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Hall effect on MHD flow of visco-elastic fluid layer heated for below saturating a porous medium

Rovin Kumar and Vijay Mehta

Abstract
In this paper the problem of visco-elastic fluid layer heated from below in the presence of uniform vertical magnetic field with Hall current in porous medium is discussed here and obtained a dispersion relation. Using normal mode analysis, from this dispersion relation, we observed that the medium permeability k_1 has destabilizing effect, the magnetic field has stabilizing effect and this stabilizing effect is independent of Hall current, Hall parameter has destabilizing effect and the sufficient condition for the non-existence of over-stability are given as $\beta_e < \frac{1}{6\pi^2 P_m}$, $P_m < \sqrt{2}$, and $2P_m < EP_r < \frac{1}{\sqrt{2}}$.

Keywords: magnetic field, hall current, porous medium, visco-elastic, MHD flow

Introduction

In technological areas there are some important class of fluid, called non-Newtonian fluid, are also being studied extensively because of their practical applications, such as fluid film lubrication, analysis of Polymers in chemical engineering etc. the micropolar fluid is famous case for non-Newtonian fluid as EI-Bory (2005). Also another example for non-Newtonian fluid is viscoelastic fluid. A detailed theoretical investigation has recently begun for the viscoelastic prototype designated liquid B. Walters (1964) [3] and Beard and Walters (1964) [20]. Sen (1978) [13] studied the behavior of unsteady free convection flow of a viscoelastic fluid past an infinite porous plate with constant section. Singh and Singh (1983) [18] have studied the magneto-hydrodynamic flow of viscoelastic fluid past an accelerated plate. The flow of viscoelastic and electrically conducting fluid past an infinite plate has been studied by Sherief and Ezaat (1994) [16].

The viscoelastic fluid subclass of microstructure flows, exhibits a great deal of influence on the normal and shear stresses in flow films. Some interesting flow characteristics of the following film, just to name a few, include:

1. The current state of stresses is a function of past history.
2. Various phenomena including elastic recoiling, creeping and stress relaxation can occur
3. The relation between the stress and velocity field is highly, non-linear, even in situation where the history of the strain is highly repetitive.

The stability problem of a falling film of viscoelastic fluid has been studied Gupta (1967) [7] studied the stability of a small amplitude falling fluid of second order. Shagfeh *et al.* (1989) demonstrated that the viscoelastic property has destabilizing effect on the film flow for small Reynolds number. However, the viscoelastic property possesses a primarily stabilizing effect on the film flow for moderate Reynolds number. Andersson and Dahi (1999) [1] studied the gravity-driven flow of a viscoelastic liquid film along a vertical wall.

Saleh *et al* (2010) [12] examined heat and mass transfer in MHD viscoelastic fluid flow through a porous medium over a stretching sheet with chemical reaction. Sonth *et al.* (2012) [19] studied heat and mass transfer in a viscoelastic fluid over an accelerated surface with heat source/ sink and viscous dissipation. E. Omokhuale *et al.* (2012) [10] studied the effects of concentration and Hall current on unsteady flow of a viscoelastic fluid in a fixed plate.

Po-Jen Cheng *et al.* (2007) studied the surface waves on viscoelastic magnetic fluid film flow down a vertical column. Rita Chaudhary *et al.* (2013) studied viscoelastic unsteady MHD flow between two horizontal parallel plates with Hall current. I.S. Shiva Kumara *et al.* (2007) [17] studied the convective instabilities in viscoelastic fluid saturated porous medium with through flow. R.C. Sharma *et al.* (1999) [15] studied the effect of suspended practices on thermal instability in Rivlin-Ericksen elastico-viscous fluid and Kirti Prakash *et al*. (1999) studied the effects of suspended practices, rotation and variables gravity field on the thermal instability of Rivlin-Ericksen elastico-viscous fluid in porous medium.

In view of the fact that the study of visco-elastic fluid in a porous medium may find applications in geophysics and chemical technology. However, in this paper I have made an attempt to examine the effect of Hall current on MHD flow of visco-elastic (Rivlin-Ericksen) be fluid layer heated from below on porous medium and to the best of my knowledge this problem is uninvestigated so far.

2. Mathematical Formulation

Consider an infinite, horizontal, incompressible electrically non-conducting visco-elastic fluid layer of thickness *d*. A Cartesian coordinate system $(x,\ y,\ z)$ is chosen such that origin it at the lower boundary and the *z*-axis is vertically upward. This fluid layer is assumed to be flowing through on isotropic and homogeneous porous medium of porosity $\in$ and medium permeability *k*. The lower boundary at $z=0$ and the upper boundary at $z=d$ are maintained at constant but different temperature T_0 and T_1 such that a steady adverse temperature gradient $\beta=\left|\frac{dT}{dz}\right|$ is maintained. The whole system is acted upon by a gravity field $\vec{g}=(0,\ 0,\ -\ g)$ and a strong uniform magnetic field $\vec{H}=(0,\ 0,\ H_0)$ is applied along *z*-axis.

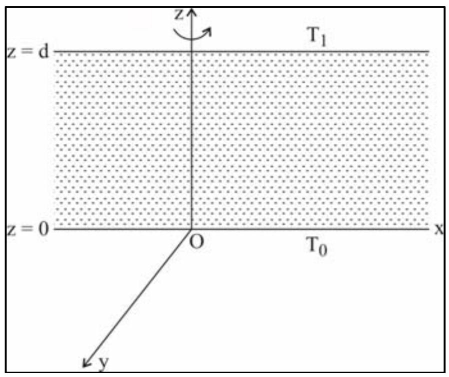

Fig 1

Here, we have taken Rivlin-Ericksen visco-elastic fluid in which when the fluid parameters a porous medium, the gross effect is a represented by Darcy's low and the usual viscous term in the momentum equation is replaced by the resistance term $\left[-\frac{1}{k}\left[\mu+\mu^1\frac{\partial}{\partial t}\right]\vec{q}\right]$. Also both boundaries are considered to be free and perfect conductor of heat. For an isotropic medium the surface porosity is $\in$ so that $1-\in$ is the fraction that is occupied by solid.

Within Boussinesq approximation, the equation governing the motion of micropolar fluid saturating. Porous medium following G. Lebon (1981) [9]. Lukaszewicz (1999), Kirti Prakash *et al* (1999) for above model are as follows:

The equation of continuity for an incompressible fluid is

$$\nabla .\ \vec{q}=0 \qquad \ldots(1)$$

The equation of momentum is

$$\frac{\rho_O}{\in}\left[\frac{\partial\vec{q}}{\partial t}+\frac{1}{\in}(\vec{q}\,.\nabla)\,\vec{q}\right]$$

$$=-\nabla p+\mu\nabla^2\vec{q}-\frac{1}{k}\left(\mu+\mu'\frac{\partial}{\partial t}\right)\vec{q}-\rho g\hat{e}_z+\frac{\mu_e}{4\pi}(\nabla\times\vec{H})\times\vec{H} \qquad \ldots(2)$$

Where, $P,\ \rho,\ \rho_O,\ \vec{q},\ \mu,\ \mu',\ k,\ \mu_e$ and $\hat{e}_z$ denote respectively, pressure, fluid density, reference density, filter velocity, viscosity, visco-elasticity, medium permeability, magnetic permeability and unit vector in *z*-direction.

The equation of energy is

$$\left[\rho_o C_V + \rho_s C_s (1-\in)\right]\frac{\partial T}{\partial t} + \rho_o\, C_V(\vec{q}\,.\nabla)T = \chi_T\,\nabla^2 T \qquad \ldots(3)$$

And the equation of state is

$$\rho = \rho_o\,[1-\alpha(T-T_o)] \qquad \ldots(4)$$

Where $C_V, C_S, \chi_T, \rho_S, \alpha, T$ and T_0 denote respectively specific heat at constant volume, heat capacity of solid (Porous Material Matrix), thermal conductivity, density of solid matrix, coefficient of thermal expansion, temperature and reference temperature.
The Maxwell's equation in the presence of Hall current yield

$$\in\frac{\partial \vec{H}}{\partial t} = \nabla\times(\vec{q}\times\vec{H}) + \in\gamma_m\,\nabla^2\vec{H} - \frac{\in}{4\pi e\, n_e}\nabla\times[(\nabla\times\vec{H})\times\vec{H}] \qquad \ldots(5)$$

and $\nabla.\vec{H} = 0$ …(6)

Where $\vec{H} = (0,\ 0,\ H_o)$, H_o is a constant, $n_e =$ electron density and e = charge on electron and γ_m is the magnetic viscosity.

3. Basic State of the Problem

The basic state is given by

$$\vec{q} = \vec{q}_b(0,\ 0,\ 0),\ \rho = \rho_b(z) \text{ and } P = P_b(z)$$

Under this basic state equation (1) to (6) become

$$\frac{dP_b}{dz} + \rho_b\, g = 0 \qquad \ldots(7)$$

$$T = -\beta z = T_0 \qquad \ldots(8)$$

and $\rho_b = \rho_o(1+\alpha\beta z)$ …(9)

4. Perturbation Equations

$$\vec{q} = \vec{q}_b + \vec{q}\,' = \vec{q}\,',\ P = P_b + P',\ \rho = \rho_b + \rho',\ \vec{H} = \vec{H}_b + \vec{h},\ T = T_b + \theta \qquad \ldots(10)$$

Where $\vec{q}\,', P', \rho', \vec{h}$ and θ are the perturbations in $\vec{q}, P, \rho, \vec{H}$ and T respectively.
Using (7) to (10), equations (1) to (6) becomes

$$\nabla q' = 0 \qquad \ldots(11)$$

$$\frac{\rho_o}{\in}\left[\frac{\partial \vec{q}\,'}{\partial t} + \frac{1}{\in}(\vec{q}\,'.\nabla)\vec{q}\,'\right] = -\nabla P' + \mu\nabla^2\vec{q}\,' - \frac{1}{k}\left(\mu + \mu'\frac{\partial}{\partial t}\right)\vec{q}\,'$$

$$-\rho' g\,\hat{e}_z + \frac{\mu_e}{4\pi}(\nabla\times\vec{h})\times\vec{H}_b + \frac{\mu_e}{4\pi}(\nabla\times\vec{h})\times\vec{h} \qquad \ldots(12)$$

$$\left[\rho_o C_V + \rho_s C_s (1-\in)\right]\frac{\partial \theta}{\partial t} + \rho_o C_V(\vec{q}\,'\nabla)\theta + \rho_o\, C_V(\vec{q}\,'\nabla)T_b = \chi_T\nabla^2\theta \qquad \ldots(13)$$

$$\in\frac{\partial \vec{h}}{\partial t} = \nabla\times(\vec{q}\,'\times\vec{h}) + \nabla\times(\vec{q}\,'\times\vec{H}_b) + \in\gamma_m\,\nabla^2 h$$

$$-\frac{\in}{4\pi\, en_e}\nabla\times\left[(\nabla\times\vec{h})\times\vec{h} + (\nabla\times\vec{h})\times\vec{H}_b\right]$$

$$\nabla\,\vec{h} = 0 \qquad \ldots(15)$$

and $\rho' = -\rho_o\alpha\beta\Theta$ …(16)

In order to linearlize above equation, ignoring the terms $(\vec{q}\,',\nabla)\ q',\ (q'.\nabla)\theta,\ (\nabla\times\vec{h})\times\vec{h},\ \nabla\times(\vec{q}\,'\times\vec{h})$ we have

$$\nabla q' = 0 \qquad \ldots(17)$$

$$\frac{\rho_o}{\in}\left[\frac{\partial \vec{q}'}{\partial t}\right] = -\nabla P' + \mu\nabla^2 \vec{q}' - \frac{1}{k}\left(\mu + \mu' + \frac{\partial}{\partial t}\right)q' - \rho' g\,\hat{e}_z + \frac{\mu_e H_o}{4\pi}(\nabla \times \vec{h}) \times \hat{e}_z \quad \ldots(18)$$

$$\left[\in + \frac{\rho_s C_v(1-\in)}{\rho_o C_v}\right]\frac{\partial \theta}{\partial t} - W'\beta = \frac{\chi_T \nabla^2 \theta}{\rho_o C_v} \quad \ldots.(19)$$

$$\in \frac{\partial \vec{h}}{\partial t} = H_o \nabla \times (\vec{q}' \times \hat{e}_z) + \gamma_m \nabla^2 \vec{h} - \frac{\in H_o}{4\pi\, e n_e}\nabla \times [(\nabla \times \vec{h}) \times \hat{e}_z] \quad \ldots(20)$$

$$\nabla \vec{h} = 0 \quad \ldots(21)$$

and $\rho' = -\rho_o\,\alpha\,\beta\theta$ …(22)

Converting the equation (17) to (22) into non-dimensional form by the following transformation and dropping the stars,

$x = dx^*,\ y = dy^*,\ z = dz^*,\ \vec{q}' = \frac{k_T}{d}q^*,\ t = \frac{\rho_o d^2}{\mu}t^*,\ \theta = \beta d\theta^*,\ P' = \frac{\mu k_T}{d^2}P^*,\ \vec{h} = H_o h^*,$ where $k_T = \frac{\chi_T}{\rho_o C_v}$ is the thermal diffusivity, we have

$$\nabla \vec{q} = 0 \quad \ldots(23)$$

$$\frac{1}{\in}\frac{\partial \vec{q}}{\partial t} = -\nabla P + \nabla^2 \vec{q} - \frac{1}{k_1}\left(1 + F\frac{\partial}{\partial t}\right)\vec{q} + R\theta\,\hat{e}_z + Q(\nabla \times \vec{h}) \times \hat{e}_z \quad \ldots(24)$$

Similarly

$$EP_r\frac{\partial \theta}{\partial t} = \nabla^2\theta + \omega \quad \ldots(25)$$

$$\in P_r\frac{\partial \vec{h}}{\partial t} = \frac{\partial \vec{q}}{\partial z} + \frac{\in P_r}{P_m}\nabla^2 \vec{h} - \in \beta_e^{\gamma_2}\frac{\partial}{\partial z}(\nabla \times \vec{h}) \quad \ldots(26)$$

Where $R = \frac{\rho_o\, g\,\alpha\,\beta\, d^4}{\mu\, k_T}$ is the thermal Rayleigh number

$Q = \frac{\mu_e H_o^2 d^2}{4\pi\,\mu k_T}$ is the Chandrasekhar number

$k_1 = \frac{k}{d^2},\ E = \in\frac{P_s C_s(1-\in)}{\rho_o C_v},\ P_r\ \frac{\mu}{\rho_o k_T}$ is the Prandtl number, $P_m = \frac{\mu}{\rho_o \gamma_m}$ is the magnetic Prandtl number, $F = \frac{\mu'}{\rho_o d^2}$ is the viscoelastic Parameter $\beta e = \left(\frac{H_o}{4\pi k_T\, e n_e}\right)^2$ is the Hall parameter and $W = \vec{q}\,.e_z^n$

5. Boundary Conditions

Both boundary are taken to be free and perfectly heat conducting, then we have $W = \frac{d^2W}{dz^2} = 0,\ \theta = 0,$ at $z = 0$ and $z = 1$ …(27)

6. Dispersion Relation

Taking curl on the sides (24), we have

$$\frac{1}{\in}\frac{\partial}{\partial t}(\nabla \times \vec{q}) = \nabla^2(\nabla \times \vec{q}) - \frac{1}{k_1}\left[1 + F\frac{\partial}{\partial t}\right](\nabla \times \vec{q})$$

$$+R\left[\frac{\partial \theta}{\partial y}\hat{e}_x + \frac{\partial \theta}{\partial x}\hat{e}_y\right] + Q\frac{\partial}{\partial z}(\nabla \times \vec{h}) \quad \ldots(28)$$

Again applying curl on both sides of (28) and taking *z*-component on both sides, we have

$$\left[\frac{1}{\in}\frac{\partial}{\partial t} + \frac{1}{k_1}\left(1 + F\frac{\partial}{\partial t}\right) - \nabla^2\right]\nabla^2 W = R\nabla_1^2\theta + Q\,D(\nabla^2 h_z) \quad \ldots.(29)$$

Where $\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2},\ \vec{h}_z = \vec{h}\,.e_z,\ D = \frac{\partial}{\partial z}$

Taking *z*-component on both sides of equation (28)

$$\left[\frac{1}{\in}\frac{\partial}{\partial t}+\frac{1}{k_1}\left(1+F\frac{\partial}{\partial t}\right)-\nabla^2\right]\zeta = Q\,Dm_z \quad \text{…(30)}$$

Where $\zeta_z = (\nabla\times\vec{q})\,\hat{e}_z$ and $m_z = (\nabla\times\vec{h})\,\hat{e}_z$

$$\left[E\,P_r\frac{\partial}{\partial t}-\nabla^2\right]\theta = W \quad \text{…(31)}$$

Taking curl on both side of equation (26) and taking z-component on both side, we have

$$\in P_r\frac{\partial m_z}{\partial t} = D\zeta_z + \in\frac{P_r}{P_m}\nabla^2 m_z + \in\,\beta\,e^{1/2}D[\nabla^2 h_z] \quad \text{…(32)}$$

Taking z-component on both sides of equation (26), we have

$$\in P_r\frac{\partial h_z}{\partial t} = DW + \in\frac{P_r}{P_m}\nabla^2 h_z - \in\beta e^{1/2}D\,m_z \quad \text{…(33)}$$

Boundary condition (27) now becomes

$$W = D^2W = 0 = \zeta_z = D\zeta_z = h_z = Dm_z = \theta \text{ at } z=0 \text{ and } z=1 \quad \text{…(34)}$$

7. Normal Mode Analysis

Consider

$$[W,\ \zeta_z,\ \Theta,\ h_z,\ m_z] = [W(z),\ X(z), \Theta(z),\ B(z),\ M(z)]\exp.[ik_x x + ik_y y + \sigma t]$$

Applying above normal mode analysis to the equation (29) to (33) we have

$$\left[\frac{\sigma}{\in}\frac{\partial}{\partial t}+\frac{1}{k_1}(1+F\sigma)-(D^2-a^2)\right](D^2-a^2)W = -Ra^2\Theta + QD(D^2-a^2)B \quad \text{…(35)}$$

$$\left[\frac{\sigma}{\in}\frac{\partial}{\partial t}+\frac{1}{k_1}(1+F\sigma)-(D^2-a^2)\right]X = QDM \quad \text{…(36)}$$

$$\left[EP_r\sigma-(D^2-a^2)\right]\Theta = W \quad \text{…(37)}$$

$$\left[\in P_r\sigma - \in\frac{P_r}{P_m}(D^2-a^2)\right]M = DX + \in\beta e^{1/2}D(D^2-a^2)B \quad \text{…(38)}$$

$$\left[\in P_r\sigma - \in\frac{P_r}{P_m}(D^2-a^2)\right]B = DW - \in\beta\,e^{1/2}DM \quad \text{…(39)}$$

Where $a^2 = k_x^2 + k_y^2$ is the wave number and $\sigma = \sigma_r + i\sigma_i$ is the stability parameter.

Now, the boundary condition become

$$W = D^2W = 0 = X = DX = B = M = DM,\ \Theta = 0 \text{ at } z=0 \text{ and } z=1 \quad \text{…(40)}$$

Assuming (40) we get

$D^{(2n)}W = 0$ at $z=0,\ z=1$, where n is a positive integer

Thus the proper solution satisfying (40) can be taken as

$W = W_o\sin\pi z$, where W_o is a constant …(41)

Eliminating $\Theta,\ B,\ X$ and M from (35) to (39), we have

$$\left[\left[\frac{\sigma}{\in}+\frac{1}{k_1}(1+F\sigma)-(D^2-a^2)\right]\times\left\{\left[\in P_r\sigma-\in\frac{P_r}{P_m}(D^2-a^2)\right]^2+\in^2\beta_e D^2(D^2-a^2)\right\}\right.$$

$$\left.-QD^2\times\left[\in P_r\sigma-\in\frac{P_r}{P_m}(D^2-a^2)\right]\right]B = \left[\frac{\sigma}{\in}+\frac{1}{k_1}(1+F\sigma)-(D^2-a^2)\right]$$

$$\times\left\{\left[\in P_r\sigma-\in\frac{P_r}{P_m}(D^2-a^2)\right]-QD^2\right\}DW \quad \text{…(42)}$$

From (35) and (37), we get

$$\left[\frac{\sigma}{\in}+\frac{1}{k_1}(1+F\sigma)-(D^2-a^2)\right]\left[EP_r\sigma-(D^2-a^2)\right](D^2-a^2)W = -Ra^2W$$

$$+QD(D^2-a^2)[EP_r\,\sigma-(D^2-a^2)]B \quad \text{…(43)}$$

From (42) and (43)

$$W\left[\left[\frac{\sigma}{\in}+\frac{1}{k_1}(1+F\sigma)-(D^2-a^2)\right]\times\left\{\left[\in P_r\sigma-\in\frac{P_r}{P_m}(D^2-a^2)\right]+\in^2\beta_e D^2(D^2-a^2)\right\}\right.$$

$$-QD^2\left[\in P_r\sigma-\in\frac{P_r}{P_m}(D^2-a^2)\right]\Bigg]\times\begin{bmatrix}\left[\frac{\sigma}{\in}+\frac{1}{k_1}(1+F\sigma)-(D^2-a^2)\right]\\(E P_r\sigma-D^2-a^2)(D^2-a^2)+Ra^2\end{bmatrix}$$

$$=\left[\left[\frac{\sigma}{\in}+\frac{1}{k_1}(1+F\sigma)-(D^2-a^2)\right]\times\left\{\left[\in P_r\sigma-\in\frac{P_r}{P_m}(D^2-a^2)\right]-QD^2\right\}\right.$$

$$\left. .\,QD^2(D^2-a^2)\left[EP_r\,\sigma-(D^2-a^2)\right]\ \right]W \qquad \ldots(44)$$

Substituting the value of W from (41) into (44) and using $b=\pi^2+a^2$, we have

$$\left\{\left[\frac{\sigma}{\in}+\frac{1}{k_1}(1+F\sigma)+b\right]\times\left[\left[\in P_r\sigma+\in\frac{P_r}{P_m}b\right]^2+\in^2\pi^2\beta_e b\right]Q\,\pi^2\left(\in P_r\sigma+\frac{\in P_r}{P_m}b\right)\right\}$$

$$\times\left\{\left[\frac{\sigma}{\in}+\frac{1}{k_1}(1+F\sigma)+b\right].\left[EP_r\sigma+b\right](-b)+Ra^2\right\}$$

$$=\left[\frac{\sigma}{\in}+\frac{1}{k_1}(1+F\sigma)+b\right]\times\left\{\left[\in P_r\,\sigma+\in\frac{P_r}{P_m}b\right]+\theta\pi^2\right\}.\theta\pi^2 b(EP_r\sigma+b) \qquad \ldots(45)$$

8. Stationary Convection

In order to examine the stationary convection

We put $\sigma=0$ in (45)

$$R=\frac{b^2\left[\frac{\in P_r b}{P_m}\left(\frac{1}{k_1}+b\right)+Q\pi^2\right]^2+\in^2\pi^2 b^3\beta_e\left(\frac{1}{k_1}+b\right)^2}{a^2\left[\left(\frac{1}{k_1}+b\right)\left[\left(\frac{\in P_r b}{P_m}\right)^2+\in^2\pi^2 b\beta_e\right]+Q\pi^2\frac{\in P_r b}{P_m}\right]} \qquad \ldots(46)$$

In the absence of magnetic field i.e., $(H_o=0)$, i.e., $Q=0$ and $\beta_e=0$ then equation (46)

$$R=\frac{b^2}{a^2}\left(\frac{1}{k_1}+b\right) \qquad \ldots(47)$$

In particular, in non-porous medium $[k_1\to\infty]$ equation (47) reduces to

$$R=\frac{b^3}{a^2}$$

Which is classical value of R for Newtonian fluid as that proposed by **G. Lebon and C. Perez-Garcia**.

Equation (46) can be written as

$$R=\frac{b^2\left[\frac{\in P_r b}{P_m}\left(\frac{1}{k_1}+b\right)+Q\pi^2\right]^2+\in^2\pi^2 b^2\beta_e\left(\frac{1}{k_1}+b\right)^2}{a^2\left[\left(\frac{1}{k_1}+b\right)\left[\left(\frac{\in P_r b}{P_m}\right)^2+\in^2\pi^2 b\beta_e\right]+Q\pi^2\frac{\in P_r b}{P_m}\right]} \qquad \ldots(48)$$

In order to investigate the behavior of k_1 (Medium Permeability), Q (Magnetic field), β_e (Hall current), we find the nature of $\frac{dR}{dk_1},\frac{dR}{dQ},\frac{dR}{d\beta_e}$ respectively.

From (48)

$$\frac{dR}{dk_1}=\frac{-\frac{b^2}{a^2k_1}\left[\begin{array}{l}\left(\frac{\in^2 P_r^2b^2}{P_m^2}+\in^2\pi^2b\beta_e\right)\left(\frac{\in P_rb}{P_m}\left(\frac{1}{k_1}+b\right)+Q\pi^2\right)\\ \left[-Q\pi^2+\frac{\in P_rb}{P_m}\left(\frac{1}{k_1}+b\right)\right]+\in^2\pi^2\beta_e\left(\frac{1}{k_1}+b\right)^2\\ \left(\frac{\in^2 P_r^2b^2}{P_m^2}+\in^2\pi^2b\beta_e\right)+\frac{2\in^2 P_r^2b^2Q\pi^2}{P_m^2}\left(\frac{\in P_rb}{P_m}\left(\frac{1}{k_1}+b\right)+Q\pi^2\right)\\ +\frac{2\in^3\pi^4b^2\beta_eP_rQ}{P_m}\left(\frac{1}{k_1}+b\right)\end{array}\right]}{\left[\left(\frac{1}{k_1}+b\right)\left[\left(\frac{\in P_rb}{P_m}x\right)^2+\in^2\pi^2b\beta_e\right]+Q\pi^2\frac{\in P_rb}{P_m}\right]^2}$$

$$\frac{dR}{dk_1}<0 \text{ when } \frac{\in P_rb^2}{P_m}-Q\pi^2>0$$

Thus, the medium permeability k_1 has destabilizing effect when $Q<\frac{\in P_rb^2}{P_m\pi^2}$.

In particular, $(H_0=0)$, i.e., $Q=0$ and $\beta_e=0$

$$\frac{dR}{dk_1}=-\frac{b^2}{a^2k_1^2}$$

Which is always negative, the medium permeability has destabilizing effect without any condition.
From (48)

$$\frac{dR}{dQ}=\frac{\pi^2b^2}{a^2}\frac{\left[\begin{array}{l}\left(\frac{1}{k_1}+b\right)^2\left(\frac{\in^3 P_r^3b^3}{P_m^3}\right)+\left(\frac{1}{k_1}+b\right)^2\left(\in^3\pi^2b^2\beta_e\frac{P_r}{P_m}\right)\\ +2\left(\frac{1}{k_1}+b\right)\left(\frac{Q\pi^2\in^2 P_r^2b^2}{P_m^2}\right)+2\left(\frac{1}{k_1}+b\right)Q\pi^4\in^2 b\beta_e+Q^2\pi^4\frac{\in P_rb}{P_m}\end{array}\right]}{\left[\left(\frac{1}{k_1}+b\right)\left(\frac{\in^2 P_r^2b^2}{P_m^2}+\in^2\pi^2b\beta_e\right)+Q\pi^2\frac{\in P_r}{P_m}\right]^2}$$

$$\frac{dR}{dQ}>0$$

Thus, the magnetic field has stabilizing effect and this stabilizing effect is independent of the presence of Hall current.
From (48)

$$\frac{dR}{d\beta_e}=-b^2Q^2\pi^2\frac{\left[\in^2\pi^4b\left(\frac{1}{k_1}+b\right)+\left(\frac{1}{k_1}+b\right)^2\frac{\in^3\pi^2P_rb^2}{P_m}\right]}{\left[\left(\frac{1}{k_1}+b\right)\left(\frac{\in^2 P_r^2b^2}{P_m^2}+\in^2\pi^2b\beta_e\right)+Q\pi^2\frac{\in P_rb}{P_m}\right]^2}$$

$$\frac{dR}{d\beta_e}<0$$

Thus, the Hall current parameter has destabilizing effect

9. Oscillatory Convection

Putting $\sigma=i\sigma_i$ in (45) and separate real and imaginary parts, we have

$$\left[b+\frac{1}{k_1}+i\left(\frac{1}{\in}+\frac{F}{k_1}\right)\sigma_i\right]\times\left\{\left[\left[\frac{\in P_rb}{P_m}+i\in P_r\sigma_i\right]^2+\in^2\pi^2b\beta_e\right]+Q\pi^2\left(i\in P_r\sigma_i+\frac{\in P_rb}{P_m}\right)\right\}$$

$$\times\left\{\left[\left(b+\frac{1}{k_1}\right)+i\left(\frac{1}{\in}+\frac{F}{k_1}\right)\sigma_i\right](b+iEP_r\sigma_i)\ (-b)+Ra^2\right\}$$

$$=\left\{\left[\left(b+\frac{1}{k_1}\right)+i\sigma_i\left(\frac{1}{\in}+\frac{F}{k_1}\right)\right]\times\left[\left(\frac{\in P_r b}{P_m}+i\in P_r\sigma_i\right)+Q\pi^2\right]\right\}.Q\pi^2 b(b+iEP_r\sigma_i)$$

$$\left(b+\frac{1}{k_1}\right)\left[\frac{\in^2 P_r^2 b^2}{P_m^2}-\in^2 P_r^2\sigma_i^2+\in^2\pi^2 b\beta_e\right]-\left(\frac{1}{\in}+\frac{F}{k_1}\right)\sigma_i\left(\frac{2\in^2 P_r^2 b\sigma_i}{P_m}\right)$$

$$+i\left[\begin{array}{l}\left(b+\frac{1}{k_1}\right)\left(\frac{2\in^2 P_r^2 b\sigma_i}{P_m}\right)+\left(\frac{1}{\in}+\frac{F}{k_1}\right)\sigma_i\\ \left(\frac{\in^2 P_r^2 b^2}{P_m^2}-\in^2 P_r^2\sigma_i^2+\in^2\pi^2 b\beta_e\right)+Q\pi^2\in P_r\sigma_i\end{array}\right]+Q\pi^2\frac{\in P_r b}{P_m}$$

$$\times\left\{-b^2\left(b+\frac{1}{k_1}\right)+EbP_r\sigma_i^2\left(\frac{1}{\in}+\frac{1}{k_1}\right)+Ra^2+i\left[\begin{array}{l}-bEp_r\sigma_i\left(b+\frac{1}{k_1}\right)\\ -b^2\left(\frac{1}{\in}+\frac{1}{k_1}\right)\sigma_i\end{array}\right]\right\}$$

$$=\left[\left(b+\frac{1}{k_1}\right)\left(\frac{\in P_r b}{P_m}\right)-\sigma_i^2\in P_r\left(\frac{1}{\in}+\frac{F}{k_1}\right)+Q\pi^2\right]Q\pi^2 b^2$$

$$-\left[\in P_r\sigma_i\left(b+\frac{1}{k_1}\right)+\sigma_i\left(\frac{1}{\in}+\frac{F}{k_1}\right)\frac{\in P_r b}{P_m}\right]Q\pi^2\, bEP_r\sigma_i$$

$$+i\left[\in P_r\sigma_i\left(b+\frac{1}{k_1}\right)+\sigma_i\left(\frac{1}{\in}+\frac{F}{k_1}\right)\frac{\in P_r b}{P_m}\right]Q\pi^2 b^2$$

$$+\left[\left(b+\frac{1}{k_1}\right)\left(\frac{\in P_r b}{P_m}\right)-\sigma_i^2\in P_r\left(\frac{1}{\in}+\frac{F}{k_1}\right)+Q\pi^2\right]Q\pi^2\, b\,EP_r\sigma_i\Bigg]$$

The real part is given by

$$R=\frac{a_1\sigma_i^4+a_2\sigma_i^2+a_3}{b_1\sigma_i^2+b_2} \qquad \ldots(49)$$

Where $a_1=\left(b+\frac{1}{k_1}\right)\left(\frac{1}{\in}+\frac{1}{k_1}\right)\left(-\in^2 EP_r^3 b-\in^2 EP_r^3 b\right)+\left(\frac{1}{\in}+\frac{F}{k_1}\right)\left(\frac{-2\in^2 EP_r^3 b^2}{P_m}-\in^2 P_r^2 b^2\right)$

$$a_2=\in^2 P_r^2 b^2\left(b+\frac{1}{k_1}\right)^2+\left(\frac{1}{\in}+\frac{F}{k_1}\right)\left(b+\frac{1}{k_1}\right)\left(\frac{2\in^3 P_r^2 b^2}{P_m}\right)$$

$$+\left(b+\frac{1}{k_1}\right)\left(\frac{1}{\in}+\frac{F}{k_1}\right)\left(\frac{\in^2 EP_r^3 b^3}{P_m^2}\right)+\left(b+\frac{1}{k_1}\right)\left(\frac{1}{\in}+\frac{F}{k_1}\right)\left(\in^2\pi^2 Eb^2 P_r\beta_e\right)$$

$$+\left(\frac{1}{\in}+\frac{F}{k_1}\right)\left(\frac{Q\pi^2 E\in P_r^2 b^2}{P_m}\right)+\left(b+\frac{1}{k_1}\right)^2\left(\frac{2\in^2 EP_r^3 b^2}{P_m}\right)$$

$$+\left(b+\frac{1}{k_1}\right)\left(\frac{1}{\in}+\frac{F}{k_1}\right)\left(\frac{E\in^2 P_r^3 b^3}{P_m^2}\right)+\left(b+\frac{1}{k_1}\right)\left(\frac{1}{\in}+\frac{F}{k_1}\right)\left(E\in^2\pi^2 P_r b^2\beta_e\right)$$

$$+\left(b+\frac{1}{k_1}\right)\left(Q\pi^2 E\in P_r^2 b\right)+\left(b+\frac{1}{k_1}\right)\left(\frac{1}{\in}+\frac{F}{k_1}\right)\left(\frac{2\in^2 P_r^2 b^3}{P_m}\right)$$

$$+\left(\frac{1}{\in}+\frac{F}{k_1}\right)^2\left(\frac{\in^2 P_r^2 b^4}{P_m^2}\right)+\left(\frac{1}{\in}+\frac{F}{k_1}\right)^2\left(\in^2\pi^2 b^3\beta_e\right)+\left(\frac{1}{\in}+\frac{F}{k_1}\right)Q\pi^2\in P_r b^2$$

$$+\left(\frac{1}{\in}+\frac{F}{k_1}\right)Q\pi^2 b^2 \in P_r + \left(b+\frac{1}{k_1}\right)Q\pi^2 \in P_r^2\, bE + \left(\frac{1}{\in}+\frac{F}{k_1}\right)\left(\frac{\in P_r b}{P_m}\right)Q\pi^2 bEP_r$$

$$a_3 = -\left[\begin{array}{l}\left(b+\frac{1}{k_1}\right)^2\left(\frac{\in^2 P_r^2 b^4}{P_m^2}\right) + \left(b+\frac{1}{k_1}\right)^2\left(\frac{Q\pi^2 \in P_r b^3}{P_m}\right) \\ + \left(b+\frac{1}{k_1}\right)\left(\frac{\in P_r b}{P_m}\right)Q\pi^2 b^2 - Q^2\pi^4 b^2\end{array}\right]$$

And $$b_1 = a^2\left[\left(b+\frac{1}{k_1}\right)(\in^2 P_r^2) + \left(\frac{1}{\in}+\frac{F}{k_1}\right)\left(\frac{2\in^2 P_r^2 b}{P_m}\right)\right]$$

$$b_2 = -a^2\left[\left(\frac{\in^2 P_r^2 b^2}{P_m^2} + \in^2 \pi^2 b\beta_e\right)\left(b+\frac{1}{k_1}\right) + \frac{Q\pi^2 \in P_r b}{P_m}\right]$$

The imaginary part is given by

$$R = \frac{P_1\sigma_i^5 + P_2\sigma_i^3 + P_3\sigma_i}{q_1\sigma_i^3 + q_2\sigma_i} \qquad \ldots(50)$$

Where

$$P_1 = \left(\frac{1}{\in}+\frac{F}{k_1}\right)^2 \in^2 EP_r^3 b$$

$$P_2 = -\left[\left(b+\frac{1}{k_1}\right)^2 \in^2 EP_r^3 b + \left(b+\frac{1}{k_1}\right)\left(\frac{1}{\in}+\frac{F}{k_1}\right)\left(\frac{2\in^2 EP_r^3 b^2}{P_m}\right)\right.$$

$$+\left(\frac{1}{\in}+\frac{F}{k_1}\right)\left(b+\frac{1}{k_1}\right)\left(\in^2 P_r^2 b^2\right) + \left(\frac{1}{\in}+\frac{F}{k_1}\right)^2\left(\frac{2\in^2 P_r^2 b^3}{P_m}\right)$$

$$+\left(b+\frac{1}{k_1}\right)\left(\frac{1}{\in}+\frac{F}{k_1}\right)\left(\in^2 P_r^2 b^2\right) + \left(b+\frac{1}{k_1}\right)\left(\frac{1}{\in}+\frac{F}{k_1}\right)\left(\frac{2\in^2 EP_r^3 b^2}{P_m}\right)$$

$$\left.+\left(\frac{1}{\in}+\frac{F}{k_1}\right)^2\left(\frac{\in^2 EP_r^3 b^3}{P_m^2}\right) + \left(\frac{1}{\in}+\frac{F}{k_1}\right)^2\left(\in^2 \pi^2 EP_r b^2\beta_e\right) + 2\left(\frac{1}{\in}+\frac{F}{k_1}\right)(Q\pi^2 \in EP_r^2 b)\right]$$

$$P_3 = \left(b+\frac{1}{k_1}\right)^2\left(\frac{\in^2 EP_r^3 b^3}{P_m}\right) + \left(b+\frac{1}{k_1}\right)^2\left(\in^2 E\pi^2 P_r b^2\beta_e\right)$$

$$+\left(b+\frac{1}{k_1}\right)\left(\frac{Q\pi^2 E \in P_r^2 b^2}{P_m}\right) + \left(b+\frac{1}{k_1}\right)\left(\frac{1}{\in}+\frac{F}{k_1}\right)\left(\frac{\in^2 P_r^2 b^4}{P_m^2}\right)$$

$$+\left(b+\frac{1}{k_1}\right)\left(\frac{1}{\in}+\frac{F}{k_1}\right)\left(\in^2 \pi^2 b^3\beta_e\right) + \left(\frac{1}{\in}+\frac{F}{k_1}\right)\left(\frac{Q\pi^2 \in P_r b^3}{P_m}\right)$$

$$+\left(b+\frac{1}{k_1}\right)^2\left(\frac{2\in^2 P_r^2 b^3}{P_m}\right) + \left(b+\frac{1}{k_1}\right)\left(\frac{1}{\in}+\frac{F}{k_1}\right)\left(\frac{\in^2 P_r^2 b^4}{P_m^2}\right)$$

$$+\left(b+\frac{1}{k_1}\right)\left(\frac{1}{\in}+\frac{F}{k_1}\right)\left(\in^2 \pi^2 b^3\beta_e\right) + \left(b+\frac{1}{k_1}\right)\left(Q\pi^2 \in P_r b^2\right)$$

$$+\left(b+\frac{1}{k_1}\right)\left(Q\pi^2 \in P_r b^2\right) + \left(\frac{1}{\in}+\frac{F}{k_1}\right)\left(\frac{Q\pi^2 \in P_r b^3}{P_m}\right) + \left(b+\frac{1}{k_1}\right)\left(\frac{Q\pi^2 \in EP_r^2 b^2}{P_m}\right)$$

$$q_1 = -a^2\left(\frac{1}{\in}+\frac{F}{k_1}\right)\left(\in^2 P_r^2\right),$$

$$q_2 = a^2\left(\frac{2\in^2 P_r^2 b}{P_m}\left(b+\frac{1}{k_1}\right)+\left(\frac{1}{\in}+\frac{F}{k_1}\right)\left(\frac{\in^2 P_r^2 b^2}{P_m^2}+\in^2 \pi^2 b\beta_e\right)+Q\pi^2 \in P_r\right.$$

Eliminating R between (49) and (50)

For $f_0\sigma_i^6 + f_1\sigma_i^4 + f_2\sigma_i^2 + f_3 = 0$

Where $s=\sigma_i^2$

$$f_0 s^3 + f_1 s^2 + f_2 s + f_3 = 0 \qquad \ldots(51)$$

$$f_0 = a_1q_1 - P_1b_1,\ \ f_1 = a_2q_1 + a_1q_2 - P_2b_1 - P_1b_2,$$
$$f_2 = a_3q_1 + a_2q_2 - P_3b_1 - P_2b_2,\ \ f_3 = a_3q_2 - P_3b_2$$

Where $f_0 = a_1q_1 - P_1b_1$

$$f_0 = a^2 \in^4 P_r^4 b\left(\frac{1}{\in}+\frac{F}{k_1}\right)^2\left[\left(b+\frac{1}{k_1}\right)E\,P_r+\left(\frac{1}{\in}+\frac{F}{k_1}\right)\right]>0$$

$$f_1 = a_2q_1 + a_1q_2 - P_2b_1 - P_1b_2 > 0$$

when

$$\left.\begin{array}{l}\left(\frac{1}{\in}+\frac{1}{4\sqrt{2}}\right)<\frac{1}{\in}+\frac{F}{k_1}<\min\left\{\frac{1}{3},\frac{P_r}{\pi k_1\sqrt{\beta_e}}\right\},\ k_1<\frac{1}{2}\\ \beta_e<\frac{1}{6\pi^2 P_m},\ P_m<\sqrt{2},\ E>1,\ P_r\notin\left(\frac{1}{2},1\right),\ 2P_m<EP_r<\frac{1}{\sqrt{2}}\end{array}\right\}$$

From (51), we notice that $s=\sigma_i^2$ which is always positive, therefore the sum of roots equation of (51) is positive but this is impossible if $f_0>0$ and $f_1>0$, because the sum of roots of equation (51) is $\left(-\frac{f_0}{f_1}\right)$. Thus, $f_0>0$ and $f_1>0$ are the sufficient condition for the non-existence of over-stability.

Now $f_0>0$

and $f_1>0$

when

$$\left.\begin{array}{l}\left(\frac{1}{\in}+\frac{1}{4\sqrt{2}}\right)<\frac{1}{\in}+\frac{F}{k_1}<\min\left\{\frac{1}{3},\frac{P_r}{\pi k_1\sqrt{\beta_e}}\right\},\ k_1<\frac{1}{2}\\ \beta_e<\frac{1}{6\pi^2 P_m},\ P_m<\sqrt{2},\ E>1,\ P_r\notin\left(\frac{1}{2},1\right),\ 2P_m<EP_r<\frac{1}{\sqrt{2}}\end{array}\right\} \qquad \ldots(52)$$

Hence for the conditions given in (52), overstability cannot occur and the principle of exchange of stability (PES) is valid.

10. Conclusions

1. For Stationary Convection

(i) When $Q<\frac{\in P_r b^2}{P_m\pi^2}$, $\frac{dR}{dk_1}<0$, which implies that the medium permeability k_1 has destabilizing effect under the above condition. In the absence of Hall parameter and magnetic field, the medium permeability has destabilizing effect without any condition.

(ii) $\frac{dR}{dQ}>0$, Thus the magnetic field has stabilizing effect and this stabilizing effect is independent of Hall current.

(iii) $\frac{dR}{d\beta_e}<0$, thus the Hall parameter has destabilizing effect.

2. For Oscillatory Convection: The sufficient condition for the non-existence of overstability are given by condition

$$\left(\frac{1}{\in}+\frac{1}{4\sqrt{2}}\right)<\left(\frac{1}{\,}+\frac{F}{k_1}\right)<\min\left\{\frac{1}{3},\frac{P_r}{\pi k_1\sqrt{\beta_e}}\right\},\ k_1<\frac{1}{2}$$

$$\beta_e<\frac{1}{6\pi^2 P_m},\ P_m<\sqrt{2},\ E>1,\ P_r\notin\left(\frac{1}{2},1\right),\ 2P_m<EP_r<\frac{1}{\sqrt{2}}$$

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