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The stress-intensity factors for two Griffith-cracks opened by forces at crack faces in an isotropic stress-free rectangle

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Abstract
The exact expressions for normal stress and for crack shape has been obtained by using Fourier transform method. It is observed that normal stress possess Cauchy type singularity at crack tips while crack shape function is smooth.

Keywords: Stress-intensity factors (SIF), crack opening displacement (C.O.D.), Fourier transforms, wedge or rivets

Introduction
Now a days, the fail-safe-design concepts play an important role in manufacturing of machines etc. These design concepts depend mainly on crack-shape and stress-intensity factors. The rectangular plates are very common in use. Therefore, the title problem is important to investigate the displacement & stress-components.

In 1920 AA Griffith advanced the adequate theory of cracks which requires consideration of cohesion of molecular force acting near the edge of the crack. He gave the energy balance theory first time. In brittle materials, surface energy is the only source of dissipation.

There is good account of crack problems in [1, 2, 3]. These encompasse rectangular, cylindrical and spherical shape type bodies. The cracks in the medium are discontinuity in the continuum. Therefore, the crack problems in mathematical theory of elasticity generates mixed-boundary value problems.

Sneddon [4] used transform method in crack problem for the first time. Srivastava and Lowengrub [5] used triple integral equation in solving two cracks problem in infinite medium. Parihar and Kushwaha [6] extended the problem to infinite isotropic strip with rigidly lubricated edges. Kushwaha [7] extended to rectangular domains with rigidly lubricated edges. Kushwaha [8] had done for crack opening due to body forces in orthotropic infinite medium.

Parihar and Sowdamini [9] had done three crack problem in infinite isotropic medium. The authors of [9] had extended the problem to orthotropic infinite medium in [10]. There are some more work in [11-14].

The title problem is important for fail-safe designs. Thus we see that the cracks occupy the region $y = 0, b < |x| < c$. The rectangle is of length $2a$ and width 2δ . The cracks lie on x-axis while the y-axis passes through the middle of width. Thus the physical problem is reduced to mathematical mixed-boundary value problem as below.

$$\sigma_{xx}(\pm a, y) = \sigma_{xy}(\pm a, y) = 0, 0 \le |y| \le \delta \tag{1.1}$$

$$\sigma_{yy}(x, \pm\delta) = \sigma_{xy}(x, \pm\delta) = 0, 0 \le |x| \le a \tag{1.2}$$

$$\sigma_{xy}(x, 0^{\pm}) = 0, 0 \le |x| \le a \tag{1.3}$$

$$\sigma_{yy}(x, 0^{\pm}) = -p(x), b < |x| < c \tag{1.4}$$

$$u_y(x, 0^{\pm}) = 0, \{0 \le |x| < b, c \le |x| \le a \tag{1.5}$$

It is assumed that plane-strain conditions prevail. The geometry, see figure 1, of the problem is symmetrical, therefore, the solution space will reduce to $[0,a]\cup[0,\delta]$ and the boundary conditions (1.1) – (1.5) will reduce to,

$$\sigma_{xx}(x,y)=\sigma_{xy}(a,y)=0, 0\le y\le\delta \tag{1.6}$$

$$\sigma_{yy}(x,\delta)=\sigma_{xy}(x,\delta)=0, 0\le x\le a \tag{1.7}$$

$$\sigma_{xy}(x,0)=0, 0\le x\le a \tag{1.8}$$

$$\sigma_{yy}(x,0)=-p(x), b<x<c \tag{1.9}$$

$$u_y(x,0)=0, 0\le x\le b, c\le x\le a \tag{1.10}$$

It is being checked throughout, see Burnishton [15].

$$u_y(x,0^{\pm})>0, b<x<c \tag{1.11}$$

Which means that crack really opens out. It means that crack faces do not meet each other except at crack tips.
The plan of the paper is as follows: Section-1 gives importance of the problem and reduction to mixed-boundary value problem of mathematics. Section-2 formulates the problem. Section-3 reduces the problem to triple series equation. Solution of triple series equation is given in term of Fredholm integral equation of second kind in section 4. Physical quantities are given in terms of solution of triple series. The solution of Fredholm integral equation for special loading is given in section 6. Discussions and conclusions are reported in section-7. The references are at the last.

Formulation
The solution of above mentioned mathematical problem in (1.6) – (1.10) is obtained by solving the following equations of equilibrium.

$$\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\sigma_{xy}}{\partial y}=0,\quad \frac{\partial\sigma_{xy}}{\partial x}+\frac{\partial\sigma_{yy}}{\partial y}=0 \tag{2.1}$$

with the stress-strain relations as

$$\sigma_{ij}=2e_{ij}+\frac{2(1+\eta)}{1-2\eta}e_{kk}\delta_{kk}, i,j=x,y \tag{2.2}$$

Where we have chosen μ (Modulus of rigidity) as unit of measurement for stress and η is Poisson ratio.

$$\delta_{ij}\begin{cases}1, i=j\\ 0, i\ne j\end{cases},\quad e_{ij}=(u_{i,j}+u_{j,i})$$

$$e_{kk}=u_{x,x}+u_{y,y},$$

Where (,) over quantities refere to differential and e_{ij} are strain components and u_i, are displacement components and σ_{ij} are stress components. The solutions of (2.1) is sought through Airy's function method [16]. The solution of (2.1) is assumed in terms of displacement components which will satisfy (2.1) through (2.2). We take

$$u_x(x,y)=\frac{2(1+\eta)}{E}\left[\sum_{n=1}^{\infty}\frac{\sin(\alpha_n x}{\alpha_n}\left\langle(1-\eta)G_{,yy}+\eta\alpha_n^2G\right\rangle+\frac{1}{2}u_{xc}(x,0)+\sum_{m=1}^{\infty}\cos(\beta_m y)u_{xc}(x,\beta_m)\right] \tag{2.3}$$

$$u_y(x,y)=\frac{1}{2}u_{ye}(0,y)+\sum_{n=1}^{\infty}u_{yc}(\alpha_n,y)\cos(\alpha_n x)+\sum_{m=1}^{\infty}\frac{\sin\beta_m y}{\beta_m^2}\left[(1-\eta)H_{,xxx}+\eta\beta_m^2H_{,x}\right]\frac{2(1+n)}{E} \tag{2.4}$$

with
$$\alpha_n=\frac{n\pi}{a}=\pi q, \beta_m=\frac{m\pi}{\delta}, u_{xc}(x,\beta_m)=\frac{(1-\eta)H_{,xx}+\eta\beta_m^2H}{\beta_m}$$

$$u_{yc}(\alpha_n,y)=[(1-\eta)G_{,xxx}+\eta\alpha_n^2G_{,x}]\frac{2(1+\eta)}{E\alpha_n^2} \tag{2.5}$$

$$G(\alpha_n, y) = (A_n + yB_n)\cos(\alpha_n y) + (C_n + yD_n)\sinh(\alpha_n y) \tag{2.6}$$

$$H(x, \beta_m) = \left(E_m + xF_m\right)\sinh(\beta_m x) \tag{2.7}$$

where A_n, B_n, C_n, D_n and E_m, F_m are six constants which are to be determined by given six conditions.

Reduction to triple series equation

The boundary conditions second of (1.6) – (1.7) and (1.8) after using (2.1) – (2.4), we get,

$$E_m = -\frac{a_2}{a_1}F_m,\ A_n a_3 + B_n a_4 + D_n a_5 = 0,\ \alpha_n G = -B_n, \tag{3.1}$$

And first of (1.6) – (1.7) give

$$F_m a_6 = \sum_{n=1}^{\infty}(-1)^n \alpha_n^2[A_n a_9 + a_{11}B_n + a_{12}D_n], \tag{3.2}$$

$$\sum_{m=1}^{\infty} a_3 a_{18}(-1)^{m-1}F_m = B_n a_{19} + D_n a_{20} \tag{3.3}$$

Where

$$\left.\begin{aligned} &a_1 = \beta_m\cosh(\beta_m a), a_2 = a a_1 + \sinh(\beta_m a), a_3 = \alpha_n\sinh(\alpha_n\delta) \\ &a_4 = \delta a_3 - \cosh(\alpha_n\delta), a_5 = \alpha_n\delta\cosh(\alpha_n\delta) + \sinh(\alpha_n\delta) \\ &a_6 = \left(a - \frac{a_2}{a_1}\right)\beta_m + 2\cosh(\beta_m a) \end{aligned}\right\} \tag{3.4}$$

$$\left.\begin{aligned} (a_7, a_8) = \Bigg[&\frac{2(-1)^n\alpha_n\delta\left\langle\cosh(\alpha_n\delta), \sinh(\alpha_n\delta)\right\rangle}{\alpha_n^2 + \beta_m^2} + 2\alpha_n^2(1-(-1)^n)(\alpha_n^2 - \beta_m^2) \\ &\left\langle\sinh\alpha_n\delta, \cosh(\alpha_n\delta)\right\rangle / \left(\alpha_n^2 + \beta_m^2\right)\Bigg] \end{aligned}\right\} \tag{3.5}$$

$$\left.\begin{aligned} &a_9 = \frac{(-1)^m\alpha_n\delta\cosh\alpha_n\delta}{\alpha_n^2 + \beta_m^2}, a_{10} = \frac{\alpha_n\left\langle 2 - (-1)^m\right\rangle\cosh(\alpha_n\delta)}{\alpha_n^2 + \beta_m^2} \\ &a_{11} = a_7 + 2a_{10}, a_{12} = a_8 + 2a_9, a_{13} = \beta_m\frac{\left\langle 2 - (-1)^m\cosh(\beta_m a)\right\rangle}{\alpha_n^2 + \beta_m^2} \end{aligned}\right\} \tag{3.6}$$

$$a_{14} = a_1 a_7\delta - a_{12}a_{13}, a_{15} = \alpha_n^2\cosh(\alpha_n\delta), a_{16} = \alpha a_{15}, \tag{3.7}$$

$$a_{18} = \beta_m^2 a_{14}/a_1, a_{17} = \alpha_n\delta\sinh(\alpha_n\delta), a_{19} = a_3 a_{16} - a_4 a_{15} \tag{3.8}$$

$$\left.\begin{aligned} &a_{20} = a_3 a_{17} - a_5 a_{15}, a_{21} = (a_{11}a_3 - a_n a_9)/a_3 \\ &a_{22} = (a_3 a_{12} - a_5 a_9)/a_3, a_{33} = a_{22} - a_{21}a_{20} \\ &a_{24} = a_6 - \sum_{n=1}^{\infty}(-1)^n\alpha_n^2 a_3 a_{12}a_{18} \end{aligned}\right\} \tag{3.9}$$

Thus through five relations (3.1)-(3.3) we can find five constants in terms of sixth. Now the mixed-boundary conditions (1.9) – (1.10) and using above relations between constants we get the following triple series equation.

$$\frac{\phi_0}{2} + \sum_{n=1}^{\infty}\phi_n\cos(\alpha_n x) = 0, 0 \le x \le b, c \le x \le a, \tag{3.10}$$

$$\sum_{n=1}^{\infty} \alpha_n \phi_n \cos(\alpha_n x) = P_3(x), b < x < c \tag{3.11}$$

$$P_3(x) = P_2(x) + \sum_{n=1}^{\infty} \alpha_n \phi_n \left(\frac{a_{21} - a_{23}}{a_{23}} \right) \cos(\alpha_n x), \tag{3.12}$$

$$P_2(x) = -p(x) + \sum_{m=1}^{\infty} (x - T_1) \frac{\phi_m \sinh(\beta_m x)}{a_{23}} \tag{3.13}$$

$$\phi_n = a_{23} F_m, \phi_0 = F_0, T_1 = a + \beta_m^{-1} \tanh(a\beta_m) \tag{3.14}$$

Thus the solution of physical problem is reduced to the solution of triple series equation (3.10) – (3.11).

Solution of Triple series Equations

To solve the triple series equation (3.10) – (3.11) we use the method of Parihar [17]. We assume the trial solution as,

$$\alpha_n \phi_n = 2\int_b^c g(t) \sin(\alpha_n t) dt \tag{4.1}$$

$$\phi_0 = 2\int_b^c t\, g(t) dt \tag{4.2}$$

and using the relation

$$\frac{qt}{2} + \sum_{n=1}^{\infty} \frac{\sin(\alpha_n t) \cos \alpha_n x}{n} = \begin{cases} a/2, & t > x \\ a/4, & t = x \\ 0, & t < x \end{cases} \tag{4.3}$$

then the substitution of (4.1) – (4.2) into (3.10) and using (4.3), it is satisfied identically if,

$$\int_b^c g(t) dt = 0 \tag{4.4}$$

The substitution of (4.1) into (3.11) and using the series

$$\sum_{n=1}^{\infty} \frac{\sin(\alpha_n t) \sin(\alpha_n x)}{n} = \frac{q}{2} \log \left| \frac{\sin q(x - t)}{\sin q(x + t)} \right|, \tag{4.5}$$

and then using [17] for inversion, we get

$$g(t) = \frac{1}{a^2 \theta(t)} \left[\Delta_0(t) + \int_b^c g(x) K(x,t) dx \right], b < t < c \tag{4.6}$$

$$\Delta_0(t) = \int_b^c \frac{\sin(qx)\theta(x) p(x) dx}{G(x,t)} + D \tag{4.7}$$

with

$$G(x,t) = \cos(qx) - \cos(qt), \ \theta(x) = \left\{ \left| G(b,x) \right| \left| G(x,c) \right| \right\}^{1/2} \tag{4.8}$$

D is an arbitrary constant to be determined by (4.6) and (4.4). And

$$\left. \begin{aligned} K(y,t) = \sum_{m=1}^{\infty} \int_b^c (x - T_1) \sinh(\beta_n x) \sin(\alpha_m y) \theta_0(x,t) dx \\ + \sum_{n=1}^{\infty} \int_b^c \frac{(a_{21} - a_{23})}{a_{23}} \sin(\alpha_n y) \cos(\alpha_n x) \theta_0(x,t) dx \end{aligned} \right\} \tag{4.9}$$

$$\theta_0(x,t) = \frac{\sin(qx)\theta(x)}{G(x,t)} \tag{4.10}$$

The equation (4.6) is Fredholm integral equation of second kind.

Physical Quantities

The physical quantities of interest in fracture mechanics are crack shape and stress-intensity factors. The crack shape is nothing but graph of crack opening displacement. By knowing crack shape experimentally we can calculate required pressure to generate such crack. The stress intensity factors are defined in terms of stress components at crack tips.

Crack Shape

The crack shape will be plotted through the value of $u_y(x,0), b < |x| < c$ which will be the value of series of left hand side in equation (3.10).

$$u_y(x,0) = \frac{2(1+\eta)}{\pi E}\int_x^c g(t)\,dt, b < x < c \tag{5.1}$$

where $g(t)$ will be obtained from the solution of Fredholm integral equation of second kind.

Stress Components

The shear stress $\sigma_{xy}(x,0)$ is zero for $0 \le |x| \le a,$ therefore it will not play any role. $\sigma_{xx}(x,0)$ can be evaluated through the values of $\sigma_{yy}(x,0).$ Therefore, $\sigma_{yy}(x,0)$ is important to evaluate. It is evaluated through the value of series in (3.11) after transferring the right hand terms, except $p(x)$, to the left hand side. It is given as

$$\sigma_{yy}(x,0) = \frac{2}{a}\left[\int_b^c \frac{g(t)\sin(qt)}{G(x,t)}dt + \int_b^c g(\alpha)F_4(\alpha,x)\,d\alpha\right], 0 \le x < b,\ c < x \le a \tag{5.2}$$

Now using $g(t)$ from (4.6) and then evaluating the integrals we get,

$$\sigma_{yy}(x,0) = \begin{cases} \dfrac{\Delta_1(x)}{\theta(x)} + F_5(x),\ 0 \le x < b \\ -\dfrac{\Delta_1(x)}{\theta(x)} + F_5(x),\ c < x \le a \end{cases} \tag{5.3}$$

with

$$\Delta_1(x) = \Delta_0(x) + \int_b^c g(\alpha)K(\alpha,x)\,dx, \quad F_5(x) = \int_b^c g(\alpha)F_4(\alpha,x)\,dx \tag{5.4}$$

$$F_4(\alpha,x) = \sum_{n=1}^{\infty}(-1)^n \alpha_n^2 \sin(\alpha\,\alpha_n)\int_0^{\infty} F_3(s\alpha_n, x)\,ds \tag{5.5}$$

$$F_3(s\alpha_n, x) = \sum_{m=1}^{\infty}(x - T_1)\sinh(\beta_m x) + \sum_{n=1}^{\infty}(-1)^n \left\langle \frac{a_{21} - a_{23}}{a_{23}} \right\rangle \cos\alpha_n x \tag{5.6}$$

Stress-Intensity Factors

The stress-intensity factors at crack tip are defined as

$$\left.\begin{aligned} [K_b, M_b, N_b] &= \lim_{x \to b^-} \sqrt{b-x}\left[\sigma_{yy}(x,0), \sigma_{xx}(x,0), \sigma_{xy}(x,0)\right] \\ [K_c, M_c, N_c] &= \lim_{x \to c^+} \sqrt{x-c}\left[\sigma_{yy}(x,0), \sigma_{xx}(x,0), \sigma_{xy}(x,0)\right] \end{aligned}\right\} \tag{5.7}$$

$$N_b = N_c = 0 \tag{5.8}$$

Since $\sigma_{yy}(x,0)$ is in crack opening mode, therefore, it is more important in fracture mechanics. Now we use (5.3) in (5.7) and evaluate the limits we get,

$$K_b = \frac{\Delta_1(b)}{\psi_1(b)},\quad K_c = -\frac{\Delta_1(c)}{\psi_1(c)},\quad \psi_1(x) = [q\sin(qx)G(b,c)]^{1/2}$$

(5.9)

while $\Delta_1(x)$ will be obtained from (5.4). $F_5(x)$ does not possess the singularity at crack tips, therefore in limit it contributors zero.

Solution of Fredholm Integral Equation for Special Loading

We consider the case for constant and uniform pressure acting over crack surface. Let

$$p(x) = p_0 = \text{constant} \tag{6.1}$$

Then $\Delta_0(t) = p_0 \Delta_{01}(t)/a$ (6.2)

$$\Delta_{01}(t) = \left[-\frac{a}{2} + G(b,t) + D \right] \tag{6.3}$$

we take for first approximation as

$$g(t) = p_0 \Delta_0(t)/\theta(t) \tag{6.4}$$

$$D = \frac{a}{2} - \left[G(b,c) E\left(\frac{\pi}{2}, \mu\right) + 2 \frac{\sin\left(\frac{qb}{2}\right)}{\sin\left(\frac{qc}{2}\right)} \Pi\left(\frac{\pi}{2}, \mu_1, \mu\right) \right] / \left[\sin\left(\frac{qb}{2}\right) F\left(\frac{\pi}{2}, \mu\right) \right]$$

Then, (6.5)

$$\mu^2 = G(b,c)/G(0,c), \mu_1^2 = \frac{G(b,c)}{G(0,b)} \tag{6.6}$$

Where F, E and Π are complete elliptic integrals of first, second and third type, respectively, see Gratshteyn & Rizhik [18]. We make a substitution in the integrals which have limit of integration as (b, c),

$$G(0,t) = G(0,c) - G(b,c)\sin^2\theta \tag{6.7}$$

Then limit changes to $(0, \pi/2)$. The integrals becomes numerically tractable. The limit of interval is divided into m equal intervals defined as,

$$p_m = p_{m-1} + \frac{\pi}{2m}, m = 1, 2, \ \ldots..\infty \tag{6.8}$$

Thus,

$p_0 = t_0 = 0$.

The numerical solution of (4.6) for special loading gives m linear equations as

$$A_{ij} g_i(p_j) = -\beta_j, A_{ij} = \frac{K(p_i, t_j)}{a^2 \theta(p_i)}, i, j = 1, 2, 3 \ldots m \tag{6.9}$$

$$A_{ii} = \frac{K_i}{a^2 \theta(p_i)}, i = 1, 2, 3 \ldots m$$

With (6.10)

and $K_i = K(p_i, t_i), \beta_j = \Delta_0(t_j)/a^2\theta(t_j), b < t < c$ (6.11)

Thus knowing $g(t)$ from (6.9) – (6.11) we can easily plot crack shape from (5.1) with $\eta = 0.25.$ Similarly the stress-intensity factors are evaluated from (5.10) – (5.11) and $\Delta_1(x)$ from (5.4). The numerical results are shown graphically in next section. We could have used the method of Fox and Goodwin [19].

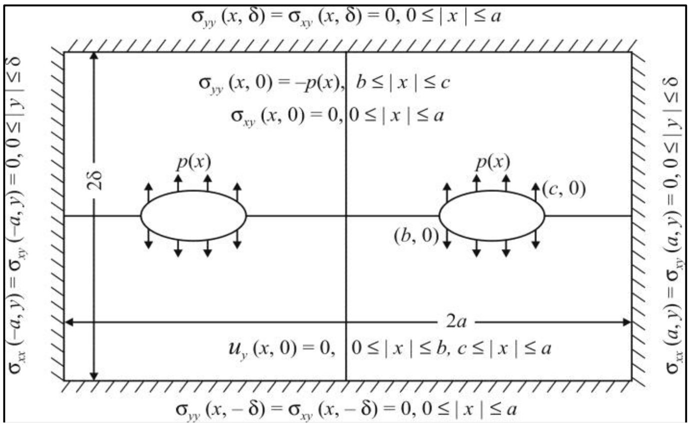

Fig 1: Geometry of Problem.

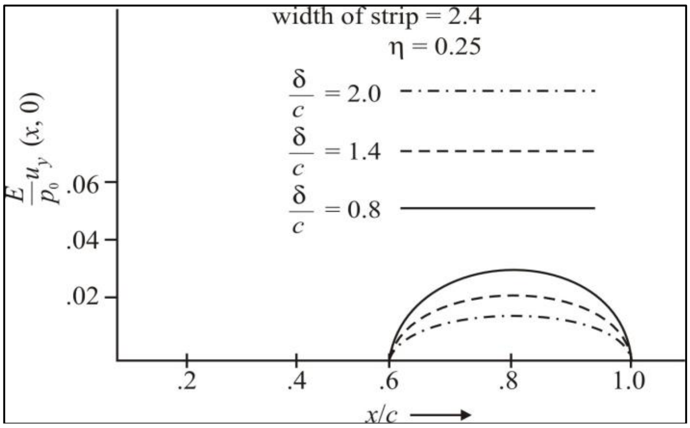

Fig 2: Crack shape $\frac{E}{p_0}u_y(x,0)$ against $\frac{x}{c}$ is plotted for crack length 0.4.

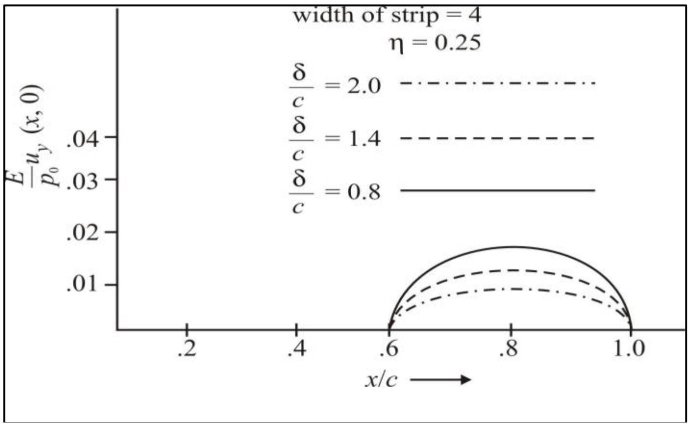

Fig 3: Crack shape $\frac{E}{p_0}u_y(x,0)$ is plotted against x/c with crack length 0.4.

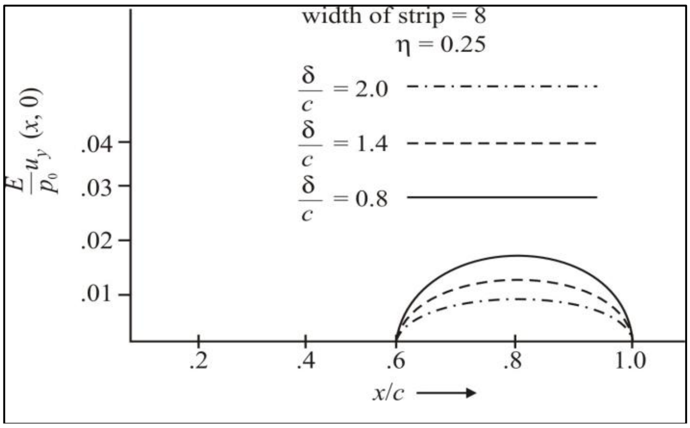

Fig 4: Crack shape $\frac{E}{p_0}u_y(x,0)$ is plotted against x/c with crack length 0.4.

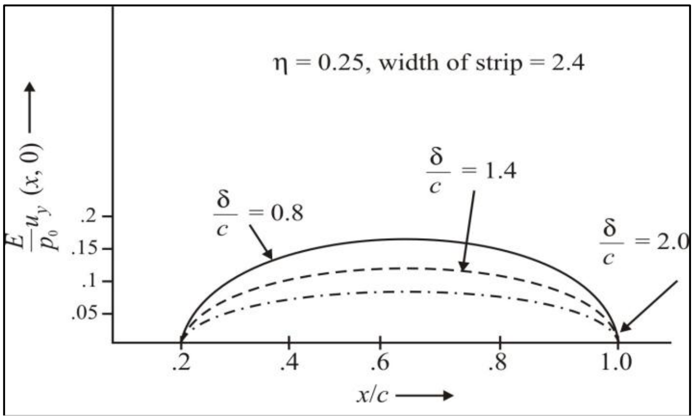

Fig 5: Crack shape $\frac{E}{p_0} u_y(x,0)$ is plotted against x/c with crack length 0.8.

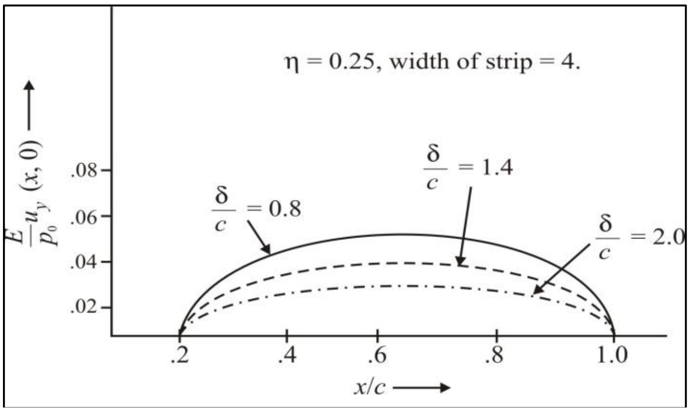

Fig 6: Crack shape $\frac{E}{p_0} u_y(x,0)$ is plotted against x/c with crack length 0.8.

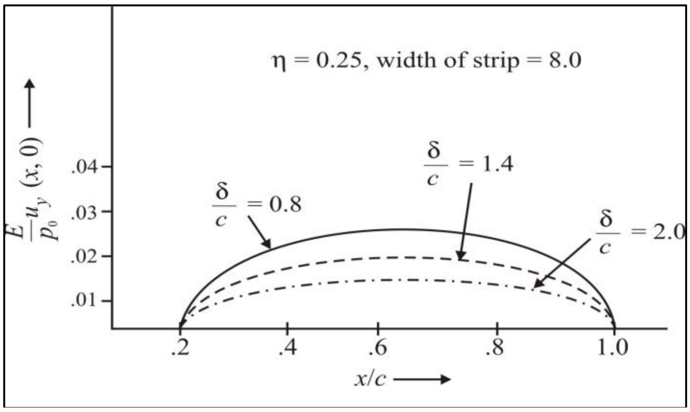

Fig 7: Crack shape $\frac{E}{p_0} u_y(x,0)$ is plotted against x/c with crack length 0.8**.**

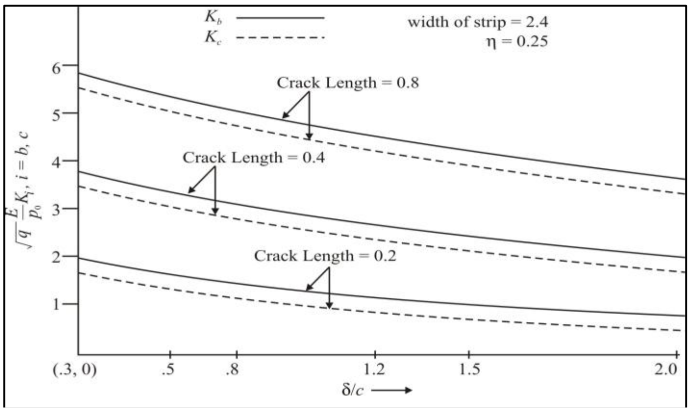

Fig 8: $\sqrt{q}\,\frac{E}{p_0} K_i, i = b, c$ are plotted against δ / c width of strip as 2.4.

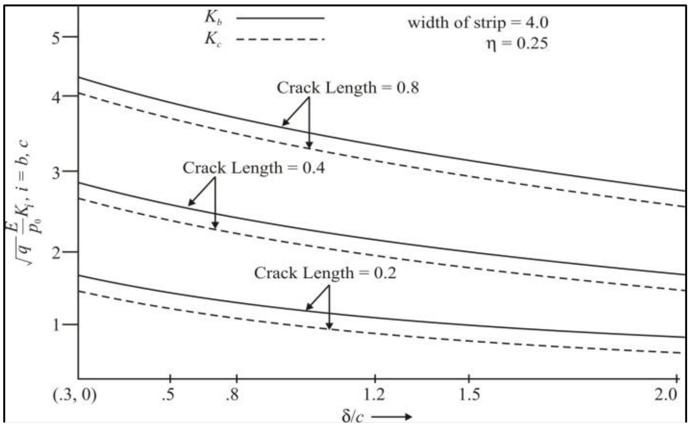

Fig 9: $\sqrt{q}\frac{E}{p_0}K_i, i = b, c$ are plotted against δ/c with width of strip as 4.0.

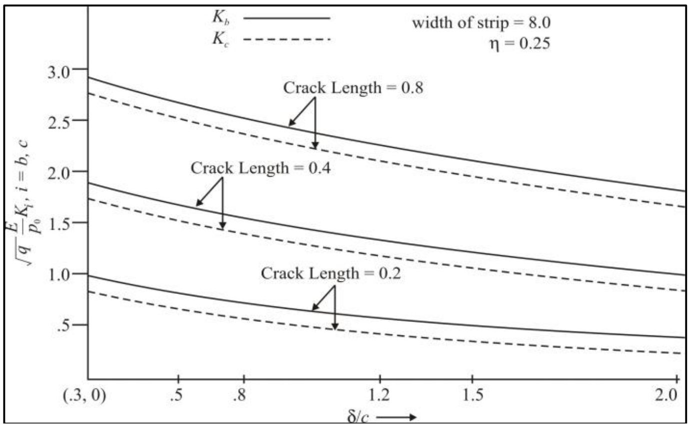

Fig 10: $\sqrt{q}\frac{E}{p_0}K_i, i = b, c$ are plotted against δ/c with width of strip as 8.0.

Discussion and Conclusion

Discussion

D_1 In getting the solution of equations of equilibrium (2.1) we used the Principle of Linear Superposition [1].

D_2 For getting the solution of Fredholm integral equation of second kind (4.6), first we expanded the non-singular kernel $K(\alpha, t)$ interms of $\{e^{-nq\delta}\}$, $n = 1, 2, \ldots m$ and then evaluated certain integrals and summed some series involved there.

D_3 When $\delta/c > 1$ we need only upto $n = 1, 2, 3, 4$ which gives fast convergence while stability of solution is also maintained. When $\delta/c < 1$ we need more terms and convergence is also slow.

D_4 We used Simpson's one third rule for numerical evaluation of integrals.

D_5 The linear system of equations given by (6.9) – (6.11) are solved by Gruss-Jordan Method.

Conclusion

C_1 As δ/c (thickness) increases the crack opening is becoming smaller (Figures 2, 3, 4)

C_2 When crack length is small then crack opening displacement is also small (Fig. 2, 3, 4, 5, 6, 7).

C_3 When length of rectangle i.e. $2a$ increases from 1, 2, 4, 8 then trend of C_2 follows. (Figures 1-7).

C_4 (a) When crack length is small the stress-intensity factors K_b, K_c are also small correspondingly.

(b) As δ/c increases the stress-intensity factors decreases. (Figures 8, 9, 10).

C_5 As $2a$ increases the stress intensity factors decreases. (Figures 8, 9, 10).

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