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# Statistical properties of the periodogram for two vector-valued stability series with missed observations

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**Abstract**
The asymptotic properties of the periodogram and the spectral density function using data window for two vector-valued stability series are investigated. Some statistical properties of covariance estimation function with missing observations are studied. The application will be studied in the economy sector.

**Keywords:** Discrete time stability processes, modified periodogram, spectral density, spectral measure, Auto covariance and data window

## 1. Introduction

Our method of proceeding is to derive the asymptotic moments of: the periodogram $I_{ab}^{(T)}(\lambda)$, the matrix of sample measures $F_{xx}^{(T)}(\lambda)$ and the matrix of sample spectral densities $f_{xx}^{(T)}(\lambda)$. Our work is mainly based on the properties of the data windows and the matrix of second order smoothing modified periodogram. The properties of the smoothing periodogram using a weight function or data window were discussed in [1, 3, 7]. We see that the idea of smoothing the periodograms using data window is an important tool in the spectral analysis of time series. Our purpose is to compare the classic results in the spectral analysis of (r+s) vector valued strictly stability time series, where all observations are available and the case where some of the observations are randomly missed, using data window.

The paper is organized as follows: Section1. Introduction, sstatistical properties of the periodogram for two vector-valued stability series will be in investigate in Section 2, in section 3 the Asymptotic properties of the Spectral Measure Matrix was discussed, the Asymptotic Moments of the Spectral Density Matrix were discussed in section 4, finally we will apply our theoretical study in the economy sector in section 5.

## 2. Statistical Properties of the Periodogram for Two Vector-Valued Stability Series

In this section we will investigate the sstatistical properties of the periodogram for two vector-
In this section we will investigate the sstatistical properties of the periodogram for two vector-
valued stability series: Consider an $(r+s)$ vector-valued stability series

$$Z(t) = [X(t) \quad Y(t)]^T \tag{2.1}$$

$t = 0, \pm 1, \pm 2, \ldots$ with $X(t)$ r- vector-valued and $Y(t)$ s- vector-valued. We assume that the series (2.1) is a strictly stability $(r+s)$ vector-valued series with components $[X_j(t) \quad Y_i(t)]^T$, $j = 1, 2, \ldots, r$, $i = 1, 2, \ldots, s$ all of whose moments exist, and we define the means

$$EX(t) = c_x, \quad EY(t) = c_y, \tag{2.2}$$

The covariances,

$$E\left\{[X(t+u)-c_x][X(t)-c_x]^T\right\}=c_{xx}(u),$$
$$E\left\{[X(t+u)-c_x][Y(t)-c_y]^T\right\}=c_{xy}(u), \quad (2.3)$$
$$E\left\{[Y(t+u)-c_y][Y(t)-c_y]^T\right\}=c_{yy}(u),$$

The second-order spectral densities can be derived as,

$$f_{xx}(\lambda)=(2\pi)^{-1}\sum_{u=-\infty}^{\infty}c_{xx}(u)\,Exp\,(-i\lambda u),$$
$$f_{xy}(\lambda)=(2\pi)^{-1}\sum_{u=-\infty}^{\infty}c_{xy}(u)\,Exp\,(-i\lambda u), \quad (2.4)$$
$$f_{yy}(\lambda)=(2\pi)^{-1}\sum_{u=-\infty}^{\infty}c_{yy}(u)\,Exp\,(-i\lambda u), \quad for\ -\infty<\lambda<\infty$$

**Assumption I.** Let $X(t)$ is a strictly stability series all of whose moments exist. For each $j=1,2,\ldots,k-1$ and any k-tuple $a_1,a_2,\ldots,a_k$ we have

$$\sum_{u_1,\ldots,u_{k-1}}\left|u_j\,c_{a_1,\ldots,a_k}(u_1,\ldots,u_{k-1})\right|<\infty,\quad k=2,3,\ldots$$

Where

$c_{a_1,\ldots,a_k}(u_1,\ldots,u_{k-1})=cum\,\{X_{a_1}(t+u_1),\,X_{a_1}(t+u_2),\ldots,\,X_{a_k}(t)\,,a_1,a_2,\ldots,a_k=\overline{1,r},\ \ ,t\in R,$

$u_1,u_2,\ldots,u_{k-1}\ \ k=2,3,\ldots$

**Assumption II.** Let $W(\alpha),\ \alpha\in[-\pi,\pi]$ be a weight function that is bounded, is symmetric about 0 and has bounded first derivative such that

$$\sum_{\alpha}W(\alpha)=1,$$

Given $B_T>0$ we then set

$W^{(T)}(\alpha)=B_T^{-1}W^{(T)}(B_T^{-1}\alpha).$

**Assumption III:** Let $d_a^{(T)}(t),\ t\in R,\ \ a=\overline{1,r}$ be bounded, is of bounded variation and vanishes for $t>T-1,\ t<0$, that is called data window.

Let

$$G_{a_1,\ldots,a_k}(\lambda)=\sum_{t=0}^{T-1}\left[\prod_{j=1}^{k}d_{a_j}^{(T)}(t)\right]\exp\left\{-i\lambda t\right\},$$

for $-\infty<\lambda<\infty$ and $a_1,\ldots,a_k=\overline{1,r}$..

Let $h_a^{(T)}(\lambda)$ be the discrete expanded finite Fourier transform is defined by

$$h_a^{(T)}(\lambda)=\left[2\pi\sum_{t=0}^{T-1}\left(d_a^{(T)}(t)\right)^2\right]^{-1/2}\ell_{\psi_a}^{(T)}(\lambda),\quad -\infty<\lambda<\infty \quad (2.5)$$

Where

$$\ell_{\psi_a}^{(T)}(\lambda)=\begin{bmatrix}\ell_{X_a}^{(T)}(\lambda)\\ \ell_{Y_a}^{(T)}(\lambda)\end{bmatrix}=\sum_{t=0}^{T-1}d_a^{(T)}(t)\psi_a(t)Exp\,\{-i\lambda t\}, \quad (2.6)$$

$$\psi_a(t)=B_a(t)Z_a(t), \quad (2.7)$$

$a = 1, 2, \ldots, \min(r, s)$, $X_a(t), Y_a(t)$ are the observations on the stable stochastic processes and $B_a(t)$ is Bernoulli sequence of random variable which is stochastically independent of $X_a(t), Y_a(t)$ and satisfies

$$B_a(t) = \begin{cases} 1 & , \text{ if } X_a(t), Y_a(t) \text{ are observed} ; \\ 0 & , \text{ otherwise} \end{cases} \tag{2.8}$$

Let $B_a(t)$ be independent and identically distributed random variables with

$$\begin{aligned} P[B_a(t) = 1] &= p_a \\ P[B_a(t) = 0] &= q_a \end{aligned} \tag{2.9}$$

where $p_a + q_a = 1$, see [9-11].

**Theorem 2.1**

Let $\psi_a(t) = B_a(t) Z_a(t)$, $a = 1, 2, \ldots, \min(r, s)$ are missed observations on the stable stochastic processes $X_a(t), Y_a(t)$, $a = 1, 2, \ldots, \min(r, s)$ and $B_a(t)$ is Bernoulli sequence of random variables which satisfies equations (2.8), (2.9).

Then

$$E\{\psi_a(t)\} = 0, \tag{2.10}$$

$$Cov\{\psi_{a_1}(t_1), \psi_{a_2}(t_2)\} = p_{a_1 a_2} \begin{bmatrix} c_{xx}(u) & c_{xy}(u) \\ c_{yx}(u) & A(\alpha) c_{xx}(u) A(\alpha)^T \end{bmatrix}, \tag{2.11}$$

The theorem are proved [13].

**Theorem 2.2**

Let $\psi_a(t) = B_a(t) Z_a(t)$, $a = 1, 2, \ldots, \min(r, s)$ are missed observations on the strictly stability discrete series which satisfy Assumption I with mean zero, $d_a(t)$, $-\infty < u < \infty$ satisfy Assumption III for $a = 1, 2, \ldots, \min(r, s)$, and let

$$I_{\psi\psi}^{(T)}(\lambda) = I_{ab}^{(T)}(\lambda) = \left[\left\{2\pi G_{ab}^{(T)}(0)\right\}^{-1} \ell_{\psi_a}^{(T)}(\lambda) \overline{\ell_{\psi_b}^{(T)}(\lambda)}\right], \tag{2.12}$$

where the bar denotes the complex conjugate. Then,

$$E\left[I_{ab}^{(T)}(\lambda)\right] = P_{ab} \begin{bmatrix} f_{a_1 a_2}(\lambda) & f_{a_1 b_2}(\lambda) \\ f_{b_1 a_2}(\lambda) & A(\lambda) f_{a_1 a_2}(\lambda) A(\lambda)^T \end{bmatrix} + \begin{bmatrix} O(T^{-1}) & O(T^{-1}) \\ O(T^{-1}) & O(T^{-1}) \end{bmatrix}, \tag{2.13}$$

where $O(T^{-1})$ is uniform in $\lambda$. And,

$$Cov\left[I_{a_1 b_1}^{(T)}(\lambda), I_{a_2 b_2}^{(T)}(\mu)\right] = \left\{G_{a_1 b_1}^{(T)}(0) G_{a_2 b_2}^{(T)}(0)\right\}^{-1} \times \left[(P^4 G_{a_1 a_2}(\lambda - \mu) \overline{G_{b_1 b_2}(\lambda - \mu)} \Phi \mathrm{H} + \right.$$

$$\left. + P^4 G_{a_1 b_2}(\lambda + \mu) \overline{G_{b_1 a_2}(\lambda + \mu)} \Phi \mathrm{H}\right] + T^{-2} M_{a_1 b_1 a_2 b_2}^{(T)}(\lambda, \mu) + O(T^{-1}), \quad (2.14)$$

Where,

$$\Phi = \begin{bmatrix} f_{a_1 a_2}(\lambda) & f_{a_1 b_2}(\lambda) \\ f_{b_1 a_2}(\lambda) & A(\lambda) f_{a_1 a_2}(\lambda) A(\lambda)^T \end{bmatrix}, \ \mathrm{H} = \begin{bmatrix} f_{a_1 a_2}(-\lambda) & f_{a_1 b_2}(-\lambda) \\ f_{b_1 a_2}(-\lambda) & A(\lambda) f_{a_1 a_2}(-\lambda) A(\lambda)^T \end{bmatrix}.$$

**Lemma 2.1.** [2]

If the data window function $d_a^{(T)}(t),\ t \in R,\ a = \overline{1,r}$ is bounded and has bounded variation and vanishes for $t > T-1,\ t < 0$, then

$$\sum_{t=0}^{T-1} d_a^{(T)}(t) \quad \sim \quad T\int_0^1 d_a^{(T)}(u)du\ , \tag{2.15}$$

where ~ is defined as

$$\frac{1}{T}\sum_{t=0}^{T-1} d_a^{(T)}(t) \quad \xrightarrow[T\to\infty]{} \quad \int_0^1 d_a^{(T)}(u)du\ , \tag{2.16}$$

for $a = \overline{1,r},\ T = 1,2,\dots$ .

**Lemma 2.2.** [2]

Suppose that $d_a^{(T)}(t), t \in R,\ a = \overline{1,r}$ be bounded by a constant L and satisfying the Lipschitz condition,

$$\left| d_a^{(T)}(t+u) - d_a^{(T)}(t) \right| \le c\left|u\right|, \tag{2.17}$$

for some constants $c,\ a = \overline{1,r},$ then,

$$\tau = \left| \sum_{t=0}^{T-1} d_{a_1}^{(T)}(t+u) d_{a_2}^{(T)}(t)\exp(-i\lambda t) - \sum_{t=0}^{T-1} d_{a_1}^{(T)}(t) d_{a_2}^{(T)}(t)\exp(-i\lambda t) \right| \le Lc\left|u\right|, \tag{2.18}$$

$-\infty < \lambda < \infty,\ u = \overline{-T,T},\ a_1, a_2 = \overline{1,r}$ .

**Lemma 2.3** [2]

Suppose that $d_a^{(T)}(t), t \in R,\ a = \overline{1,r}$ be bounded by a constant L and satisfying condition (2.17) then,

$$\left| \sum_{t=0}^{T-1} d_{a_1}^{(T)}(t) d_{a_2}^{(T)}(t)\exp(-i\lambda t) \right| \le \frac{1}{\left|\lambda/2\right|} + Lc\ , \tag{2.19}$$

for some constants $L, c$ and a point $\lambda, \lambda \in R, \lambda \ne 0, a_1, a_2 = \overline{1,r}$ .

**Corollary 2.1.**

Let $\psi_a(t) = B_a(t)Z_a(t),\ \ a = 1,2,\dots,\min(r,s)$ are missed observations on the strictly stability discrete series which satisfies Assumption I with mean zero and Let $d_a(t), -\infty < t < \infty$ be data window satisfy Assumption III for $a = 1,\dots,\min(r,s)$ .

Let

$$I_{\psi\psi}^{(T)}(\lambda) = \left[I_{ab}^{(T)}(\lambda)\right] = \left[\left\{2\pi\, G_{ab}^{(T)}(0)\right\}^{-1} \ell_a^{(T)}(\lambda)\overline{\ell_b^{(T)}(\lambda)}\right],$$

then

$$E\left[I_{ab}^{(T)}(\lambda)\right] \to p_{a_1a_2}\begin{bmatrix} f_{a_1a_2}(\lambda) & f_{a_1b_2}(\lambda) \\ f_{b_1a_2}(\lambda) & A(\lambda)\, f_{a_1a_2}(\lambda)A(\lambda)^T \end{bmatrix} \text{as } T \to \infty,\ a,b = 1,\dots,\min(r,s),\ \ \lambda \in R\ .$$

**Proof**

The prove comes directly from (2.13) by taking the limits for both sides and then using the given conditions.

In Corollary (2.2) below we make use of the Kroncker delta function

$$\delta(\lambda) = \begin{cases} 1 \quad , \ if \quad \lambda = 0, \\ 0 \quad , \ other \quad wise \ , \end{cases} \tag{2.20}$$

The statistical dependence of $I_{a_1b_1}^{(T)}(\lambda)$ and $I_{a_2b_2}^{(T)}(\mu)$ , $a_i, b_i = 1, 2, .., \min (r, s)$ , $i = 1, ..., k, , \lambda, \mu \in R$ is seen to fall off as the function $G_{ab}^{(T)}(\lambda)$ , $a, b = 1, 2, ..., \min (r, s)$, $\lambda \in R$ fall off.

**Corollary 2.2.**
Under the conditions of theorem (2.2) then,

$$\lim_{T \to \infty} Cov \left[I_{a_1b_1}^{(T)}(\lambda), I_{a_2b_2}^{(T)}(\mu)\right] = \begin{cases} P^4\delta(\lambda - \mu)\Phi H + P^4\delta(\lambda + \mu)\Phi H \ , & if \ \lambda \pm \mu = 0 \\ 0, & if \ \lambda \pm \mu \neq 0 \end{cases} \quad \text{for all}$$

$a_i, b_i = 1, 2, ... , \min (r, s)$ , $i = 1, 2$ , $\lambda, \mu \in R$

**Proof**
When $\lambda \pm \mu = 0$ , and by using the Assumption (III) then we get from (2.14)

$$Cov \left[I_{a_1b_1}^{(T)}(\lambda), I_{a_2b_2}^{(T)}(\mu)\right] = P^4\delta(\lambda - \mu)\Phi H + P^4\delta(\lambda + \mu)\Phi H + O(T^{-1})$$

In the limit, then,

$$Cov \left[I_{a_1b_1}^{(T)}(\lambda), I_{a_2b_2}^{(T)}(\mu)\right] = P^4\delta(\lambda - \mu)\Phi H + P^4\delta(\lambda + \mu)\Phi H \ ,$$

Now, when $\lambda \pm \mu \neq 0$ , $\lambda, \mu \in R$ , then take the modulus for both sides of (2.15) and then using lemma (2.3) and the boundedness of $f_{ab}(\lambda)$ , $a, b = 1, 2, ...,$ $\min (r, s)$, $\lambda \in R$ by constant $K$ , we obtain

$$\left|Cov \left[I_{a_1b_1}^{(T)}(\lambda), I_{a_2b_2}^{(T)}(\mu)\right]\right| \leq \left\{G_{a_1b_1}^{(T)}(0)G_{a_2b_2}^{(T)}(0)\right\}^{-1} \times$$
$$\times \left\{ \left[\frac{2L_1\nu_1}{|\sin(\lambda + \mu)/2|}\right]^2 K^2 + \left[\frac{2L_2\nu_2}{|\sin(\lambda - \mu)/2|}\right]^2 K^2 \right\} +$$
$$+ T^{-2}\left|T^{-2}M_{a_1b_1a_2b_2}^{(T)}(\lambda, \mu)\right| + (T^{-1}),$$

where, for some constant $K$ , we have

$$\left|T^{-2}M_{a_1b_1a_2b_2}^{(T)}(\lambda, \mu)\right| \leq K\left\{\left[\frac{2L_1\nu_1}{|\sin(\lambda + \mu)/2|}\right] + \left[\frac{2L_2\nu_2}{|\sin(\lambda - \mu)/2|}\right]\left[\frac{2L_3\nu_3}{|\sin(\lambda + \mu)/2|}\right] + \left[\frac{2L_4\nu_4}{|\sin(\lambda - \mu)/2|}\right]\right\},$$

by using lemma (2.1) we get $Cov \left[I_{a_1b_1}^{(T)}(\lambda), I_{a_2b_2}^{(T)}(\mu)\right] = O(T^{-1}) \to 0$ as $T \to \infty$ . Hence, the corollary is obtained. In the case of $\lambda = \pm\mu$ corollary (2.2) indicates the following corollary.

**Corollary 2.3.**
Under the conditions of theorem (2.2) and corollary (2.2) then,

$$\lim_{T \to \infty} D[I_{ab}^{(T)}(\lambda)] = \begin{cases} P^4\delta(\lambda - \mu)\Phi H \ , & if \ \lambda = \mu = \omega \neq 0 \\ P^4\delta(\lambda - \mu)\Phi H + P^4\delta(\lambda + \mu)\Phi H \ , & if \ \lambda = \mu = \omega = 0 \end{cases}$$

**Proof**
by substituting about $\lambda = \mu = \omega, \ \omega \in R, \ a_1 = a_2 = a, b_1 = b_2 = b, \ a, b = 1, 2, ...$ , $\min (r, s)$ into corollary (2.2),we get**,**

$$\lim_{T \to \infty} D[I_{ab}^{(T)}(\lambda)] = P^4\delta(\omega - \omega)\Phi H + P^4\delta(\omega + \omega)\Phi H \ .$$

When $\omega \neq 0$ ,by noting that $f_{ab}(\omega) = f_{ba}(-\omega)$ into $\Phi$ , H , $a, b = 1, \dots, \min(r, s)$ , $\omega \in R$ then,

$$\underset{T \to \infty}{Lim} \; D[I_{ab}^{(T)}(\lambda)] = P^4 \delta(\omega - \omega) \Phi \mathrm{H} .$$

When $\omega = 0$ then we obtain

$$\underset{T \to \infty}{Lim} \; D[I_{ab}^{(T)}(\lambda)] = 2P^4 \Phi \mathrm{H}$$

Hence the proof is complete.

**3. Asymptotic Properties of the Spectral Measure Matrix for Two Vector-Valued Stability Series**

We will study the properties of the spectral measure matrix for two vector-valued stability
Series in this section as follow,

Let the series $Z(t) = Z_a(t), \; a = 1, ..., \; \min \; (r, s), \; t \in R$ have a spectral density matrix which is defined as $f_{zz}(\lambda), \lambda \in R$ the matrix of a spectral measure is given by :

$$F_{z_a z_b}(\lambda) = \int_{-\infty}^{\lambda} f_{z_a z_b}(\alpha) d\alpha , \quad a, b = 1, 2, \dots, \min \; (r, s), \tag{3.1}$$

in view of (3.1) we can consider estimating $F_{z_a z_b}(\lambda), \lambda \in R$ by

$$F_{z_a z_b}^{(T)}(\lambda) = \int_{-\infty}^{\lambda} I_{z_a z_b}(\alpha) d\alpha , \tag{3.2}$$

where $I_{ab}^{(T)}(\lambda)$ , is defined as (2.10).

Now we will determine the asymptotic properties of $F_{z_a z_b}(\lambda)$ according to the dependence of $F_{z_a z_b}^{(T)}(\lambda), \; \lambda \in R$ on $I_{z_a z_b}^{(T)}(\lambda), \lambda \in R$ .

**Theorem 3.1**

Let $\psi_a(t) = B_a(t) Z_a(t), \;\; t = 0, \pm 1, ...$ are missed observations on the strictly stability $(r+s)$ vector valued discreet time series $Z_a(t), \;\; a = 1, \dots, \; \min \; (r, s), \; t \in R$ strictly stability and let $I_{ab}^{(T)}(\lambda), \;\; a = 1, \dots, \; \min \; (r, s), \; t \in R$ be defined by (2.10), $d_a^{(T)}(t)$ , $a = 1, \dots, \; \min \; (r, s)$, $\; t \in R$ be data window and $F_{z_a z_b}(\lambda)$ be given by (3.1) then for all $a_i, b_i = 1, 2, ..., \;\; \min \;\; (r, s)$ , $\; i = 1,.., k$ , then,

$$E\left[F_{ab}^{(T)}(\lambda)\right] = P^2 \begin{bmatrix} F_{a_1 a_2}(\lambda) & F_{a_1 b_2}(\lambda) \\ F_{b_1 a_2}(\lambda) & A(\lambda) F_{a_1 a_2}(\lambda) A(\lambda)^T \end{bmatrix} + \begin{bmatrix} O(T^{-1}) & O(T^{-1}) \\ O(T^{-1}) & O(T^{-1}) \end{bmatrix}, \tag{3.3}$$

where $O(T^{-1})$ is uniform in $\lambda$ .
and,

$$\operatorname{cov}\left\{F_{a_1 b_1}^{(T)}(\lambda_1), F_{a_2 b_2}^{(T)}(\lambda_2)\right\} = \left[\left\{G_{a_1 b_1}^{(T)}(0) G_{a_2 b_2}^{(T)}(0)\right\}^{-1}\right] \times P^4 G_{a_1 a_2 b_1 b_2}(0) \left[\int_{-\infty}^{\lambda_1} \Phi \mathrm{H} \;\; d\alpha + \int_{-\infty}^{\lambda_1} \Phi \mathrm{H} \;\; d\alpha \right] +$$
$$+ O(T^{-1}), \tag{3.4}$$

Where

$$\Phi = \begin{bmatrix} f_{a_1 a_2}(\lambda) & f_{a_1 b_2}(\lambda) \\ f_{b_1 a_2}(\lambda) & A(\lambda) f_{a_1 a_2}(\lambda) A(\lambda)^T \end{bmatrix}, \mathrm{H} = \begin{bmatrix} f_{a_1 a_2}(-\lambda) & f_{a_1 b_2}(-\lambda) \\ f_{b_1 a_2}(-\lambda) & A(\lambda) f_{a_1 a_2}(-\lambda) A(\lambda)^T \end{bmatrix}.$$

by using (2.12) and (2.13) the prove comes directly.

**Corollary 3.1**

Let $\psi_a(t) = B_a(t) Z_a(t),\ a = 1,\dots,\min\ (r,s)$ are missed observations on the strictly stability discrete series which satisfies Assumption I with mean zero and Let $d_a(t),\quad -\infty < t < \infty$, be data window satisfies Assumption III for $a = 1,\dots,\min\ (r,s)$ and let

$$I_{\psi\psi}^{(T)}(\lambda) = \left[I_{ab}^{(T)}(\lambda)\right] = \left[\left\{2\pi\, G_{ab}^{(T)}(0)\right\}^{-1} \ell_a^{(T)}(\lambda)\overline{\ell_b^{(T)}(\lambda)}\right],$$

Then

$$E\left[F_{ab}^{(T)}(\lambda)\right] \underset{T\to\infty}{\longrightarrow} P^2 \begin{bmatrix} F_{a_1a_2}(\lambda) & F_{a_1b_2}(\lambda) \\ F_{b_1a_2}(\lambda) & A(\lambda)F_{a_1a_2}(\lambda)A(\lambda)^T \end{bmatrix}, \tag{3.5}$$

for all $a,b = 1,\dots,\min\ (r,s)$.

**proof**

Formula (3.5) comes directly by taking the limits for both sides of (3.3) and the proof is complete.

The statistical dependence of $F_{a_1b_1}^{(T)}(\lambda_1)$ and $F_{a_2b_2}^{(T)}(\lambda_2)$ is seen to fall off as the function $G_{ab}^{(T)}(\lambda),\quad a,b = 1,\dots,\min\ (r,s)$ fall off. in taking the limit of (3.4) satisfies the following corollary**.**

**Corollary 3.2.**

Under the conditions of theorem (3.1) if the spectral density function $f_{ab}(x)$ is bounded by a constant $k$, $a,b = 1,\dots,\min\ (r,s)$ and continuous at point $x = \lambda$, $\lambda \in R$, then

$$\lim_{T\to\infty} Cov\left[F_{a_1b_1}^{(T)}(\lambda),\ F_{a_2b_2}^{(T)}(\mu)\right] = 0, \text{ for } a_j, b_j = 1,2,\dots,\min\ (r,s)\ \ j = 1,2,\dots,k,\ k = 1,2,\dots$$

**Proof**

Taking the modulus on both sides of (3.4), we get

$$\left|Cov\left[F_{a_1b_1}^{(T)}(\lambda), F_{a_2b_2}^{(T)}(\mu)\right]\right| \le (2\pi)\left|p^4\right| G_{a_1a_2b_1b_2}(0)\left\{G_{a_1b_1}^{(T)}(0) G_{a_2b_2}^{(T)}(0)\right\}^{-1}\left|P^4\right| G_{a_1a_2b_1b_2}(0) \times$$
$$\times \left[\int_{-\infty}^{\lambda_1} |\Phi||\mathrm{H}|\, d\alpha_1 + \int_{-\infty}^{\lambda_1} |\Phi||\mathrm{H}|\, d\alpha_1\right] + O(T^{-1}),$$

Using Assumption III and the boundedness of $f_{ab}(\lambda),\quad a,b = 1,\dots,\min\ (r,s),\ \lambda \in R$ we get

$$Cov\ \left[F_{a_1b_1}^{(T)}(\lambda), F_{a_2b_2}^{(T)}(\mu)\right] = O(T^{-1}) \underset{T\to\infty}{\longrightarrow} 0.$$

Then the corollary is obtained.

## 4. Asymptotic Moments of the Spectral Density Matrix for Two Vector-Valued Stability Series and Its Distributions

This section is concerned with constructing an estimate of $f_{zz}(\lambda)$, the matrix of second order spectral densities and study it's asymptotic moments.

Let $B_T$ be a scale vector depending on $T$. Suppose that $Z(t) = \{Z_a(t),\ a = \overline{1,(r+s)},\ t \in R$ is $r$-dimensional discrete time stability process with mean zero and let $W_{ab}^{(T)}(\alpha),\ \alpha \in R$ is a weight function which defined in Assumption II. as an estimate of $f_{zz}^{(T)}(\lambda)$, we propose

$$f_{zz}^{(T)}(\lambda) = \sum_{t=0}^{T-1} W^{(T)}(\lambda - \alpha)\mathrm{I}_{zz}^{(T)}(\alpha), \tag{4.1}$$

that is a weight average of the periodogram.

Now, we will determine the asymptotic moments of $f_{zz}^{(T)}(\lambda), \lambda \in R$ directly from theorem (2.2) because the elementary dependence of $f_{zz}^{(T)}(\lambda)$, $\lambda \in R$ on $I_{zz}^{(T)}(\lambda)$, $\lambda \in R$.

**Lemma 4.1**

Let $d_a^{(T)}(t), -\infty < t < \infty$, be data window satisfy Assumption III for $a = 1,..,\min(r,s)$, then $d_a^{(T)}(t)$ satisfies the following properties:

1. $\sum_{t_2=0}^{T-1} W_{a_2b_2}^{(T)}(\lambda_2 - \alpha_2) G_{a_1a_2}(\lambda - \mu) = 2\pi W_{a_2b_2}^{(T)}(\lambda_2 - \mu) + O(1).$
2. $\sum_{t_2=0}^{T-1} W_{a_2b_2}^{(T)}(\lambda_2 - \alpha_2) G_{a_1a_2}(\lambda + \mu)\overline{G_{b_1b_2}(\lambda + \mu)} = 2\pi\, G_{a_1a_2b_1b_2}^{(T)}(0) W_{a_2b_2}^{(T)}(\lambda_2 - \alpha_1) + O(T^{-1}).$

**Theorem 4.1**

Let $\psi_a(t) = Z_a(t) B_a(t)$, $t = 0, \pm 1,..,$ $a = 1,\ldots,\min(r+s)$ are missed observations on the strictly stability discrete series $Z_a(t)$, $a = 1,\ldots,\min(r+s)$, $t \in R$ which satisfies Assumption I with mean zero, $d_a(t)$, $a = 1,\ldots,\min(r+s)$, $t \in R$ be data window satisfies Assumption III and let $f_{zz}^{(T)}(\lambda) = \left\{ f_{ab}^{(T)}(\lambda), a,b = 1,\ldots,\min(r+s), \lambda \in R \right\}$ be given by (4.1) where $W(\alpha)$ satisfies Assumption II then,

$$E\left\{ f_{ab}^{(T)}(\lambda) \right\} = P^2 \sum_{t=0}^{T-1} W_{ab}^{(T)}(\lambda - \alpha) \begin{bmatrix} f_{a_1a_2}(\alpha) & f_{a_1b_2}(\alpha) \\ f_{b_1a_2}(\alpha) & A(\alpha) f_{a_1a_2}(\alpha) A(\alpha)^T \end{bmatrix} + \begin{bmatrix} O(T^{-1}) & O(T^{-1}) \\ O(T^{-1}) & O(T^{-1}) \end{bmatrix}, \quad (4.2)$$

and

$$\operatorname{cov}\left\{ f_{a_1b_1}^{(T)}(\lambda), f_{a_2b_2}^{(T)}(\lambda) \right\} = 2\pi P^4 \left\{ G_{a_1b_1}^{(T)}(0) G_{a_2b_2}^{(T)}(0) \right\}^{-1} G_{a_1a_2b_1b_2}(0) \times \left[ \sum_{t_1=0}^{T-1} W_{a_1b_1}^{(T)}(\alpha) W_{a_2b_2}^{(T)}(\lambda_2 - \lambda_1 + \alpha)\Phi\mathrm{H} + \right.$$

$$\left. + \sum_{t_1=0}^{T-1} W_{a_1b_1}^{(T)}(\alpha) W_{a_2b_2}^{(T)}\left(\frac{\lambda_2 + \lambda_1}{B_T} - \alpha\right)\Phi\mathrm{H} \right] + O(T^{-2}) \quad (4.3)$$

**Corollary 4.1**

Let $Z(t) = Z_a(t)$, $a = 1,\ldots,\min(r+s)$, $t \in R$ which satisfies Assumption **I** with mean zero and let $f_{zz}^{(T)}(\lambda) = \left\{ f_{ab}^{(T)}(\lambda), a,b = 1,\ldots,\min(r+s), \lambda \in R \right\}$ be given by (4.1) where $W(\alpha)$ satisfies Assumption **II** then,

$$E\left\{ f_{ab}^{(T)}(\lambda) \right\} \underset{T\to\infty}{\to} P^2 \begin{bmatrix} f_{a_1a_2}(\alpha) & f_{a_1b_2}(\alpha) \\ f_{b_1a_2}(\alpha) & A(\alpha) f_{a_1a_2}(\alpha) A(\alpha)^T \end{bmatrix},$$

if $\lambda \neq 0, \lambda \in R$ and $B_T \to 0$ as $T \to \infty$.

**Proof.**

The proof comes directly by taking the limits for both sides of formula (4.2) as $T \to \infty$.

**Corollary 4.2.**

Under the conditions of theorem (4.1) if the spectral density function $f_{ab}(x)$ is bounded by a constant $M$ **,** $a,b = 1,\ldots,\min(r+s)$ and continuous at a point $x = \lambda$, $\lambda \in R$ and $B_T \to 0$, $B_T T \to \infty$ as $T \to \infty$, then,

$$\operatorname{cov}\left\{ f_{a_1b_1}^{(T)}(\lambda), f_{a_2b_2}^{(T)}(\lambda) \right\} \xrightarrow[T\to\infty]{} 0,$$

for all $a_j, b_j = 1,\ldots,\min(r+s)$, $\lambda_j \in R$, $j = 1,\ldots,k$, $k = 1,2,..$

**Proof.**

Using Assumption III and the boundedness of $f_{ab}(\lambda)$ by constant $M$ ,we get

$$\operatorname{cov}\left\{f^{(T)}_{a_1b_1}(\lambda),\ f^{(T)}_{a_2b_2}(\lambda)\right\} = O(B_T^{-1}T^{-1}) \xrightarrow[T\to\infty]{} 0.$$

then the corollary is obtained.

## 5. Applications

We will apply our theoretical study on the production of "the national company for flour mills and feed".

### 5.1 Studying the quantity of wheat used and flour product

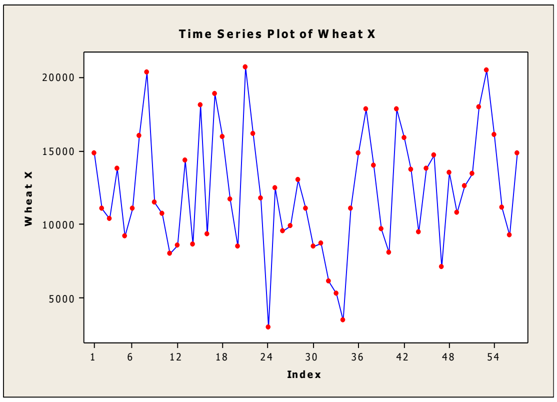
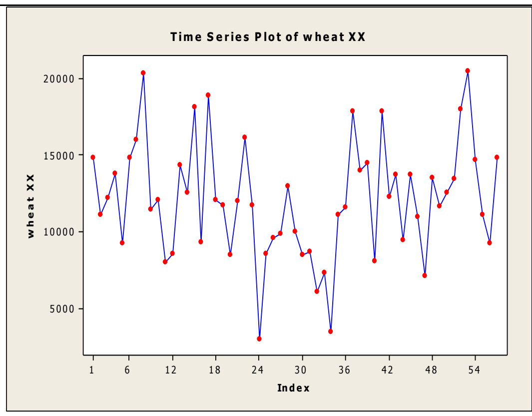
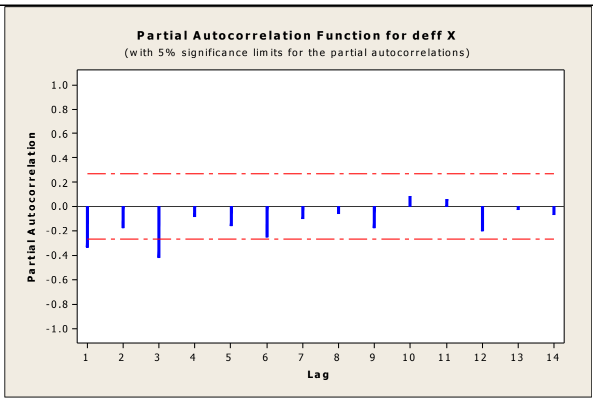
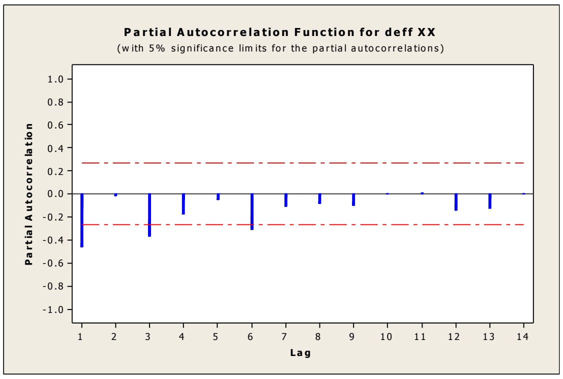
The data manipulated in this research are a monthly chronological series representing the average amount of wheat used and the amount of flour production in "the national company for flour mills and feed" for the period from January 2005 to September 2009.

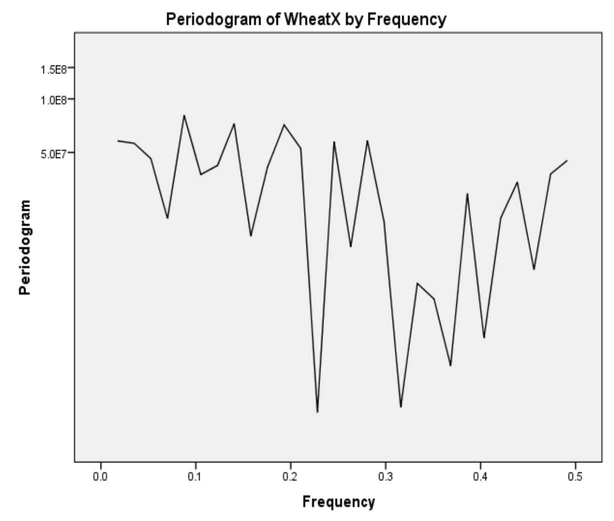
### 5.1.1 Studying the quantity of wheat used

In this study we will comparison between our results, model of strictly stability time series (wheat) with some missing observations and the classical results, where all observations are available.

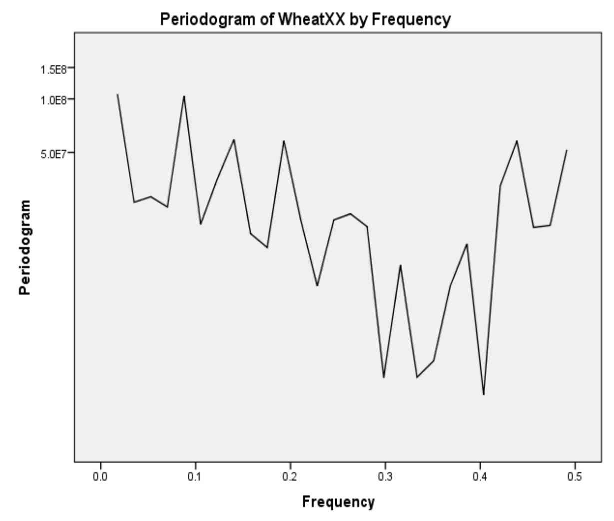
Let $X_a(t)$ be the data of the average amount of wheat used where all observations are available (classical case), suppose that there is some missing observations in a random way (our study), table 1 shows the comparison between our results, spectral analysis of strictly stability time series with some missing observations and the classical results, where all observations are available.

**Table 1:** The comparison of the results with and without missed observations of the average amount of wheat used

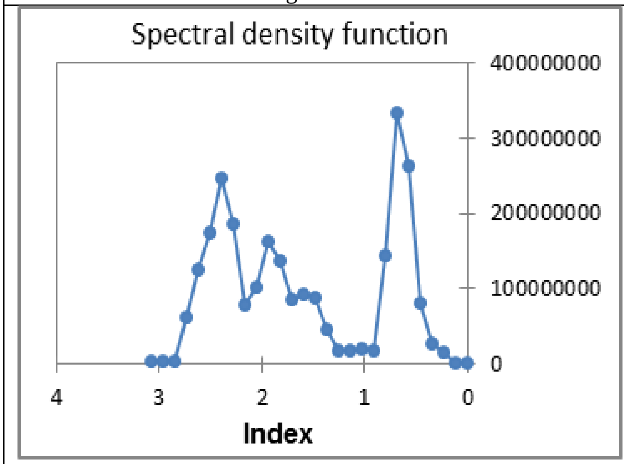
| Series without missed observations | Series with missed observations |
|---|---|
|   The Average wheat used per month |   The Average wheat used per month |
|   PACF of the first difference of wheat used |   PACF of the seasonal difference of wheat used |
| **ARIMA Model: Wheat X [ARIMA(1,1,1)]**<br>Final Estimates of Parameters<br>Type Coef SE Coef T P<br>AR 1 0.2336 0.1480 1.58 0.120<br>MA 1 0.9632 0.0827 11.64 0.00<br>Constant 10.61 31.04 0.34 0.734<br><br>Modified Box-Pierce (Ljung-Box) Chi-Square statistic<br><br>Lag 12 24 36 48<br>Chi-Square 7.2 13.5 20.8 29.7<br>DF 9 21 33 45<br>P-Value 0.620 0.888 0.952 0.961 | **ARIMA Model: Wheat X [ARIMA(1,1,1)]**<br>Final Estimates of Parameters<br>Type Coef SE Coef T P<br>AR 1 0.2433 0.1451 1.68 0.100<br>MA 1 0.9605 0.0678 14.16 0.000<br>Constant 1.98 25.97 0.08 0.939<br><br>Modified Box-Pierce (Ljung-Box) Chi-Square statistic<br><br>Lag 12 24 36 48<br>Chi-Square 8.4 24.3 40.4 52.1<br>DF 9 21 33 45<br>P-Value 0.496 0.277 0.175 0.217 |



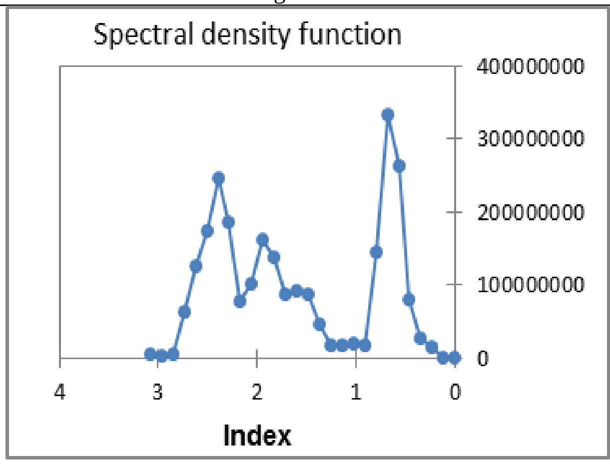
Periodogram of Wheat



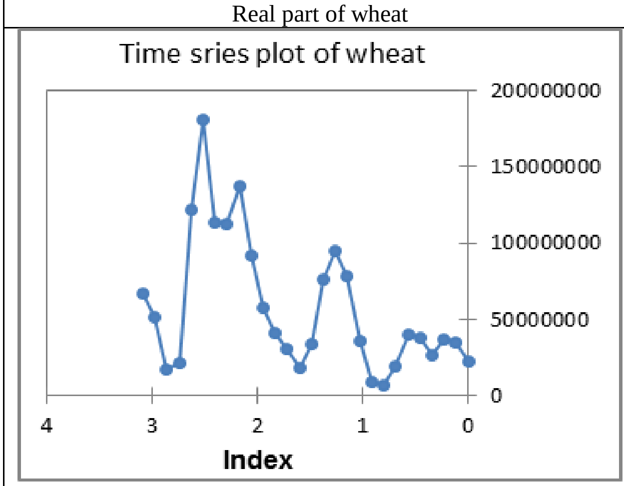
Periodogram of Wheat



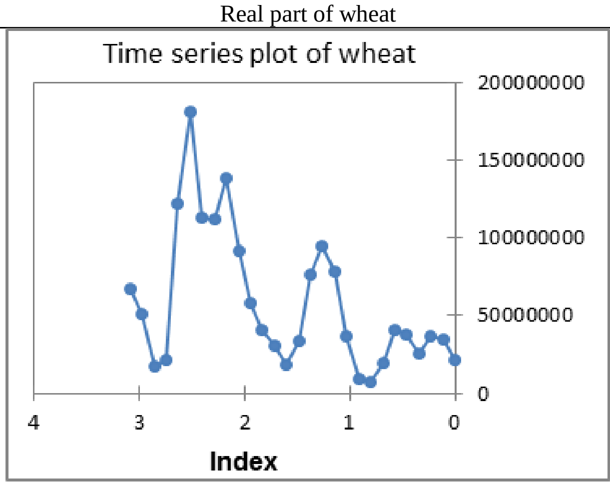
Real part of wheat



Real part of wheat



Imaginary part of wheat



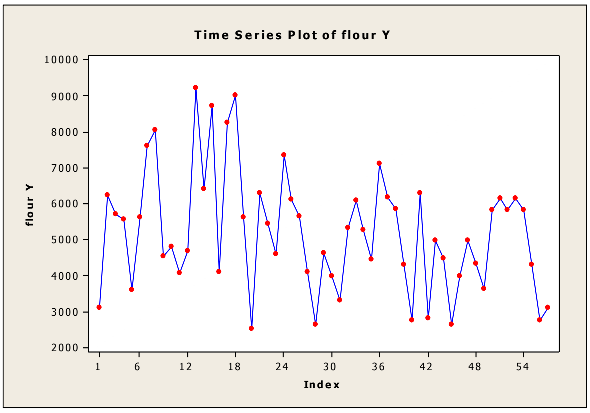
Imaginary part of wheat

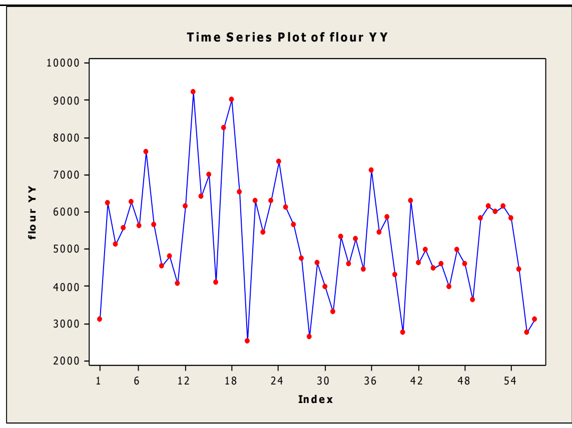
### 5.1.2 Studying the quantity of flour production

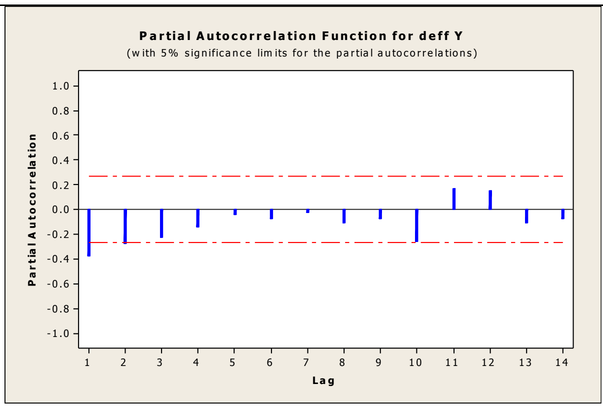
In this study we will comparison between our results, model of strictly stability time series (flour) with some missing observations and the classical results, where all observations are available.

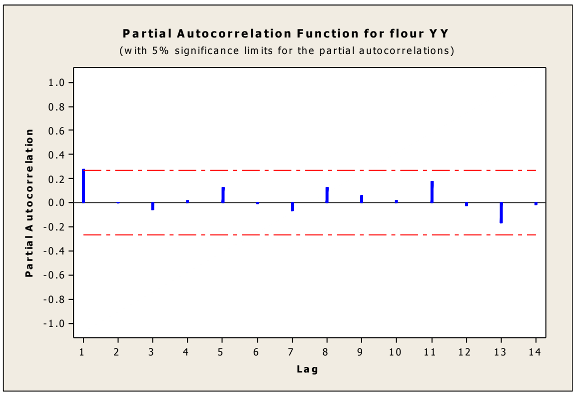
Let $Y_a(t)$ be the data of the amount of flour production, where all observations are available (classical case), suppose that there is some missing observations in a random way (our study), table 2 shows the comparison between our results, spectral analysis of strictly stability time series with some missing observations and the classical results, where all observations are available.

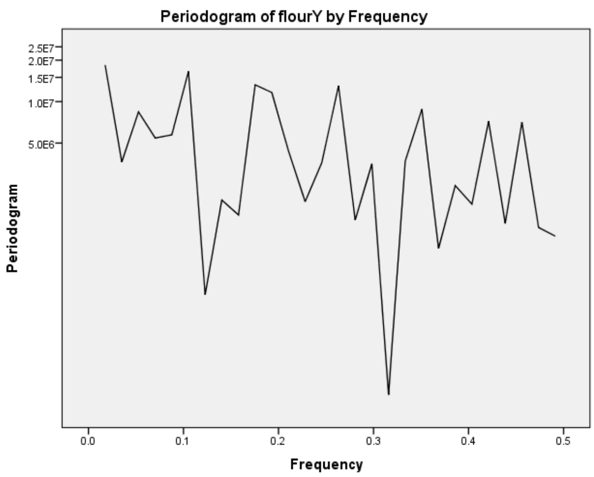
**Table 2:** The comparison of the results with and without missed observations of the amount of flour production

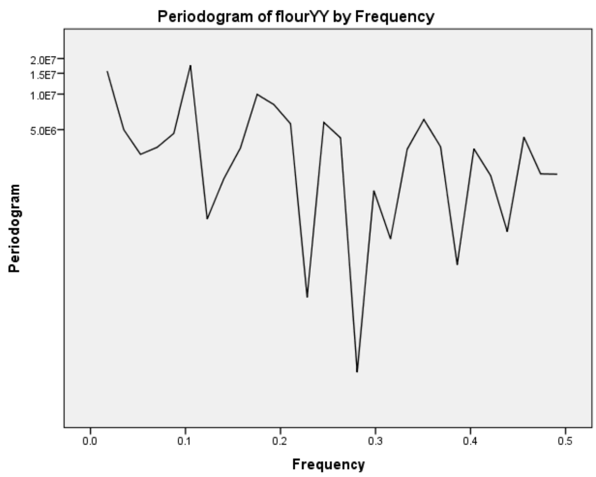
<table>
<tr><th>Series without missed observations</th><th>Series with missed observations</th></tr>
<tr><td>


The average monthly production of flour
</td><td>


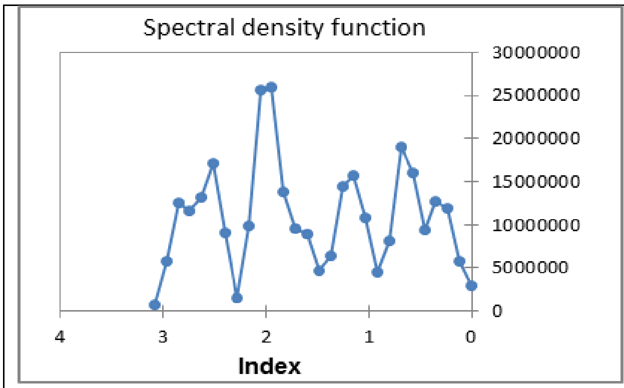
The Average monthly production of flour
</td></tr>
<tr><td>


PACF of the first difference of production of flour
</td><td>


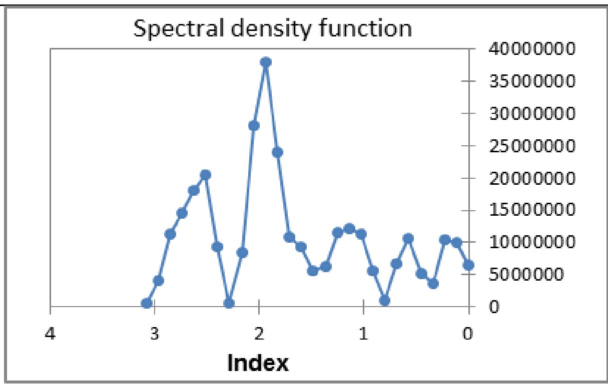
PACF of the first difference of production of flour
</td></tr>
<tr><td>
ARIMA Model: Flour Y [ARIMA(1,1,1)]<br>
Final Estimates of Parameters<br><br>
Type Coef SE Coef T P<br>
AR 1 0.1648 0.1442 1.14 0.258<br>
MA 1 0.9661 0.0834 11.59 0.000<br>
Constant -23.60 17.34 -1.36 0.179<br><br>
Modified Box-Pierce (Ljung-Box) Chi-Square statistic<br><br>
Lag 12 24 36 48<br>
Chi-Square 5.6 14.2 28.6 34.8<br>
DF 9 21 33 45<br>
P-Value 0.778 0.860 0.684 0.865
</td><td>
ARIMA Model: Flour YY [ARIMA(1,1,1)]<br>
Final Estimates of Parameters<br><br>
Type Coef SE Coef T P<br>
AR 1 0.2244 0.1436 1.56 0.124<br>
MA 1 0.9635 0.0817 11.80 0.000<br>
Constant -19.94 15.03 -1.33 0.190<br><br>
Modified Box-Pierce (Ljung-Box) Chi-Square statistic<br><br>
Lag 12 24 36 48<br>
Chi-Square 5.9 14.2 22.4 30.3<br>
DF 9 21 33 45<br>
P-Value 0.752 0.863 0.919 0.955
</td></tr>
<tr><td>


Periodogram of flour
</td><td>


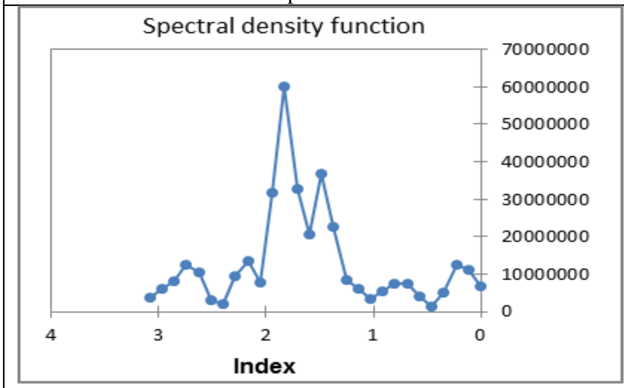
Periodogram of flour
</td></tr>
</table>



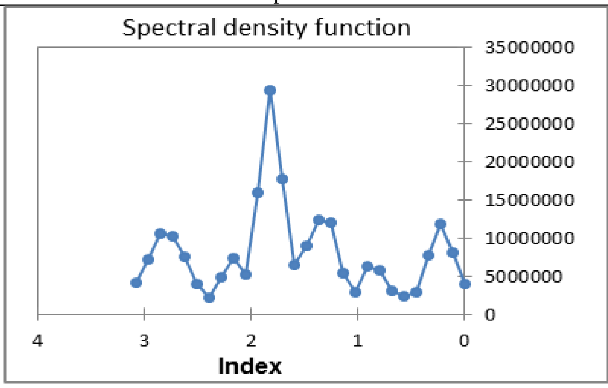

Real part of flour




Real part of flour




Imaginary part of flour




Imaginary part of flour

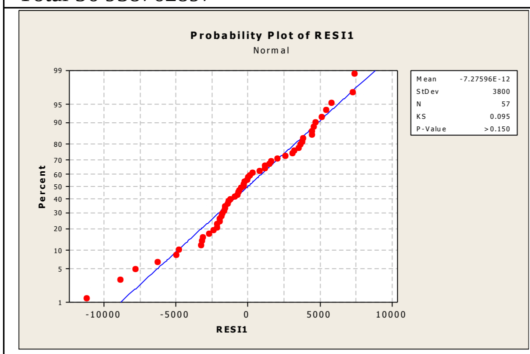
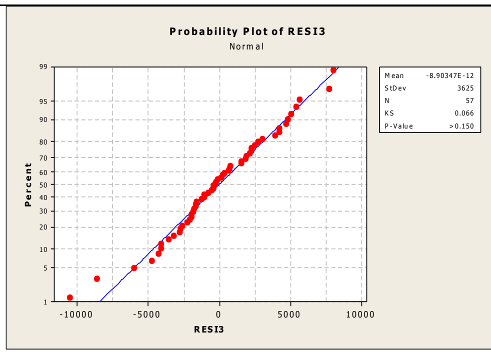
### 5.1.3 Studying the Regression between the Quantity of wheat used and flour product

In this section, we modify the regression model that represents the relationship between the average amount of wheat used and the amount of flour produced in tone from January 2005 to September 2009.

In this study we will comparison between our results with some missing observations and the classical results where all observations are available.

Let $Z(t) = [X(t) \quad Y(t)]^T$ where, $X(t)$ is the series of the average amount of flour produced and $Y(t)$ is the series of the average amount of wheat used, first we consider that the observations are available $P = 1, \psi(t) = B(t)Z(t) = pZ(t) = Z(t)$, then consider that there are some missing of observations randomly, $P = 0$. We used SPSS,MINITAB and XLSTAT to investigate our results which is shown in table 3.

**Table 3:** The comparison of the results with and without missed observations of the regression analysis

| Regression without missed observations | Regression with missed observations |
|---|---|
| The regression equation is<br>Wheat = 7453 + 0.918 flour | The regression equation is<br>wheat = 8357 + 0.698 flour |
| Predictor Coef SE Coef T P<br>Constant 7453 1686 4.42 0.000<br>flour 0.9184 0.3086 2.98 0.004 | Predictor Coef SE Coef T P<br>Constant 8357 1807 4.62 0.000<br>flour 0.6976 0.3289 2.12 0.038 |
| S = 3834.18 R-Sq = 13.9% R-Sq(adj) = 12.3% | S = 3658.26 R-Sq = 7.6% R-Sq(adj) = 5.9% |
| Analysis of Variance<br>Source DF SS MS F P<br>Regression 1 130212803 130212803 8.86 0.004<br>Residual Error 55 808550093 14700911<br>Total 56 938762897 | Analysis of Variance<br>Source DF SS MS F P<br>Regression 1 60200734 60200734 4.50 0.038<br>Residual Error 55 736056761 13382850<br>Total 56 796257495 |
|  Normal-plot of standardized Residuals |  Normal-plot of standardized Residuals |

## 6. Conclusion

1. Tables 1 and 2 show the study of time series with missed observations and the original time series and they have close results.
2. Table 3 show the study of regression model between the average amount of wheat used and the amount of flour produced with some missed observations which had the same results of the study of the classical regression model.

## 7. References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