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A novel approach of determining Stella Octangula number and Pronic number using initial value theorem in Z - Transform

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Abstract
In this communication, we evaluate Stella Octangula number and Pronic number by applying the initial value theorem in Z – Transform.

Keywords: Inverse Z – Transform, Pronic number, Stella Octangula number

Notations

$SO_n = n(2n^2 - 1)$ = Stella Octangula number of rank n .

$\text{Pro}_n = n(n+1)$ = Pronic number of rank n.

Introduction

Life is full of patterns; especially Mathematics is also full of patterns. The main aim of Number theory is to discover interesting and unexpected relationships. In [1-7], theory of numbers was discussed. The Z – Transform is useful for the manipulation of discrete date sequences and has acquired a new significance in the formulation and analysis of discrete-time systems. It is used extensively today in the areas of applied mathematics, digital signal processing, control theory, population science, and economics. These discrete models are solved with difference equations in a manner that is analogous to solving continuous models with differential equations. In [8-10], Z-Transform methods were analysed. Recently in [11], the sequence of m-gonal numbers and octahedral numbers was developed.

In this communication, we evaluate Stella Octangula number and Pronic number by using initial value theorem in Z-Transform.

Definition

If the function u_n is defined for discrete values $(n = 0, 1, 2, \cdots)$ and $u_n = 0$ for $n < 0$, then its Z – Transform is defined to be $Z(u_n) = U(z) = \sum u_n z^{-n}$. The inverse Z – Transform is written as $Z^{-1}[U(z)] = u_n$.

Initial Value Theorem

If $Z(u_n) = U(Z)$, then $u_0 = \underset{z \to \infty}{Lt} U(z)$.

Method of Analysis

The process of finding the Stella Octangula number and Pronic number by using initial value theorem in Z – Transform is given in the following theorems.

Theorem 1:

$$Z^{-1}\left[\frac{z^3+10z^2+z}{(z-1)^4}\right]=SO_n$$

Proof

Assume that

$$U(z)=\frac{z^3+10z^2+z}{(z-1)^4}.$$

By initial value theorem, we have

$$u_0=\underset{z\to\infty}{Lt}\ U(z)=0$$

$$u_1=\underset{z\to\infty}{Lt}\ \left[z\left(U(z)-u_0\right)\right]=\underset{z\to\infty}{Lt}\ \left[z\left(\frac{z^3+10z^2+z}{(z-1)^4}\right)-0\right]=1$$

$$u_2=\underset{z\to\infty}{Lt}\ \left[z^2\left(U(z)-u_0-u_1z^{-1}\right)\right]=\underset{z\to\infty}{Lt}\ z^2\left[\frac{z^3+10z^2+z}{(z-1)^4}-\frac{1}{z}\right]=14=2\left(2\times2^2-1\right)$$

$$u_3=\underset{z\to\infty}{Lt}\ \left[z^3\left(U(z)-u_0-u_1z^{-1}-u_2z^{-2}\right)\right]=\underset{z\to\infty}{Lt}\ z^3\left[\frac{z^3+10z^2+z}{(z-1)^4}-\frac{1}{z}-\frac{14}{z^2}\right]=51=3\left(2\times3^2-1\right)$$

$$u_4=\underset{z\to\infty}{Lt}\ \left[z^4\left(U(z)-u_0-u_1z^{-1}-u_2z^{-2}-u_3z^{-3}\right)\right]=\underset{z\to\infty}{Lt}\ z^4\left[\frac{z^3+10z^2+z}{(z-1)^4}-\frac{1}{z}-\frac{14}{z^2}-\frac{51}{z^3}\right]=124=4\left(2\times4^2-1\right)$$

$$u_5=\underset{z\to\infty}{Lt}\ \left[z^5\left(U(z)-u_0-u_1z^{-1}-u_2z^{-2}-u_3z^{-3}-u_4z^{-4}\right)\right]$$

$$=\underset{z\to\infty}{Lt}\ z^5\left[\frac{z^3+10z^2+z}{(z-1)^4}-\frac{1}{z}-\frac{14}{z^2}-\frac{51}{z^3}-\frac{124}{z^4}\right]=245=5\left(2\times5^2-1\right)$$

Continuing the above process, we get

$$u_n=\underset{z\to\infty}{Lt}\ \left[z^n\left(U(z)-u_0-u_1z^{-1}-u_2z^{-2}-u_3z^{-3}-\cdots-u_{n-1}z^{-(n-1)}\right)\right]$$

$$=\underset{z\to\infty}{Lt}\ z^n\left[\frac{z^3+10z^2+z}{(z-1)^4}-\frac{1}{z}-\frac{14}{z^2}-\frac{51}{z^3}-\cdots\right]$$

$$u_n=n\left(2n^2-1\right)=SO_n$$

Thus,

$$Z^{-1}\left[\frac{z^3+10z^2+z}{(z-1)^4}\right]=SO_n$$

Theorem 2:

$$Z^{-1}\left[\frac{2z^2}{(z-1)^3}\right]=\text{Pro}_n$$

Proof

Assume that

$$U(z)=\frac{2z^2}{(z-1)^3}.$$

By initial value theorem, we have

$$u_0=\underset{z\to\infty}{Lt}\ U(z)=0$$

$$u_1=\underset{z\to\infty}{Lt}\ \left[z\left(U(z)-u_0\right)\right]=\underset{z\to\infty}{Lt}\ \left[z\left(\frac{2z^2}{(z-1)^3}\right)-0\right]=2=1(1+1)$$

$$u_2 = \underset{z\to\infty}{Lt}\left[z^2\left(U(z) - u_0 - u_1 z^{-1}\right)\right] = \underset{z\to\infty}{Lt}\ z^2\left[\frac{2z^2}{(z-1)^3} - \frac{2}{z}\right] = 6 = 2(2+1)$$

$$u_3 = \underset{z\to\infty}{Lt}\left[z^3\left(U(z) - u_0 - u_1 z^{-1} - u_2 z^{-2}\right)\right] = \underset{z\to\infty}{Lt}\ z^3\left[\frac{2z^2}{(z-1)^3} - \frac{2}{z} - \frac{6}{z^2}\right] = 12 = 3(3+1)$$

$$u_4 = \underset{z\to\infty}{Lt}\left[z^4\left(U(z) - u_0 - u_1 z^{-1} - u_2 z^{-2} - u_3 z^{-3}\right)\right] = \underset{z\to\infty}{Lt}\ z^4\left[\frac{2z^2}{(z-1)^3} - \frac{2}{z} - \frac{6}{z^2} - \frac{12}{z^3}\right] = 20 = 4(4+1)$$

$$u_5 = \underset{z\to\infty}{Lt}\left[z^5\left(U(z) - u_0 - u_1 z^{-1} - u_2 z^{-2} - u_3 z^{-3} - u_4 z^{-4}\right)\right]$$

$$= \underset{z\to\infty}{Lt}\ z^5\left[\frac{2z^2}{(z-1)^3} - \frac{2}{z} - \frac{6}{z^2} - \frac{12}{z^3} - \frac{20}{z^4}\right] = 30 = 5(5+1)$$

Continuing the above process, we get

$$u_n = \underset{z\to\infty}{Lt}\left[z^n\left(U(z) - u_0 - u_1 z^{-1} - u_2 z^{-2} - u_3 z^{-3} - \cdots - u_{n-1} z^{-(n-1)}\right)\right]$$

$$= \underset{z\to\infty}{Lt}\ z^n\left[\frac{2z^2}{(z-1)^3} - \frac{2}{z} - \frac{6}{z^2} - \frac{12}{z^3} - \cdots\right]$$

$$u_n = n(n+1) = \text{Pro}_n$$

Thus,

$$Z^{-1}\left[\frac{2z^2}{(z-1)^3}\right] = \text{Pro}_n$$

Conclusion

In this communication, we find Stella Octangula number and Pronic number by applying the initial value theorem in Z – Transform. To conclude that, one can find the other special numbers by using various properties of Z – Transform.

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