

# International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452  
Maths 2017; 2(6): 282-285  
© 2017 Stats & Maths  
www.mathsjournal.com  
Received: 06-09-2017  
Accepted: 07-10-2017

**Subham Biswal**  
PG Scholar,  
Department of AH Statistics &  
Computer Applications, MVC,  
Chennai, Tamil Nadu, India

**M Thirunavukkarasu**  
Dean, Veterinary College and  
Research Institute, Tirunelveli,  
Tamil Nadu, India

**R Venkataramanan**  
Assistant Professor,  
PGRIAS, Kattupakkam,  
Chennai, Tamil Nadu, India

**A Serma Saravana Pandian**  
Assistant Professor,  
Department of AH Economics,  
MVC, Chennai, Tamil Nadu,  
India

**Correspondence**  
**M Thirunavukkarasu**  
Dean, Veterinary College and  
Research Institute, Tirunelveli,  
Tamil Nadu, India

## Factors influencing lactation curve parameters in dairy farms

**Shubham Biswal, M Thirunavukkarasu, R Venkataramanan and A Serma Saravana Pandian**

### Abstract

The lactation curve parameters describing the shape of lactation curve were obtained using Ali and Schaeffer (ALI) model. For determining the effect of various factors affecting lactation curve parameters, least squares analysis was conducted. The results revealed that parity had significant influence on the increasing slope of lactation curve, indicating that cows in third parity produced more milk than other cows. Period (year) had no significant effect on any of the parameters, indicating that there was uniform management of the cows throughout the period of study. Non-significant effect of season of calving implied that the influence of climatic conditions was negligible under optimal feeding and management conditions and also animals had more adaptability to climatic variations. Breed had significant influence on the increasing slope of lactation curve. Farm had significant influence on parameters a, b and d, while age at first calving had no significant influence on any of the parameters.

**Keywords:** Dairy farming, lactation parameters, factors influencing

### Introduction

The knowledge of lactation curve parameters in dairy cattle is important for decisions on herd management and selection strategies, for determining optimum strategies for insemination and replacement of dairy cows and for genetic evaluation of dairy cows for improvement of milk production traits (Macciotta *et al.*, 2005) [14]. There are several factors which influence lactation curves in dairy cattle which include genetic background, period of calving, feeding, environmental conditions, herd, parity, season, age at calving and health status of animal. (Macciotta *et al.*, 2006) [13]. This study was conducted done with the objective of assessing the factors influencing the lactation curve parameters in dairy farms.

### Methodology

For the study, daily milk yield data on 259 lactations of cows belonging to these genetic groups were collected for a period of eleven years (2005-2015) from the records maintained at two farms of Tamil Nadu Veterinary and Animal Sciences University (TANUVAS), viz. Post Graduate Research Institute in Animal Sciences (PGRIAS), Kattupakkam and University Research Farm (URF), Chennai. Various lactation curve models were fitted in the study, for assessing their ability to explain the lactation and lactation curve in different breeds of dairy cattle: Parabolic exponential model, Inverse quadratic polynomial (IQP), IQP Modified model, Gamma function, Quadratic model, Quadratic cum log model, Wilmink model, Ali and Schaeffer model (ALI), Mixed log model, Grossman model and Cubic model, as detailed in Table 1.

**Table 1:** Lactation curve models fitted for different dairy cattle breeds

Name of the Model		Model Description	Source
Parabolic exponential model		$Y_t = a \exp(bt - ct^2)$ $\ln(Y_t) = \ln(a) + bt - ct^2 + e$	Sikka (1950)
Inverse quadratic polynomial (IQP)		$Y_t = t(a + bt + ct^2)^{-1}$	Nelder (1966)
IQP Modified Model		$Y_t^{-1} = a + bt^{-1} + ct + e$	Yadav <i>et al.</i> (1977a, b) [22,23]
Gamma function		$Y_t = at^b e^{-ct}$ $\ln(Y_t) = \ln(a) + b \ln(t) - ct + e$	Wood (1967)
Quadratic model		$Y_t = a + bt - ct^2$	Dave (1971)
Quadratic cum log model		$Y_t = a + bt + ct^2 + d \ln(t) + e$	Malhotra <i>et al.</i> (1980)
Wilmink model	4 models with $k=0.050, 0.065, 0.610, \text{ and } 0.100$	$Y_t = a + b e^{-kt} + ct + e$	Wilmink (1987)
Ali and Schaeffer model (ALI)		$Y_t = a + bx + cx^2 + d \log(1/x) + f \log(1/x)^2 + e$	Ali and Schaeffer, (1987)
Mixed log model		$Y_t = a + bt^{1/2} + c \log(t) + e$	Guo and Swalve (1995a,b)
Grossman model		$Y_t = at^b e^{-ct} (1 + u \sin(x) + v \cos(x))$ $\ln(Y_t) = \ln(a) + b \ln(t) + ct + u \sin x + v \cos x + e$	Grossman <i>et al.</i> (1986)
Cubic model		$Y_t = a + bt + ct^2 + dt^3 + e$	Dag <i>et al.</i> (2005)

where,  $Y_t$  = Daily milk yield,  $t$  = days in milk,  $e$  = random error, and  $a, b, c, d, f, u$  and  $v$  - regression coefficients. In Wilmink model,  $k$  is a constant term, while in ALI model,  $x = t/305$  and in Grossman model,  $x = \text{Day of year}$ .  $\ln$  is natural logarithm to base  $e$ .

Among these models, the best model of fit for lactation curve was identified using the criteria such as: highest Coefficient of determination ( $R^2$ ) and Adjusted  $R^2$  ( $R^2_{adj}$ ), lowest Root Mean Square Error (RMSE), and Durbin Watson (DW) coefficient which ranges from 0 to 4 (a DW value nearer to two indicates non auto-correlation, a value toward zero indicates positive auto-correlation and a value toward four indicate negative auto-correlation).

**Least Square Analysis for identifying the factors influencing lactation curve parameters**

For determining the effect of various factors affecting lactation parameters, least squares analysis was employed using General Linear Model (GLM) procedure in IBM SPSS Statistics@20.0. Various factors influencing lactation curve parameters considered for this study were parity, period, season, breed, farm and age at first calving. The effect of factors *viz.*, breed, period of calving, season of calving and age at first calving on lactation parameters was studied. Breed, year, season, parity and farm were included as fixed classes, while age at first calving was included as fixed covariate in GLM. Period (year) of calving was classified into 11 different periods (years) as 2005, 2006, ..., 2015. Four different seasons were used - Summer (March - May), South west monsoon (June - August), North east monsoon (September - November) and Winter (December - February). The following mathematical models were used for determining different factors affecting lactation curve parameters:

**First parity lactation parameters**

$$Y_{ijkl} = \mu + B_i + P_j + S_k + b(A_{ijkl}) + e_{ijkl}$$

where, first lactation parameter of the  $i^{\text{th}}$  cow that calved in  $i^{\text{th}}$  breed,  $j^{\text{th}}$  period and  $k^{\text{th}}$  season  
 $\mu$  overall mean, when equal subclass frequencies exist  
 $B_i$  effect of  $i^{\text{th}}$  breed  
 $P_j$  effect of  $j^{\text{th}}$  year ( $j = 1$  to 5)  
 $S_k$  effect of  $k^{\text{th}}$  season ( $k = 1$  to 4)  
 $b$  partial regression of the first lactation trait ( $Y$ ) on age at first calving ( $A$ )  
 $A_{ijkl}$  age at first calving for the corresponding  $Y_{ijk}$  observation  
 $e_{ijkl}$  random errors NID ( $0, \sigma^2_e$ )

**Lactation parameter pooled over lactations**

$$Y_{ijklmn} = \mu + B_i + P_j + S_k + O_l + F_m + e_{ijklmn}$$

where,  $n^{\text{th}}$  observation in  $i^{\text{th}}$  breed,  $j^{\text{th}}$  period,  $k^{\text{th}}$  season,  $l^{\text{th}}$  parity

$Y_{ijklm}$  and  $m^{\text{th}}$  farm  
 $\mu$  overall mean, when equal subclass frequencies exist  
 $B_i$  effect of  $i^{\text{th}}$  breed ( $i = 1$  to 6)  
 $P_j$  effect of  $j^{\text{th}}$  year ( $j = 1$  to 11)  
 $S_k$  effect of  $k^{\text{th}}$  season ( $k = 1$  to 4)  
 $O$  effect of  $l^{\text{th}}$  parity ( $l = 1$  to 8)  
 $F$  effect of  $m^{\text{th}}$  farm ( $m = 1, 2$ )  
 $e$  random errors NID ( $0, \sigma^2_e$ )

**Spearman Rank Correlation**

Friedman's test was used to rank different models according to the mean rank obtained by this test. The mean ranks obtained through Friedman's test for all the five breeds were then subjected to Spearman's rank correlation to understand the similarity of ranking.

**Results and Discussion**

Among the models fitted, the best model of fit for lactation curve was identified using the criteria specified in the Methodology, i.e., highest  $R^2$  and  $R^2_{adj}$ , lowest RMSE, and status of DW coefficient from 0 to 4. These coefficients are presented in Table 2.

**Table 2:** Statistical parameters for identifying the best lactation curve model

Lactation Curve Model	$R^2$	$R^2_{adj}$	RMSE	DW
Gamma	0.69	0.69	0.20	0.85
Wilmink	0.73	0.68	0.86	0.91
Cubic	0.74	0.73	0.78	1.03
Grossman	0.67	0.66	0.21	0.85
Inverse quadratic polynomial	0.74	0.74	2.28	0.87
Inverse quadratic polynomial modified	0.45	0.45	0.08	0.76
Mixed log	0.70	0.70	0.84	0.92
Ali and Schaeffer	0.74	0.74	0.77	1.09
Quadratic	0.70	0.70	0.98	0.95
Quadratic cum log	0.73	0.73	0.79	1.03
Parabolic exponential	0.68	0.68	0.20	0.90

The  $R^2$  and the adjusted  $R^2$  values were the highest in Cubic, IQP and ALI models (0.74), which meant that all these models were able to explain 74 per cent of variation in daily milk yield. Besides this, considering the RMSE and DW coefficients, the Ali and Schaeffer model was identified to be the best fit for constructing the lactation curve for all the breeds. Spearman Rank correlation among various breeds for

mean rank obtained through Friedman's test was very high with a range of 0.994 to 1 (Table 3). This was indicative of highly similar ranking in all the breeds. That is, here again,

the Ali and Schaeffer model was found to be the best model based on mean rank in all the breeds studied.

**Table 3:** Spearman Rank correlation among various breeds for mean rank obtained through Friedman's test

	Jersey Cross	Gir	Sahiwal	Tharparkar	Rathi	Deoni
Jersey Cross	-	0.998**	0.999**	0.998**	0.995**	1.000**
Gir	0.998**	-	0.999**	0.996**	0.994**	0.998**
Sahiwal	0.999**	0.999**	-	0.997**	0.996**	0.999**
Tharparkar	0.998**	0.996**	0.997**	-	0.989**	0.998**
Rathi	0.995**	0.994**	0.996**	0.989**	-	0.995**
Deoni	1.000**	0.998**	0.999**	0.998**	0.995**	-

\*\* Correlation is highly significant  $p < 0.01$

Based on  $R^2$ ,  $R^2_{adj}$ , RMSE and DW coefficients, Ali and Schaeffer model was identified to be the best fit for all the breeds. Hence, least squares means for different parameters of this best model - Ali model were estimated and presented in Table 4. The overall means for a, b, c, d and f were 4.337, -2.770, 3.447, 2.114 and -0.598, respectively. Parity had significant influence ( $p < 0.05$ ) on parameter d. Parameter d represented increasing slope of lactation curve which indicated that cows in third parity produced more milk than cows in other parities. Kocak and Ekiz (2008) [12], Atashi *et al.* (2009) [2], Bahashwan *et al.* (2014) [3] and Darfour-Oduro *et al.* (2014) [6] also reported that parity had a significant influence on the lactation curve parameters.

Period (year) had no significant effect on any parameters of ALI model which indicated that there was uniform management of the cows throughout the period of study. Singh and Bhat (1978) [18] reported that all the components of parabolic exponential except b (linear constant which

measures the average slope of the curve) were significantly affected by period and month of calving. However, Dedkova and Nemcova (2003) [8] and Bouallegue *et al.* (2015) [4] found that the year of calving had significant influence on the shape of lactation curve traits.

Similarly, season also had no significant effect on the curve parameters. Non-significant effect of season of calving implied that the influence of climatic conditions was negligible under optimal feeding and management conditions, besides the fact that all breeds of animals had more adaptability to climatic variations. Yadav *et al.* (1977a) [22] also reported earlier that season had no effect on all the components of inverse polynomial function. However, they reported a significant influence of season of calving on all the parameters of Wood function. Atashi *et al.* (2009) [2] and Bouallegue *et al.* (2015) [4] also found that there was significant influence of season of calving on shape of the lactation curve.

**Table 4:** Least square means ( $\pm$ S.E) of parameters of Ali and Schaeffer model

Details	N	Mean $\pm$ S.E				
		a	b	c	d	f
<b>Overall mean</b>	412	4.337 $\pm$ 3.745	-2.770 $\pm$ 6.431	3.447 $\pm$ 5.660	2.114 $\pm$ 2.136	-0.598 $\pm$ 1.156
<b>Parity</b>		NS	NS	NS	*	NS
1	143	3.704 $\pm$ 3.732	-3.109 $\pm$ 6.409	1.624 $\pm$ 3.436	2.109 $\pm$ 2.129	-0.035 $\pm$ 1.152
2	120	0.451 $\pm$ 3.639	3.152 $\pm$ 6.248	-1.245 $\pm$ 3.349	4.000 $\pm$ 2.075	-1.668 $\pm$ 1.123
3	74	-0.398 $\pm$ 3.920	6.634 $\pm$ 6.731	-4.051 $\pm$ 3.608	4.944 $\pm$ 2.236	-0.601 $\pm$ 1.210
4	38	-0.339 $\pm$ 4.717	5.068 $\pm$ 8.100	-1.977 $\pm$ 4.342	4.146 $\pm$ 2.690	-0.741 $\pm$ 1.456
5	21	4.403 $\pm$ 5.928	-3.667 $\pm$ 10.180	2.492 $\pm$ 5.457	1.394 $\pm$ 3.381	-0.507 $\pm$ 1.830
6	7	4.710 $\pm$ 9.568	-5.870 $\pm$ 16.431	5.198 $\pm$ 8.807	2.582 $\pm$ 5.457	-0.833 $\pm$ 2.954
7	5	8.835 $\pm$ 11.182	-5.063 $\pm$ 19.202	-1.895 $\pm$ 10.293	-0.176 $\pm$ 6.370	-0.334 $\pm$ 3.452
8	4	13.335 $\pm$ 12.450	-19.303 $\pm$ 21.379	8.789 $\pm$ 11.460	-3.085 $\pm$ 7.101	-0.063 $\pm$ 3.844
<b>Period</b>		NS	NS	NS	NS	NS
2005	29	5.710 $\pm$ 5.877	-3.121 $\pm$ 10.093	0.313 $\pm$ 5.410	1.306 $\pm$ 3.352	-0.327 $\pm$ 1.815
2006	39	2.799 $\pm$ 5.086	0.589 $\pm$ 8.734	-1.605 $\pm$ 4.682	2.865 $\pm$ 2.901	-4.429 $\pm$ 1.570
2007	57	1.618 $\pm$ 4.669	0.174 $\pm$ 8.018	1.150 $\pm$ 4.298	3.471 $\pm$ 2.663	-0.142 $\pm$ 1.441
2008	36	-1.484 $\pm$ 5.314	9.172 $\pm$ 9.125	-5.549 $\pm$ 4.891	5.497 $\pm$ 3.031	-0.517 $\pm$ 1.640
2009	35	9.786 $\pm$ 5.609	-11.654 $\pm$ 9.632	5.168 $\pm$ 5.163	-1.248 $\pm$ 3.199	0.197 $\pm$ 1.732
2010	28	7.785 $\pm$ 5.860	-7.169 $\pm$ 10.063	2.806 $\pm$ 5.394	0.806 $\pm$ 3.342	-0.052 $\pm$ 1.800
2011	26	10.767 $\pm$ 6.044	-15.236 $\pm$ 10.370	7.168 $\pm$ 5.564	-0.694 $\pm$ 3.447	0.011 $\pm$ 1.866
2012	46	3.570 $\pm$ 5.025	-1.872 $\pm$ 8.629	0.073 $\pm$ 4.625	1.774 $\pm$ 2.866	-0.323 $\pm$ 1.551
2013	55	7.729 $\pm$ 4.950	-8.760 $\pm$ 8.500	3.424 $\pm$ 4.557	-0.127 $\pm$ 2.823	0.044 $\pm$ 1.528
2014	44	3.057 $\pm$ 5.234	0.072 $\pm$ 8.988	-0.932 $\pm$ 4.818	3.053 $\pm$ 2.985	-0.094 $\pm$ 1.616
2015	17	-3.626 $\pm$ 7.080	7.336 $\pm$ 12.158	0.271 $\pm$ 6.517	6.556 $\pm$ 4.038	-0.942 $\pm$ 2.186
<b>Season</b>		NS	NS	NS	NS	NS
Winter	101	5.335 $\pm$ 4.401	-3.581 $\pm$ 7.558	1.178 $\pm$ 4.051	1.656 $\pm$ 2.510	-0.247 $\pm$ 1.359
Summer	117	5.349 $\pm$ 4.128	-2.459 $\pm$ 7.089	-0.755 $\pm$ 3.800	1.542 $\pm$ 2.355	0.192 $\pm$ 1.275
South west	92	6.422 $\pm$ 4.378	-7.493 $\pm$ 7.518	3.460 $\pm$ 4.030	0.908 $\pm$ 2.497	-1.619 $\pm$ 1.352
North east	102	0.244 $\pm$ 4.342	2.453 $\pm$ 7.456	0.586 $\pm$ 3.996	4.351 $\pm$ 2.476	-0.717 $\pm$ 1.340
<b>Breed</b>		NS	NS	NS	*	NS
Jersey cross	259	8.631 $\pm$ 3.951	-6.138 $\pm$ 6.784	-0.342 $\pm$ 3.636	0.705 $\pm$ 2.253	-0.480 $\pm$ 1.220
Gir	31	8.560 $\pm$ 6.086	-5.540 $\pm$ 10.451	-0.487 $\pm$ 5.602	0.558 $\pm$ 3.471	-0.380 $\pm$ 1.879
Sahiwal	71	9.531 $\pm$ 5.148	-6.137 $\pm$ 8.840	0.345 $\pm$ 4.738	0.710 $\pm$ 2.936	-0.460 $\pm$ 1.589
Tharparkar	16	7.239 $\pm$ 6.997	-8.328 $\pm$ 12.016	4.110 $\pm$ 6.441	0.512 $\pm$ 3.991	-0.412 $\pm$ 2.160
Rathi	15	7.224 $\pm$ 7.355	-8.059 $\pm$ 12.630	4.128 $\pm$ 6.770	0.458 $\pm$ 4.195	-0.411 $\pm$ 2.271
Deoni	20	7.631 $\pm$ 6.931	-6.140 $\pm$ 11.901	0.352 $\pm$ 6.379	0.695 $\pm$ 3.953	-0.450 $\pm$ 2.140

Farm		**	**	NS	*	NS
PGRIAS	236	-3.630±5.684	9.183±9.761	-2.055±5.232	6.048±3.242	-1.203±1.755
URF	176	12.305±3.419	-14.723±5.870	4.289±3.147	-1.820±1.950	0.007±1.055

NS – Non-Significant ( $p>0.05$ ), \* Significant ( $p<0.05$ ), \*\* Highly significant ( $p<0.01$ )

Breed had significant influence ( $p<0.05$ ) on lactation curve parameter  $d$ , which represented the increasing slope of lactation curve and indicated that Sahiwal and Jersey crossbreds were more persistent in milk production. Farm type had highly significant ( $p<0.01$ ) influence on parameters  $a$  and  $b$  and significant influence ( $p<0.05$ ) on parameter  $d$ . Age at first calving had no significant influence on any of the parameters, except parameter  $d$ . However, Rao and Sundaresan (1979)<sup>[17]</sup> reported a significant influence of age at first calving on the shape of lactation curve of Sahiwal cows using the Gamma function and Dedkova and Nemcova (2003)<sup>[8]</sup> found that cows with lower age at calving showed best persistency.

### Summary and Conclusions

The results of the study to assess the factors associated with lactation parameter in organised dairy farms revealed that parity had significant influence ( $p<0.05$ ) on parameter ' $d$ ', which represented increasing slope of lactation curve, indicating the fact that cows in third parity produced more milk than cows in other parities. Period (year) had no significant effect on any of the parameters of the lactation model which indicated that there was uniform management of the cows throughout the period of study. Non-significant effect of season of calving implied that the influence of climatic conditions was negligible under optimal feeding and management conditions and also animals had more adaptability to climatic variations. Breed had significant influence ( $p<0.05$ ) on lactation curve parameter ' $d$ ', which represented the increasing slope of lactation curve and indicated that Sahiwal and Jersey crossbreds were more persistent in milk production. Farm had significant influence on parameters  $a$ ,  $b$  and  $d$ , while age at first calving had no significant influence on any of the parameters.

### References

1. Ali TE, Schaeffer LR. Accounting for covariances among test day milk yields in dairy cows. *Can. J. Anim. Sci.* 1987; 67:637-644.
2. Atashi H, Sharbabak MM, Sharbabak HM. Environmental factors affecting the shape components of the lactation curves in Holstein dairy cattle of Iran. *Livestock Res. for Rural Dev.* 2009; 21(5) (accessed on July 18, 2016, from [www.lrrd.org/lrrd21/5/atas21060.htm](http://www.lrrd.org/lrrd21/5/atas21060.htm)).
3. Bahashwan S, Alfadli S. Dhofari cow's potentiality of milk production and lactation curve. *Net J. Agri. Sci.* 2014; 2(2):74-78.
4. Bouallegue M, Steri R, M'hamdi N, Ben Hamouda M. Modelling of individual lactation curves of Tunisian Holstein-Friesian cows for milk yield, fat, and protein contents using parametric, orthogonal and spline models. *J. Anim. and Feed Sci.* 2015; (24):11-18.
5. Dag B, Keskin I, Mikailsoy F. Application of different models to the lactation curves of unimproved Awassi ewes in Turkey. *South African J. Anim. Sci.*, 2005; 35(4):238-243.
6. Darfour-Oduro K, Hagan BA, Asafu-Adjaye A. Lactation curves of Friesian-Sanga and Sanga cows in Ghana. *Livestock Res. for Rural Dev.* 2014; 26(10) (accessed on July 18, 2016, from [www.lrrd.org/lrrd26/10/darf26188.htm](http://www.lrrd.org/lrrd26/10/darf26188.htm)).
7. Dave BK. First lactation curves of Indian water Buffalo. *JNKVV Res. J.* 1971; 5:93.
8. Dedkova L, Nemcova E, Factors affecting the shape of lactation curves of Holstein cows in the Czech Republic. *Czech J. Anim. Sci.* 2003; 48:395-402.
9. Grossman M, Kuck AL, Norton HW. Lactation Curves of Purebred and Crossbred Dairy Cattle. *J Dairy Sci.* 1986; 69:195-203.
10. Guo Z, Swalve HH. Modelling of the lactation curve as a sub- model in the evaluation of test day records. Interbull open meeting 7-8 September 1995 Prague Czech Republic, 1995a.
11. Guo Z, Swalve HH. Comparison of different lactation curve sub- models in test day models. *Proc. Interbull Mtg. Vienna Austria Interbull Bull. No.* 1995b; 16.
12. Kocak O, Ekiz B. Comparison of different lactation curve models in Holstein cows raised on a farm in the south-eastern Anatolia region. *Arch. Tierz. Dummerstorf*, 2008; 51(4):329-337.
13. Macciotta NPP, Dimauro C, Catillo G, Coletta A, Cappio-Borlino A. Factors affecting individual lactation curve shape in Italian river buffaloes. *Livestock Sci.* 2006; 104:33-37.
14. Macciotta NPP, Vicario D, Cappio-Borlino A, Detection of different shapes of lactation curve for milk yield in dairy cattle by empirical mathematical models. *J. Dairy Sci.* 2005; 88:1178-1191
15. Malhotra PK, Singh RP, Singh RN. Estimating lactation curve in Karan-Swiss cattle. *Indian J. Anim. Sci.* 1980; 50(10):799.
16. Nelder JA. Inverse polynomial, a useful group of multi-factor response functions. *Biometrics.* 1966; 22:128-41.
17. Rao MK, Sundaresan D. Influence of environment and heredity on the shape of lactation curves in Sahiwal cows. *J Agri. Sci.* 1979; 92(2):393-401.
18. Singh B, Bhat PN. Models of lactation curves for Haryana cattle. *Indian J. Anim. Sci.* 1978; 48(11):791-797.
19. Sikka LC. A study of lactation as affected by lactation and environment. *J Dairy Res.* 1950; 17:231.
20. Wilmink JBM. Adjustment of test-day milk fat and protein yield for age season and stage of lactation. *Livestock Prod. Sci.* 1987; 16:335-348.
21. Wood PDP. Algebraic model of the lactation curve in cattle. *Nature*, 1967; 216:164-65.
22. Yadav MC, Katpatal BG, Kaushik SN. Components of inverse polynomial function of lactation curve and factors affecting them in Haryana and its Friesian crosses. *Indian J Anim. Sci.* 1977a; 47(12):777-781.
23. Yadav MC, Katpatal BG, Kaushik SN. Study of lactation curve in Haryana and its Friesian crosses. *Indian J Anim. Sci.*, 1977b; 47(10):607-609.