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Analysis of a two-unit cold standby system with instructions for repair given only for one unit per visit of the expert repairman

Rashmi Gupta and Suresh Kumar Gupta

Abstract
Profit analysis of a two-unit cold standby system with an expert and assistant is investigated. Whenever a unit fails, both the repairmen arrive but the expert repairman may not in a position to devote more time to the system due to his busy schedule in other systems. The expert gives instructions for repair of only one failed unit at a time and then goes back even if second unit fails during his stay. He comes to the system again whenever assistant requires instructions for repair for the other failed unit. Various measures of system effectiveness have been obtained by making use of semi-Markov processes and regenerative point technique. Graphs have been plotted for a special case.

Keywords: Cold standby system, expert repairman

Introduction

Recently, Kumar *et al.* [1] have studied a two-unit cold standby system with instruction time. They have carried out the analysis under the supposition that whenever a unit fails, the expert repairman comes to the system with an assistant and starts repairing the unit himself. During the time of repairing of this unit, if the second unit also fails, the expert leaves the repair of former unit and starts giving instruction to his assistant for second unit. And after completion of instructions, the expert resumes the repair of former unit, while the assistant starts repairing the second unit, after receiving instructions from expert. However, there may be situations when expert may not devote much time to the system due to his busy schedule in other systems and hence goes back after giving instructions to his assistant for repair of each of the units i.e. the whole system is repaired by the assistant repairman.

Hence, in the present paper, we investigate the stochastic analysis of two-unit cold standby system wherein the whole system is repaired by assistant only after getting the instructions from the expert who arrives at the system immediately when required. The system is studied by considering a model in which the expert gives instructions for repair of only one failed unit at a time and then goes back even if second unit fails during his stay. He comes to the system again whenever assistant requires instructions for repair of other failed unit.

The techniques of regenerative processes and semi-Markov processes are used to determine expressions for the various reliability characteristics of the system effectiveness as mean time to system failure (MTSF), steady-state availability of the system, the total fraction of busy time of expert and his assistant repairman per unit time and expected number of visits by the expert repairman. Profit is calculated using the above measures. Graphs pertaining to a particular case are also plotted.

1. The system consists of two identical units. Initially one is operative and the other is kept as cold standby. The standby unit cannot fail.
2. The system becomes inoperable on the failure of both the units.
3. When the expert repairman is called on to do the job it takes negligible time to reach the system.
4. The assistant repairman repairs the failed unit perfectly on the instructions given to him by the expert.
5. The payment to expert is made for his visits as well as for the time for which he remains busy in giving instructions.

6. No payment is made for the visits of assistant repairman while the payment is made for the time for which the assistant remains busy in repairing.
7. Each unit has an exponential distribution of time to failure while the distribution of repair time for the assistant repairman and instruction time are taken as general.
8. All the random variables are mutually independent.

Notation and States of the System

λ Constant failure rate of an operative unit
a,e Assistant repairman, expert repairman respectively.
F(t), f(t) cdf and pdf of time to repair by assistant repairman.
I(t), i(t) cdf and pdf of time when expert gives instructions to his assistant.
The other symbols, not given here may be seen in Ref. [2].
Symbols for the states of the system
o/cs operative/cold standby
F_{wri} unit in F-mode and waiting for repair while expert gives instructions to his assistant
F_{Wri} unit in F-mode and waiting for repair continued from earlier state while expert gives instructions to his assistant.
Fwi Failed and waiting for instructions.
F_{ra} unit in F-mode and under repair of assistant repairman.
F_{Ra} unit in F-mode and under repair of assistant repairman continued from earlier state.

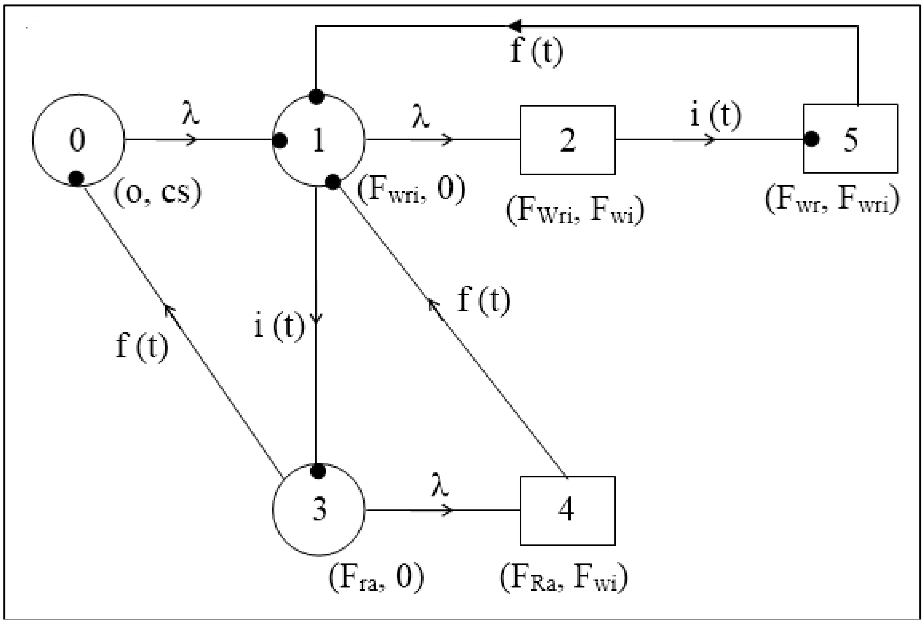
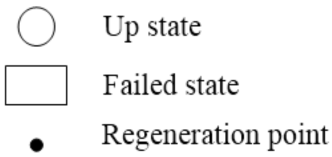


Fig 1: State Transition Diagram



Transition Probabilities and Sojourn Times

Thus considering the above symbols, possible states of the system and the transitions into the states are shown in Fig. 1. The epoch of entrance into the states 0, 1, 3, 5 are regenerative points and thus 0, 1, 3, 5 are regenerative states. States 2, 4, 5 are down states.

The non-zero elements of transition probability matrix (t.p.m.) are:

$p_{01} = p_{51} = 1$, $\quad p_{13} = i^*(\lambda)$, $\quad p_{15}^{(2)} (= p_{12}) = 1 - i^*(\lambda)$

$p_{30} = f^*(\lambda)$, $\quad p_{31}^{(4)} (= p_{34}) = 1 - f^*(\lambda)$

with,

$$p_{13} + p_{15}^{(2)} = p_{30} + p_{31}^{(4)} = 1$$

For the mean sojourn time in state S_i, its values for various states are obtained by using the formula

$$\mu_i = \int P(T_i > t)\, dt$$

$$\mu_0 = \frac{1}{\lambda}, \qquad \mu_1 = \frac{1-i^*(\lambda)}{\lambda}, \quad \mu_3 = \frac{1-f^*(\lambda)}{\lambda}$$

$\mu_5 = \int \bar{F}(t)\,dt = \int tdF(t) = \varepsilon_1$ (Mean Repair time of Assistant)

The unconditional mean time taken by the system to transit for any regenerative state i, when it (time) is counted from the epoch of entrance into that state is, mathematically, stated as

$m_{01} = \mu_0$,

$m_{13} + m_{15}^{(2)} = \int tdI(t) = \varepsilon_1$ = Mean instruction time

$m_{30} + m_{31}^{(4)} = \int tdF(t) = \varepsilon_2 = \text{Mean repair time of assistant}$

$m_{51} = \int tdF(t)$

Mean Time to System Failure

To determine it, for our system, we regard the failed states 2, 4 and 5 as absorbing. By probabilistic arguments we have

$\phi_0(t) = Q_{01}(t)$ Ⓢ $\phi_1(t)$

$\phi_1(t) = Q_{13}(t)$ Ⓢ $\phi_3(t) + Q_{12}(t)$

$\phi_3(t) = Q_{30}(t)$ Ⓢ $\phi_0(t) + Q_{34}(t)$ …(1-3)

By taking Laplace-Stieltjes transforms (LST) of equations (1-3) and solving them for $\phi_0^{**}(s)$, we have

$$\phi_0^{**}(s) = \frac{N_1(s)}{D_1(s)} \quad \text{…(4)}$$

where

$$N_1(s) = Q_{01}^{**}(s)[Q_{12}^{**}(s) + Q_{13}^{**}(s).Q_{34}^{**}(s)]$$

and

$$D_1(s) = [1 - Q_{01}^{**}(s)Q_{13}^{**}(s)Q_{30}^{**}(s)]$$

Now the MTSF, given that the system started at the beginning of state 0, is

$$T_0 = \lim_{s\to 0}\frac{1-\phi_0^{**}(s)}{s} \quad \text{…(5)}$$

Using L'Hospital's rule and putting in the value of $\phi_0^{**}(s)$ from equation (4), we get

$$T_0 = \frac{N_1}{D_1}$$

where

$N_1 = \mu_0 + \mu_1 + \mu_3\, p_{13}$

$D_1 = 1 - p_{13}\, p_{30}$

Availability Analysis

$A_0(t) = M_0(t) + q_{01}(t)$ © $A_1(t)$

$A_1(t) = M_1(t) + q_{13}(t)$ © $A_3(t) + q_{15}^{(2)}(t)$ © $A_5(t)$

$A_3(t) = M_3(t) + q_{30}(t)$ © $A_0(t) + q_{31}^{(4)}(t)$ © $A_1(t)$

$A_5(t) = q_{51}(t)$ © $A_1(t)$ ...(6-9)

where

$M_0(t) = e^{-\lambda t}$, $\quad M_1(t) = e^{-\lambda t}\bar{I}(t)$, $M_3(t) = e^{-\lambda t}\bar{F}(t)$

Taking Laplace transforms (LT) of (6-9) and solving for $A_0^*(s)$, we have (dropping s for brevity)

$$A_0^*(s) = \frac{N_2(s)}{D_2(s)} \qquad \dots (10)$$

where

$$N_2(s) = M_0^*(s)[1 - q_{13}^*(s)q_{31}^{(4)*}(s) - q_{15}^{(2)*}(s)q_{51}^*(s)][M_1^*(s) + M_3^*(s)q_{13}^*(s)]$$

and

$$D_2(s) = 1 - q_{13}^*(s)q_{31}^{(4)*}(s) - q_{15}^{(2)*}(s)q_{51}^*(s) - q_{01}^*(s)q_{13}^*(s)q_{30}^*(s)$$

The steady-state availability of the system is given by

$$A_0 = \lim_{t\to\infty} A_0(t) = \lim_{s\to 0}[s\,A_0^*(s)] = \frac{N_2}{D_2} \qquad \dots (11)$$

where

$N_2 = \mu_1 + p_{13}(\mu_1 p_{30} + \mu_3)$
and
$D_2 = \mu_0 p_{13} p_{30} + (\varepsilon_2 + \varepsilon_1)$

Busy Period Analysis of Assistant Repairman

$$B_0^a(t) = q_{01}(t) © B_1^a(t)$$
$$B_1^a(t) = q_{13}(t) © B_3^a(t) + q_{15}^{(2)}(t) © B_5^a(t)$$
$$B_3^a(t) = W_3(t) + q_{30}(t) © B_0^a(t) + q_{31}^{(4)}(t) © B_1^a(t)$$
$$B_5^a(t) = W_5(t) + q_{51}(t) © B_1^a(t) \qquad \dots (12\text{-}15)$$

where

$$W_3(t) = e^{-\lambda t}\bar{F}(t) + [\lambda e^{-\lambda t} © 1]\bar{F}(t) = \bar{F}(t)$$
$$W_5(t) = \bar{F}(t)$$

Taking LT of (12-15) and solving them for $B_0^{a*}(s)$, we have (dropping s for brevity)

$$B_0^{a*}(s) = \frac{N_3(s)}{D_2(s)} \qquad \dots (16)$$

where

$$N_3(s) = q_{01}^*(s)[W_3^*(s)q_{13}^*(s) + W_5^*(s)q_{15}^{(2)*}(s)]$$

and $D_2(s)$ is already specified.
In steady state, the total fraction of time for which the assistant repairman is busy, is given by

$$B_0^a = \lim_{s\to 0}[s\,B_0^{a*}(s)] = \frac{N_3}{D_2} \qquad \dots (17)$$

where

$N_3 = \varepsilon_2$

and D_2 is already specified.

Expected Instruction Time Analysis (Busy Period Of An Expert Repairman)

$$I_0(t) = q_{01}(t)©I_1(t)$$
$$I_1(t) = S_1(t) + q_{13}(t)©I_3(t) + q_{15}^{(2)}(t)©I_5(t)$$
$$I_3(t) = q_{30}(t)©I_0(t) + q_{31}^{(4)}(t)©I_1(t)$$
$$I_5(t) = q_{51}(t)©I_1(t) \quad \text{… (18-21)}$$

where

$$S_1(t) = e^{-\lambda t}\bar{I}(t) + [\lambda e^{-\lambda t}©1]\bar{I}(t) = \bar{I}(t)$$

Taking LT of (18-21) and solving for $I_0^*(s)$, we have (dropping s for brevity)

$$I_0^*(s) = \frac{N_4(s)}{D_2(s)} \quad \text{… (22)}$$

where

$$N_4(s) = W_1^*(s)q_{01}^*(s)$$

and $D_2(s)$ is already specified.
In a steady state, the total fraction of time for which the expert repairman is busy in giving instructions to his assistant, is given by

$$I_0 = \lim_{s\to 0}[s\,I_0^*(s)] = \frac{N_4}{D_2} \quad \text{… (23)}$$

where

$N_4 = \varepsilon_1$

and D_2 is already specified.

Expected Number of Visits by the Expert Repairman

$V_0(t) = Q_{01}(t)$ Ⓢ $[1 + V_1(t)]$
$V_1(t) = Q_{13}(t)$ Ⓢ $V_3(t) + Q_{15}^{(2)}(t)$ Ⓢ $V_5(t)$
$V_3(t) = Q_{30}(t)$ Ⓢ $V_0(t) + Q_{31}^{(4)}(t)$ Ⓢ $[1 + V_1(t)]$
$V_5(t) = Q_{51}(t)$ Ⓢ $[1+ V_1(t)]$ … (24-27)

Taking LST of equations (24-27) and solving for $V_0^{**}(s)$, we have (dropping s for brevity)

$$V_0^{**}(s) = \frac{N_5(s)}{D_2(s)} \quad \text{… (28)}$$

where $N_5(s) = Q_{01}^{**}(s)$

and $D_2(s)$ is already specified.
In a steady state, the number of visits per unit of the expert repairman is given by

$$V_0 = \lim_{s\to 0}[s\,V_0^{**}(s)] = \frac{N_5}{D_2} \quad \text{… (29)}$$

where $N_5 = 1$ and D_2 is already specified.

Profit Analysis

The expected total profit in a steady state is

$$P = K_0 A_0 - K_1 B_0^a - K_2 I_0 - K_3 V_0 \qquad \ldots (34)$$

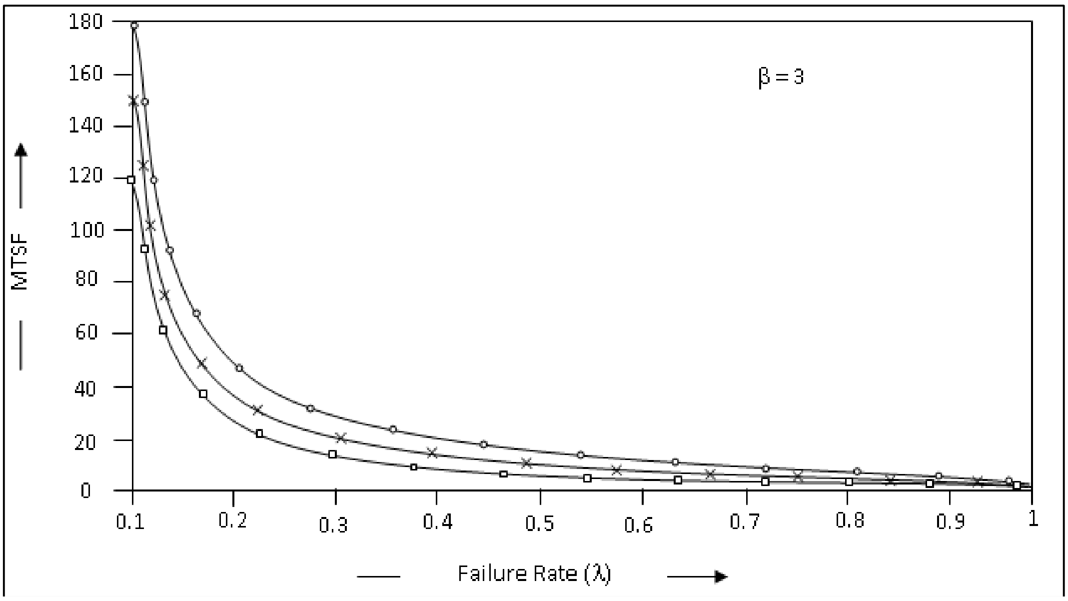
where K_0 is the revenue per unit up time of the system, K_1 is the cost per unit time for which the assistant repairman is busy, K_2 is the cost per unit time for which expert repairman is busy in giving instructions to his assistant, K_3 is the cost per visit for the expert repairman.

Graphical Representation

For a graphical representation the following particular case is considered :

$f(t) = \alpha \exp(-\alpha t)$, $i(t) = \beta \exp(-\beta t)$.

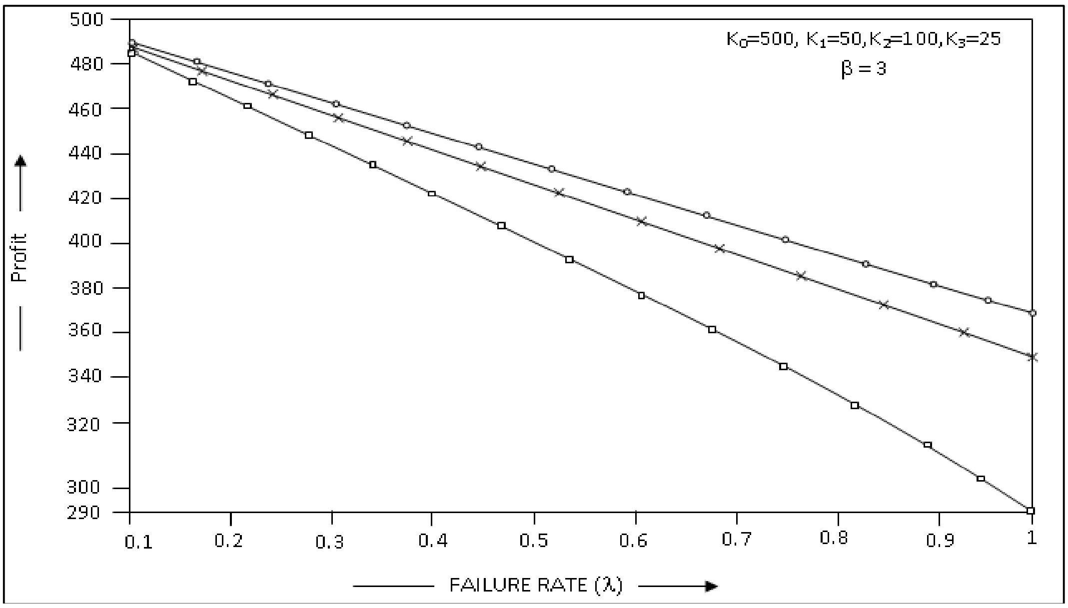
Figure 2 shows that behaviour of MTSF with respect of λ for α = 1.5, 2.5 and 3.5 while other parameters are kept fixed. Though the curves of this graph, we conclude that the MTSF decreases with an increase in λ. However, for increasing values of α, the MTSF tends to be higher.



□ α = 1.5 × α = 2.5 O α = 3.5

Fig 2: Steady–state behaviour of MTSF w.r.t. λ

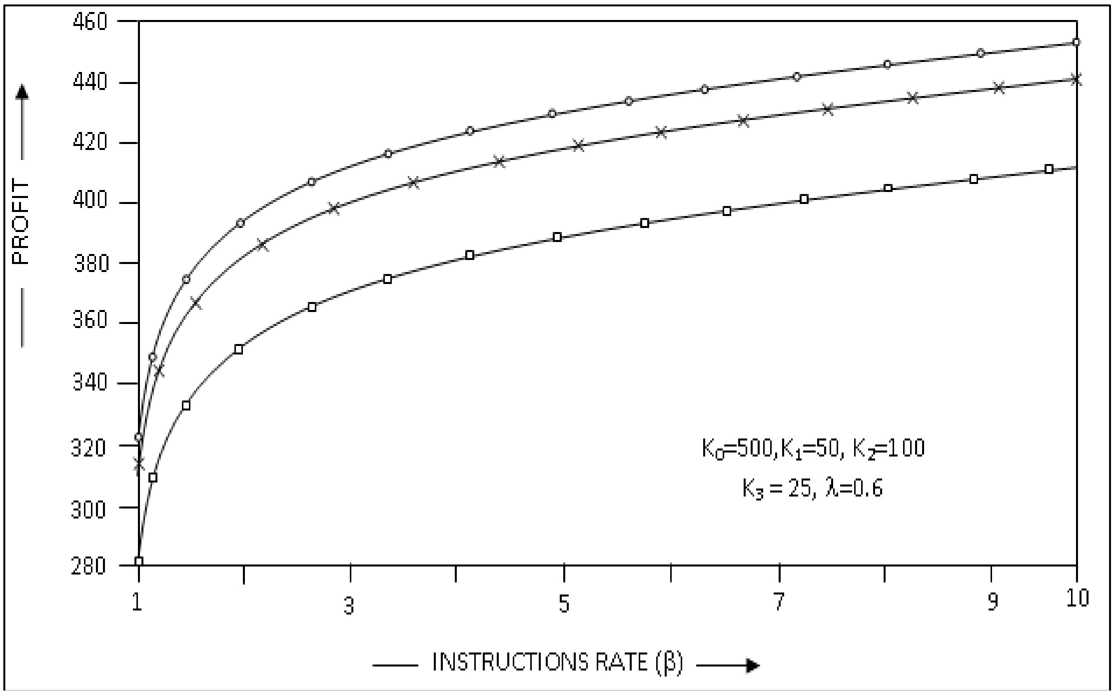
Figure 3 shows the behaviour of profit with respect to α for α = 1.5, 2.5, 3.5, while other parameters are kept fixed. Though the curves of this graph, we conclude that there is a uniform decrease in profit with an increase in λ and it increases with an increase in α.



□ α = 1.5 × α = 2.5 O α = 3.5

Fig 3: Steady State behaviour of profit w.r.t. λ

Figure 4 shows the behaviour of profit with respect to instructions rate β for α = 1.5, 2.5 and 3.5, while other parameters are kept fixed. Through the curves of this graph, we conclude that for initial value of β, profit increases slowly upto some point of β and then for higher values of β it remains constant for α = 1.5, 2.5 and 3.5 ; but for increasing the value of α, profit increases.



□ α = 1.5 × α = 2.5 O α = 3.5

Fig 4: Steady–state behaviour of profit w.r.t. β

References

1. Ashok Kumar, Suresh K. Gupta and R.K. Tuteja, Cost benefit analysis of a two-unit cold standby system with instruction time, IAPQR Transactions. 1997; 22(2):127-133.
2. R. K. Tuteja and Gulshan Taneja, Cost-benefit analysis of a two server, two-unit warm standby system with different types of failures, Microelectron. Reliab. 1992; 32:1353-1359
3. Gupta R, Goel LR. Profit analysis of a two-unit priority standby system with administrative delay in repair, Int. J. of System Sci. 1989; 20:1703
4. Goyal V, Murari K. Cost analysis of a two-unit standby system with two types of repairmen, Microelectron. Reliab. 1984; 24:849-855.
5. Sinha SM, Kapil DVS. 2-unit redundant system with delayed switchover and two types of repairs, IEEE Trans. Reliab. 1979; R-28:417.
6. Rander MC, Suresh K Gupta, Ashok Kumar. Cost Analysis of a two dissimilar cold standby system with preventive maintenance and replacement of standby system, Microelectron Reliab. 1994; 34(1):171-174.
7. Ashok Kumar, Suresh K Gupta, Gulshan Taneja. Probabilistic analysis of a two-unit cold standby system with instructions at need, Microelection. Reliab. 1995; 5(35):829-832.
8. Kumar A, Gupta SK, Taneja G. Comparative study of the profit of a two server system including patience time and instruction time, 1996; 36(10):1595-1601.