda}{\alpha_1 + \lambda}$, $p_{31}^{(7)} = \frac{\lambda}{\alpha_1 + \lambda}$, $p_{40} = \frac{\alpha_3}{\alpha_3 + \lambda}$, $p_{48} = \frac{\lambda}{\alpha_3 + \lambda}$

, $p_{41}^{(8)} = \frac{\lambda}{\alpha_3 + \lambda}$,

$p_{50} = \frac{\alpha_4}{\alpha_4 + \lambda}$, $p_{59} = \frac{\lambda}{\alpha_4 + \lambda}$, $p_{51}^{(9)} = \frac{\lambda}{\alpha_4 + \lambda}$,

$\mu_0 = \frac{1}{\lambda}$, $\mu_1 = \frac{1}{\gamma}$, $\mu_2 = \frac{1}{\alpha + \lambda}$ $\mu_3 = \frac{1}{\alpha_1 + \lambda}$, $\mu_4 = \frac{1}{\alpha_3 + \lambda}$, $\mu_5 = \frac{1}{\alpha_4 + \lambda}$, $\mu_{10} = \frac{1}{\alpha_1}$, $\mu_{11} = \frac{1}{\alpha_3}$, $\mu_{12} = \frac{1}{\alpha_4}$

$k_1 = \frac{1}{\alpha}$

On the basis of the numerical values taken as
$p_1 = 0.5$, $q_1 = 0.5$, $a = 0.5$, $b_2 = 0.45$, $b_3 = 0.05$, $\gamma = 10$,
$\alpha = 1$, $\alpha_1 = 2.5$, $\alpha_3 = 1$, $\alpha_4 = 5$, $\lambda = 0.05$,
The values of various measures of system effectiveness are obtained as

**Mean time to system failure (MTSF) =** 340.4345
**Availability ($A_0$) =** 0.9105324
**Busy period of the ordinary repairman ($B_0$) =** 0.0459564
**Busy period of the expert repairman (repair time only) ($B_0^e$) =** 0.01493584
**Expected instruction time ($IT_0$)** = 0.0045856
**Expected number of visits by the ordinary repairman ($V_0$)** = 0.0459564
**Expected number of visits by the expert repairman ($V_0^e$)** = 0.0659972
**Busy period of the expert repairman (replacement time only)($B_0^R$)** =0.00022978
**Expected number of replacements ($RP_0$) =** 0.001148911

**Graphical Interpretation**
The above particular case is considered for the graphical interpretation.

Figs. 1.2 and 1.3 show the behaviour of MTSF and Availability with respect to failure rate (λ). It is observed that both MTSF and Availability get decreased with increase in the values of failure rate (λ) and have higher values for higher values of repair rate (α).

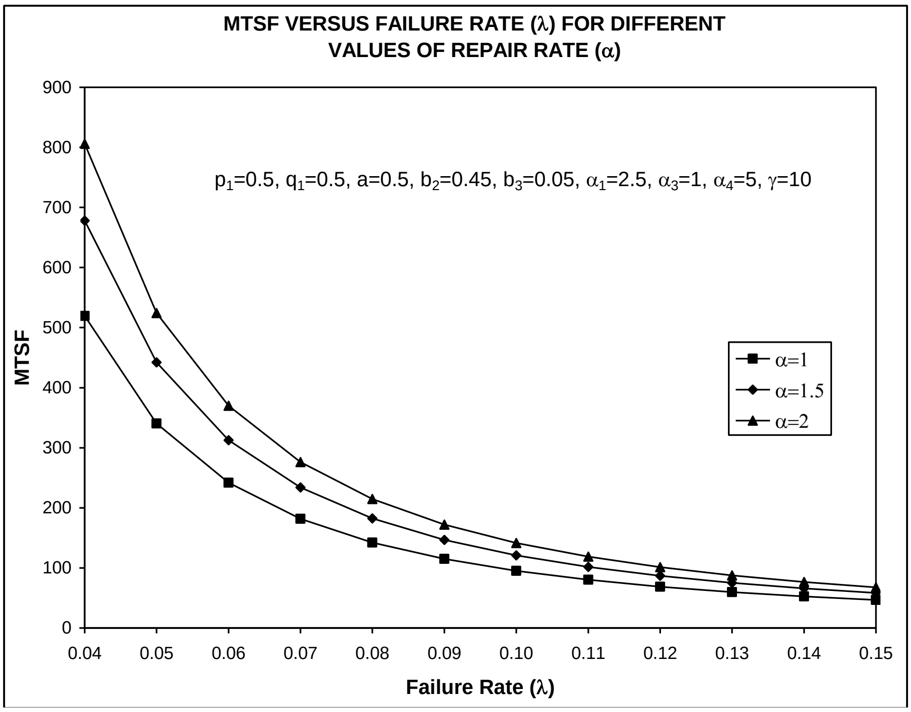


**Fig. 1.2**

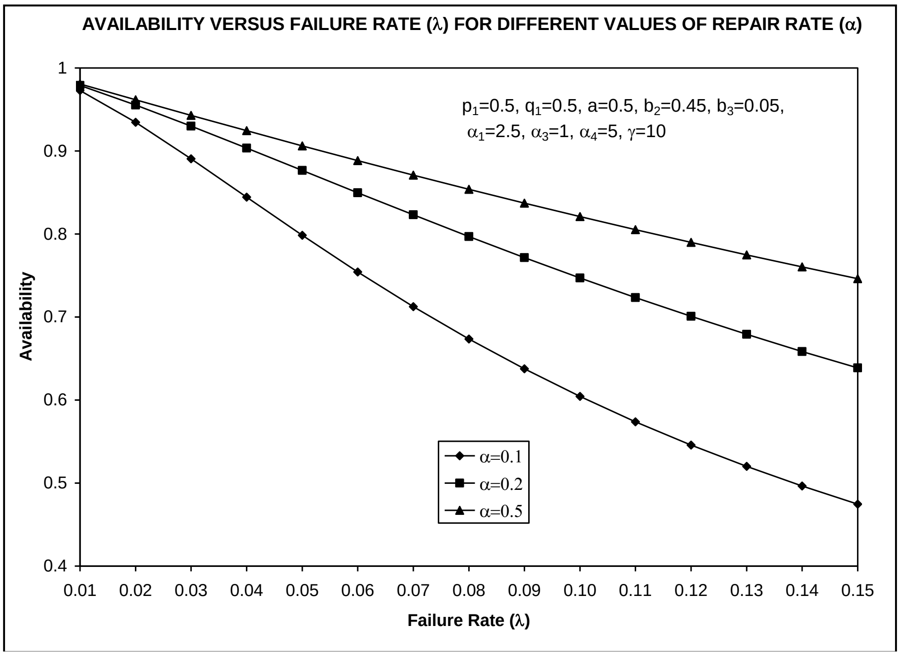


**Fig 1.3**

Fig. 1.4 reveals the pattern of the profit with respect to failure rate (λ) for different values of repair rate (α). The profit decreases as the failure rate (λ) increases and is higher for higher values of α. Following is also observed from the graph:

(i) For α = 1, the profit is positive or zero or negative according as λ < or = or > 0.0964.

(ii) For α = 1.5, the profit is positive or zero or negative according as λ < or = or > 0.106.

(iii) For α = 2, the profit is positive or zero or negative according as λ < or = or > 0.112.

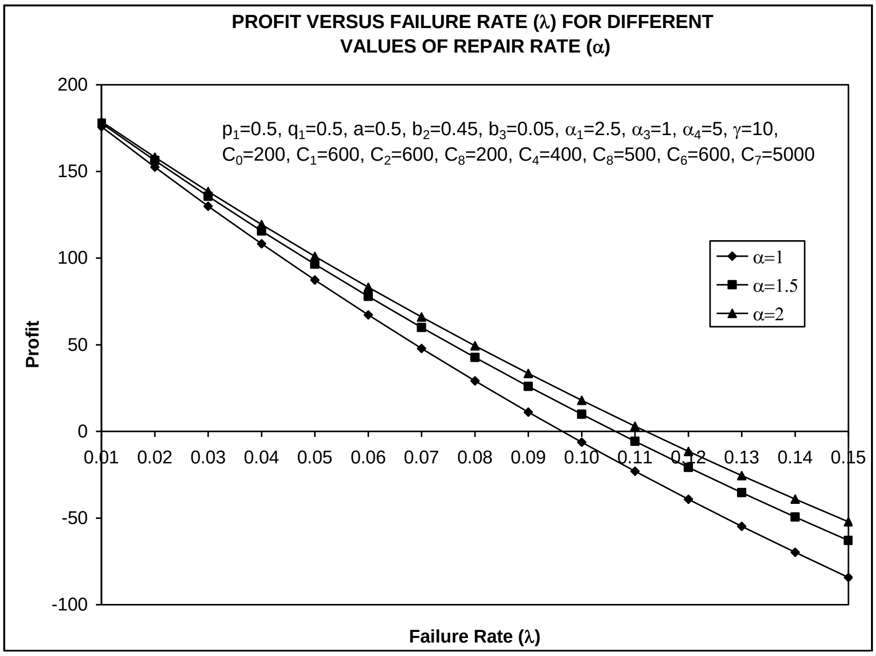


**Fig. 1.4**

Fig. 1.5 reveals the pattern of the profit with respect to revenue ($C_0$) for different values of cost ($C_2$). The profit increases with the increase in the values of revenue ($C_0$) and is lower for higher values of cost ($C_2$). Following is also observed:

(i) For $C_2 = 4000$, the profit is positive or zero or negative according as $C_0$ > or = or < 159.80.
(ii) For $C_2 = 6000$, the profit is positive or zero or negative according as $C_0$ > or = or < 192.6.
(iii) For $C_2 = 8000$, the profit is positive or zero or negative according as $C_0$ > or = or > 225.4.

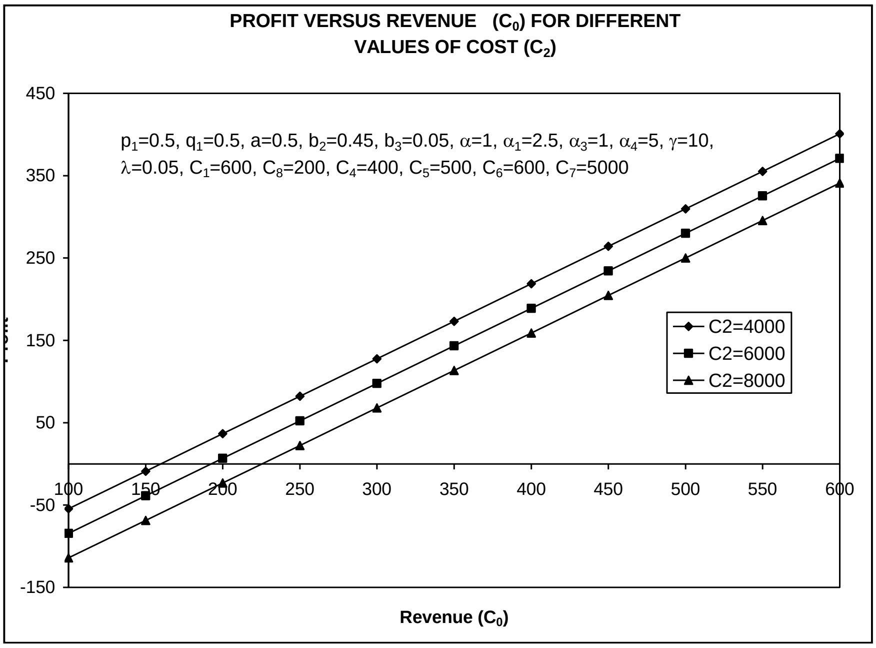


**Fig. 1.5**

Fig. 1.6 depicts the behaviour of the profit with respect to cost ($C_5$) for different values of instruction rate ($\gamma$). The profit decreases with the increase in the values of cost ($C_5$) and becomes higher for higher values of instruction rate ($\gamma$). Following inferences can also be made through the graph:

(i) For $\gamma = 1$, the profit is positive or zero or negative according as $C_5$ < or = or > 742.4.
(ii) For $\gamma = 2$, the profit is positive or zero or negative according as $C_5$ < or = or > 768.5.
(iii) For $\gamma = 10$, the profit is positive or zero or negative according as $C_5$ < or = or > 789.4.

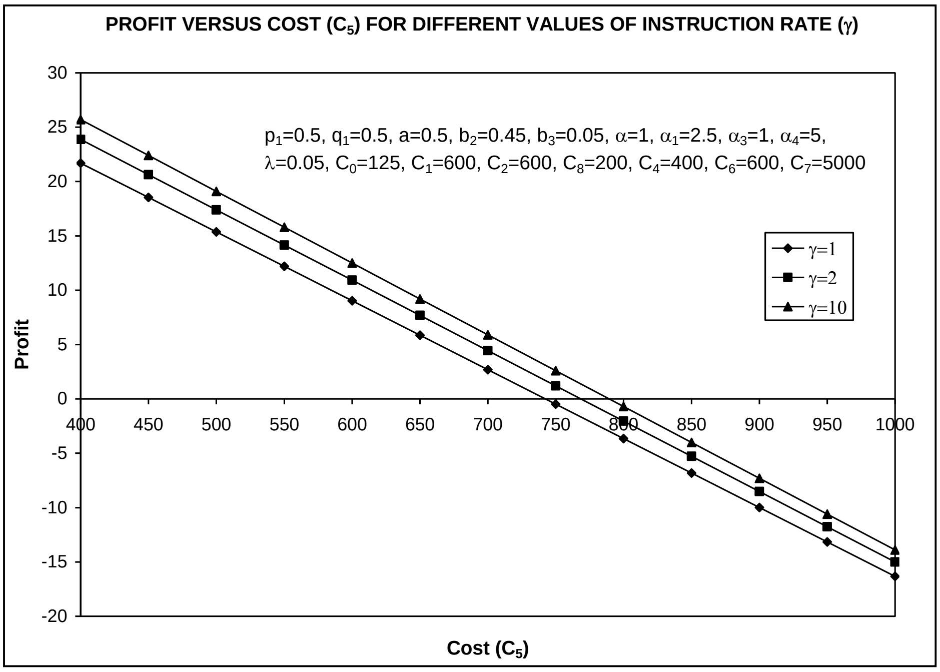


**Fig 1.6**

Fig. 1.7 shows that behaviour of the profit with respect to replacement cost ($C_7$) for different values of repair rate ($\alpha_4$). The profit decreases as the cost ($C_7$) increases and becomes higher for higher values of repair rate ($\alpha_4$). Following is also observed through the graph:

(i) For $\alpha_4 = 1$, the profit is positive or zero or negative according as $C_7$ < or = or > 9198.2.

(ii) For $\alpha_4 = 2$, the profit is positive or zero or negative according as $C_7$ < or = or > 9533.3.

(iii) For $\alpha_4 = 10$, the profit is positive or zero or negative according as $C_7$ < or = or > 9802.7.

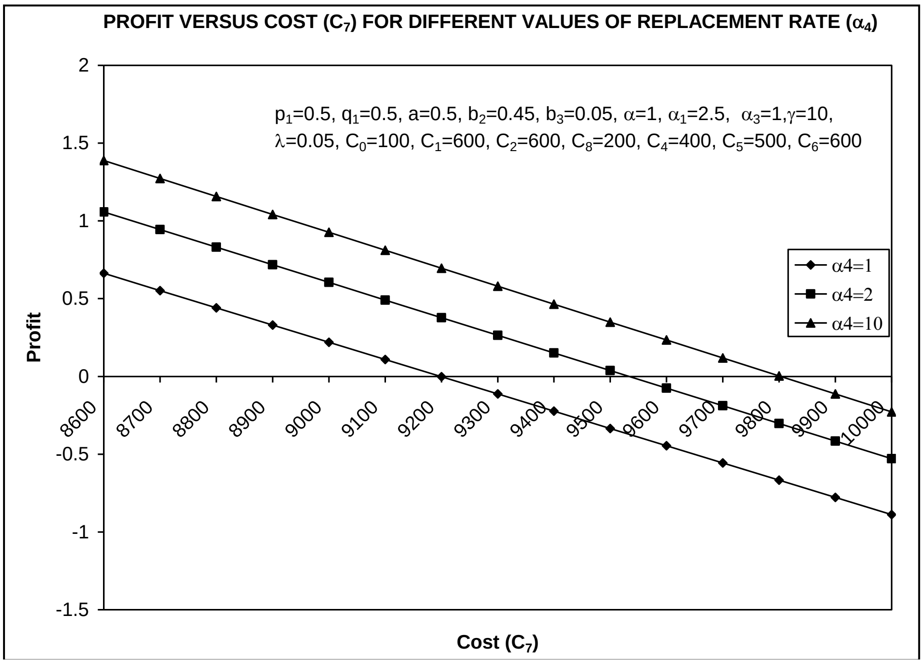


**Fig. 1.7**

Fig. 1.8 depicts the behaviour of the profit with respect to probability ($p_1$) for different values of probability (a). The profit increases with increase in the values of probability ($p_1$) and becomes higher for higher values of probability (a). Following inferences can be made.

For a = 0.2, 0.4 and 0.6, the system is profitable only if $p_1$ > 0.454, 0.413, 0.374 respectively.

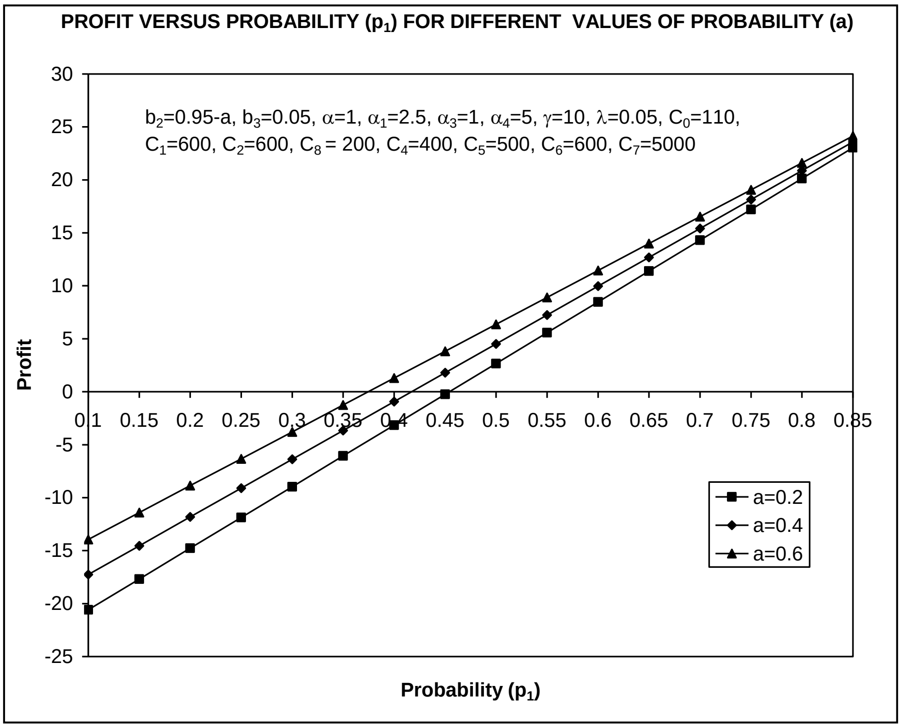


**Fig 1.8**


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