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Reliability analysis of a model with regard to undertaking the failed unit by ordinary or expert repairman with the concept of instruction time

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Abstract

The present paper deals with two-unit cold standby system with instruction and two of the three types of repair policy. The two types of repair policy used are: resume repair policy and policy adopted at the more degraded stage due to damage made by the ordinary repairman during try for repair. The purpose of taking the idea of instruction in the paper is to avoid the possibility of incorrect process of repair done by the ordinary repairman. In this model it is assumed that every failed unit first goes under the repair of ordinary repairman who starts repair after getting the instructions from the expert.

Various measures of system effectiveness have been obtained by making use of semi-Markov processes and regenerative point technique. Graphical study for a particular case is also made.

Keywords: Reliability analysis, undertaking the failed, expert repairman, instruction time

Introduction

Various systems are used under different situations in the industries/companies/ factories/organisations. Considering many types of situations, a large number of reliability models on two-unit standby systems have been discussed by most of the researchers in the field of reliability. The situation of two types of repairman and a repair policy wherein the repair is started by the expert from the more degraded stage caused by the ordinary repairman due to mishandling is undealt so far in the field.

The idea of two types of repairman i.e., one ordinary and the other expert have been discussed by various authors including [3-5]. But engaging an expert repairman for repair may be costly. Such cost can be reduced by introducing the idea of instruction which are given by the expert to the ordinary repairman before the start of repair by the ordinary repairman. So, the stay and cost of expert repairman can be reduced. The idea of instruction was first introduced by Kumar *et al*. [6]. Later on, it was discussed by some other authors including [7-8] who studied it together with concepts of accidents, two of repairmen, etc. There may be situations wherein the ordinary repairman is not only able to do some complex repairs but also may adopt incorrect process. In order to nullify the possibility of incorrect process and to reduce the stay of the expert repairman, the idea of introduction time is introduced in the present paper also together with the new aforesaid repair policy.

Thus, the present paper investigates two-unit cold standby system with instruction and two of the three types repair policy. On the failure of an operative unit, an expert repairman comes to give instruction to the ordinary repairman for repairing it. After getting instruction, the ordinary repairman repairs the unit by adopting the correct process. However during repair, he may damage the unit due to mishandling or some other causes and in this case repeat repair policy (type-II) is adopted by the expert repairman. Chances are there that even after getting instruction he may not be able to complete the repair successfully and hence resume repair policy is adopted by the expert. If at the time of imparting instruction for a failed unit other unit is operative, it is assumed that instruction time finishes before the failure of the latter. In this model it is assumed that every failed unit first goes under the repair of ordinary repairman who starts repair after getting the instructions from the expert.

The system has been analysed by making use of semi–Markov processes and regenerative point technique. Various measures of system effectiveness have been obtained. Study through graphs is also made for a particular case.

Notations

λ : constant failure rate of a unit
p_1 : probability that the ordinary repairman is able to complete the repair
q_1: probability that the ordinary repairman is unable to complete the repair
a: probability that the ordinary repairman does not damage the unit
b_2: probability that the ordinary repairman damages the unit.
g(t), G(t): p.d.f. and c.d.f. of the repair time of the ordinary repairman
i(t),I(t): p.d.f. and c.d.f. of time when expert gives instruction to the ordinary repairman
$g_1(t)$, $G_1(t)$: p.d.f. and c.d.f. of repair time of the expert repairman when resume repair policy is adopted
$g_2(t)$, $G_2(t)$: p.d.f. and c.d.f. of repair time of the expert repairman when repeat repair policy (type-I) is adopted
$g_3(t)$, $G_3(t)$: p.d.f. and c.d.f. of repair time of the expert repairman when repeat repair policy (type-II) is adopted.

Symbols for the state of the system are

o : operative unit
cs : cold standby unit
F_{ei} : failed unit under instructions of expert repairman
F_{ur} : failed unit under repair of ordinary repairman
F_R : repair of the failed unit by the ordinary repairman is continuing from previous state
F_{re_1} : Failed unit under repair of the expert repairman when resume repair policy is adopted
F_{Re_1} : Repair of the failed unit by the expert repairman is continuing from the previous state under resume repair policy.
F_{re_2} : Failed unit under repair of the expert repairman when repeat repair policy (type-I) is adopted
F_{Re_2} : Repair of the failed unit by the expert repairman is continuing from the previous state when repeat repair policy (type-I) is adopted
F_{re_3} : Failed unit under repair of the expert repairman when repeat repair policy (type-II) is adopted
F_{Re_3} : Repair of the failed unit by the expert repairman is continuing from the previous state when repeat repair policy (type-II) is adopted
F_w: failed unit waiting for repair

Every failed unit first goes under the repair of ordinary repairman who starts repair after getting instructions from expert.

Transition Probabilities and Mean Sojourn Times

The transition diagram showing the various states of the system is shown as in Fig 1.1. The epochs of entry into states 0, 1, 2, 3, 4, 8 and 9 are regeneration points and thus 0, 1, 2, 3, 4, 8 and 9 are regenerative states. States 5, 6, 7, 8 and 9 are failed states.

The non–zero elements $p_{ij} = \lim_{s\to 0} q_{ij}^*(s)$ are:

$p_{01} = p_{12} = 1$, $p_{20} = p_1\, g^*(\lambda)$, $p_{23} = q_1\, a\, g^*(\lambda)$

$p_{24} = q_1\, b_2 g^*(\lambda)$, $p_{25} = 1 - g^*(\lambda)$, $p_{21}^{(5)} = p_1(1-g^*(\lambda))$

$p_{28}^{(5)} = q_1\, a(1-g^*(\lambda))$, $p_{29}^{(5)} = q_1\, b_2(1 - g^*(\lambda))$

$p_{30} = g_1^*(\lambda)$, $p_{36} = 1- g_1^*(\lambda)$, $p_{31}^{(6)} = 1 - g_1^*(\lambda)$

$p_{40} = g_3^*(\lambda)$, $p_{47} = 1 - g_3^*(\lambda)$, $p_{41}^{(7)} = 1-g_3^*(\lambda)$

$p_{81} = p_{91} = 1$

By these transition probabilities it can be verified that

$p_{01} = p_{12} = p_{81} = p_{91} = 1$, $p_{20} + p_{23} + p_{24} + p_{25} = 1$, $p_{20} + p_{23} + p_{24} + p_{21}^{(5)} + p_{28}^{(5)} + p_{29}^{(5)} = 1$

$p_{30} + p_{36} = 1$, $p_{30} + p_{31}^{(6)} = 1$, $p_{40} + p_{47} = 1$, $p_{40} + p_{41}^{(7)} = 1$

The mean sojourn time (μ_i) in state i are

$\mu_0 = \dfrac{1}{\lambda}$, $\mu_1 = -i^{*\prime}(0)$, $\mu_2 = \dfrac{1-g_2^*(\lambda)}{\lambda}$, $\mu_3 = \dfrac{1-g_1^*(\lambda)}{\lambda}$, $\mu_4 = \dfrac{1-g_3^*(\lambda)}{\lambda}$, $\mu_8 = -g_1^{*\prime}(0)$, $\mu_9 = -g_3^{*\prime}(0)$

The unconditional mean time taken by the system to transit for any state j when it is counted from epoch of entrance into state i is mathematically stated as:

$$m_{ij} = \int_0^\infty t\, q_{ij}(t)\, dt = -q_{ij}^{*\prime}(0)$$

Thus, $m_{01} = \mu_0$; $m_{12} = \mu_1$

$m_{20} + m_{23} + m_{24} + m_{25} = \mu_2$

$m_{20} + m_{23} + m_{24} + m_{21}^{(5)} + m_{28}^{(5)} + m_{29}^{(5)} = k_1$

$m_{30} + m_{36} = \mu_3$; $m_{30} + m_{31}^{(6)} = \mu_8$

$m_{40} + m_{47} = \mu_4$; $m_{40} + m_{41}^{(7)} = \mu_9$

$m_{81} = \mu_8$; $m_{91} = \mu_9$

Mean Time to System Failure (MTSF) = $T_0 = N/D$

Steady-State-Availability Analysis = $A_0 = N_1/D_1$

Busy Period Analysis of the Ordinary Repairman = $B_0 = N_2/D_1$

Busy Period Analysis of the Expert Repairman (Repair Time Only) = $B_0^e = N_3/D_1$

Expected Instruction Time = $IT_0 = N_4/D_1$

Expected Number of Visits by the Ordinary Repairman = $V_0 = N_5/D_1$

Expected Number of Visits by the Expert Repairman = $V_0^e = N_6/D_1$

where

$N = \mu_0 + \mu_1 + \mu_2 + p_{23}\,\mu_3 + p_{24}\mu_4$

$D = 1 - p_{20} - p_{23}\, p_{30} - p_{24}\, p_{40}$

$N_1 = [p_{20} + p_{23}\, p_{30} + p_{24}\, p_{40}]\mu_0 + \mu_1 + \mu_2 + p_{23}\mu_3 + p_{24}\mu_4$

$D_1 = [p_{20} + p_{23}\, p_{30} + p_{24}\, p_{40}]\mu_0 + k_1 + (1 + p_{23})\mu_8 + (1 + p_{24})\mu_9$

$N_2 = k_1$

$N_3 = (p_{23} + p_{28}^{(5)})\mu_8 + (p_{24} + p_{29}^{(5)})\mu_9$

$N_4 = \mu_1$

$N_5 = 1$

$N_6 = 1 + p_{23}\, p_{30} + p_{24}\, p_{40}$

Profit Analysis

The expected total profit incurred to the system in steady-state is given by

$P_{52} = C_0A_0 - C_1B_0 - C_2 B_0^e - C_4V_0 - C_5V_0^e - C_8IT_0$

where

C_0 = revenue per unit up time of the system

C_1 = cost per unit time for which ordinary repairman is busy

C_2 = cost per unit time for which expert repairman is busy in repairing the unit

C_4 = cost per visit of the ordinary repairman

C_5 = cost per visit of the expert repairman

C_8 = cost per unit time for which expert repairman is busy in giving the instructions to the ordinary repairman

Particular Case

For graphical interpretation, the following particular case is considered.

$g(t) = \alpha e^{-\alpha t}$; $g_1(t) = \alpha_1 e^{-\alpha_1 t}$ $g_3(t) = \alpha_3 e^{-\alpha_3 t}$; $i(t) = \gamma e^{-\gamma t}$

Therefore, we get

$p_{01} = p_{12} = 1$, $p_{20} = \dfrac{p_1\alpha}{\alpha+\lambda}$, $p_{23} = \dfrac{q_1 a\alpha}{\alpha+\lambda}$, $p_{24} = \dfrac{q_1 b_2 \alpha}{\alpha+\lambda}$, $p_{21}^{(5)} = \dfrac{p_1\lambda}{\alpha+\lambda}$, $p_{28}^{(5)} = \dfrac{q_1 a\lambda}{\alpha+\lambda}$, $p_{29}^{(5)} = \dfrac{q_1 b_2\lambda}{\alpha+\lambda}$, $p_{25} = \dfrac{\lambda}{\alpha+\lambda}$, $p_{30} = \dfrac{\alpha_1}{\alpha_1+\lambda}$, $p_{36} = \dfrac{\lambda}{\alpha_1+\lambda}$, $p_{31}^{(6)} = \dfrac{\lambda}{\alpha_1+\lambda}$, $p_{40} = \dfrac{\alpha_3}{\alpha_3+\lambda}$, $p_{47} = \dfrac{\lambda}{\alpha_3+\lambda}$, $p_{41}^{(7)} = \dfrac{\lambda}{\alpha_3+\lambda}$, $\mu_0 = \dfrac{1}{\lambda}$, $\mu_1 = \dfrac{1}{\gamma}$, $\mu_2 = \dfrac{1}{\alpha+\lambda}$, $\mu_3 = \dfrac{1}{\alpha_1+\lambda}$, $\mu_4 = \dfrac{1}{\alpha_3+\lambda}$, $\mu_8 = \dfrac{1}{\alpha_1}$, $\mu_9 = \dfrac{1}{\alpha_3}$, $k_1 = \dfrac{1}{\alpha}$

Using the above equations, we can have the expressions for the MTSF and profit for this particular case.

On the basis of the numerical values taken as:

$p_1 = 0.5$, $q_1 = 0.5$, $a = 0.5$, $b_2 = 0.5$, $\gamma = 10$, $\alpha = 1$, $\alpha_1 = 2.5$, $\alpha_3 = 1$, $\lambda = 0.05$

The values of various measures of system effectiveness are obtained as:

Mean time to system failure (MTSF) = 335.9117

Availability (A_0) = 0.9322464

Busy period of the ordinary repairman (B_0) = 0.0463804

Busy period of the expert repairman (B_0^e) = 0.01623314

Expected instruction time (IT_0) = 0.004628042

Expected number of visits by the ordinary repairman (V_0) = 0.04638042

Expected number of visits by the expected repairman

(V_0^e) = 0.067724

Graphical Interpretation

For the graphical interpretation, the mentioned particular case is considered. Fig. 1.2 and 1.3 show the behaviour of MTSF and Availability with respect of failure rate (λ). It is clear from the graph that MTSF and Availability both get decrease with increase in the values of failure rate and both are higher for higher values of repair rate (α).

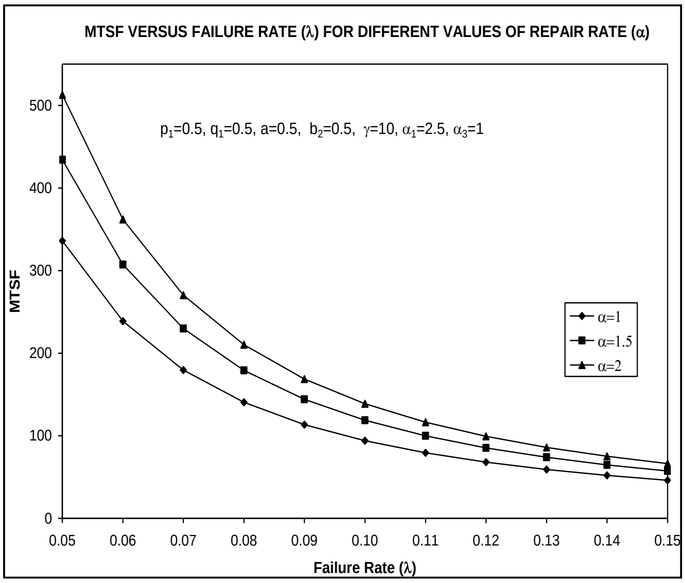

Fig 1.2

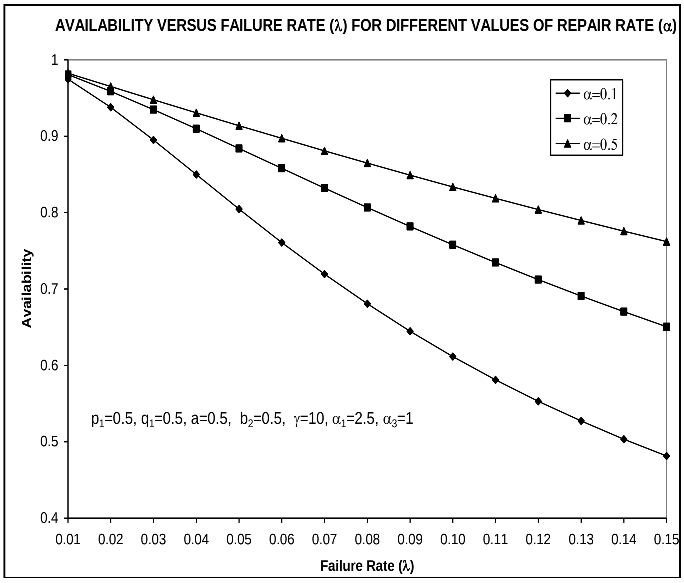

Fig 1.3

Fig. 1.4 reveals the pattern of the profit with respect to failure rate (λ) for different values of repair rate (α). It is observed that the profit decreases with the increase in the values of λ and is higher for higher values of repair rate (α). Following is also observed from the graph:

(i) For $\alpha = 1$, the profit is positive or zero or negative according as λ < or = or > 0.101. Thus, the system is not profitable if $\lambda \geq 0.101$.

(ii) For $\alpha = 1.5$, the profit is positive or zero or negative according as λ <or = or> 0.112. Thus, the system is not profitable if $\lambda \geq 0.112$.

(iii) For $\alpha = 2$, the profit is positive or zero or negative according as λ < or = or > 0.119. Thus, the system is not profitable if $\lambda \geq 0.119$.

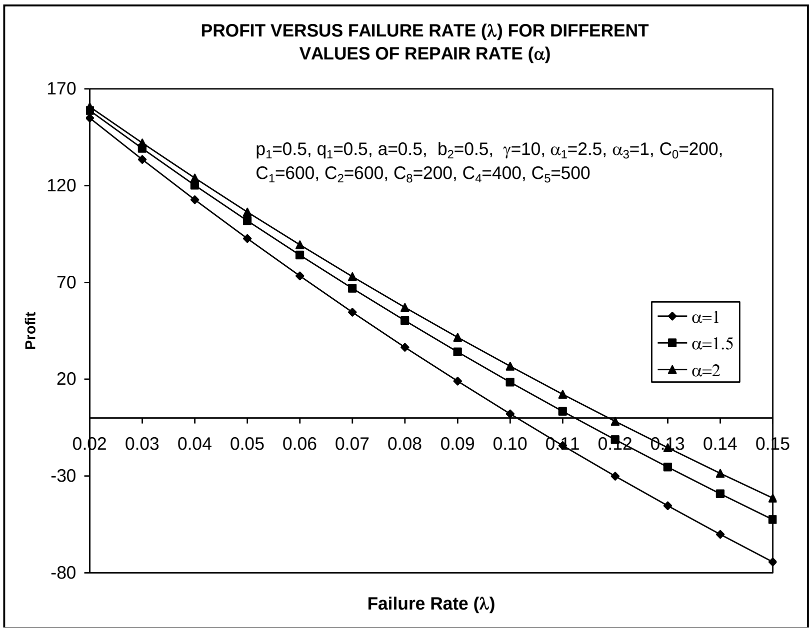

Fig 1.4

Fig. 1.5 shows the behaviour of the profit with respect to revenue (C_0) for different values of cost (C_2). The profit increases as C_0 increases and becomes lower for higher values of C_2. Following conclusions can be drawn:

(i) For $C_2 = 1000$, the profit is positive or zero or negative when C_0 > or = or < 106.08.

(ii) For $C_2 = 3000$, the profit is positive or zero or negative when C_0 > or = or < 141.4.

(iii) For $C_2 = 5000$, the profit is positive or zero or negative according as C_0 > or = or < 173.86.

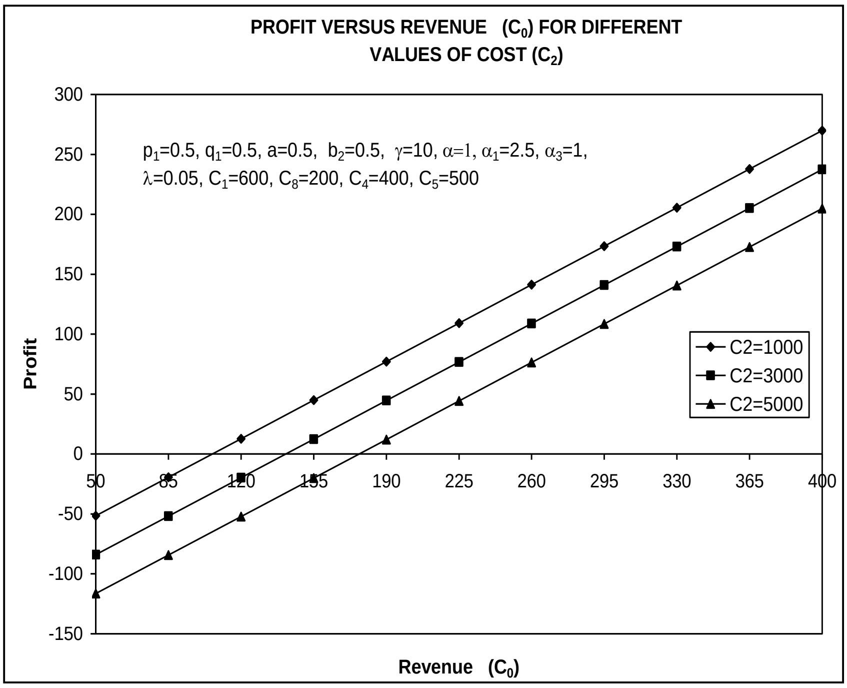

Fig 1.5

Fig. 1.6 depicts the behaviour of the profit with respect to cost (C_5) for different values of instruction rate (γ). It is clear that the profit decreases as cost (C_5) increases and becomes higher for higher values of instruction rate (γ). Following can be concluded from the graph:

(i) For $\gamma = 1$, the profit is positive or zero or negative according as C_5 < or = or > 806.05.

(ii) For $\gamma = 2$, the profit is positive or zero or negative according as C_5 < or = or > 831.75.

(iii) For $\gamma = 10$, the profit is positive or zero or negative according as C_5 < or = or > 852.3.

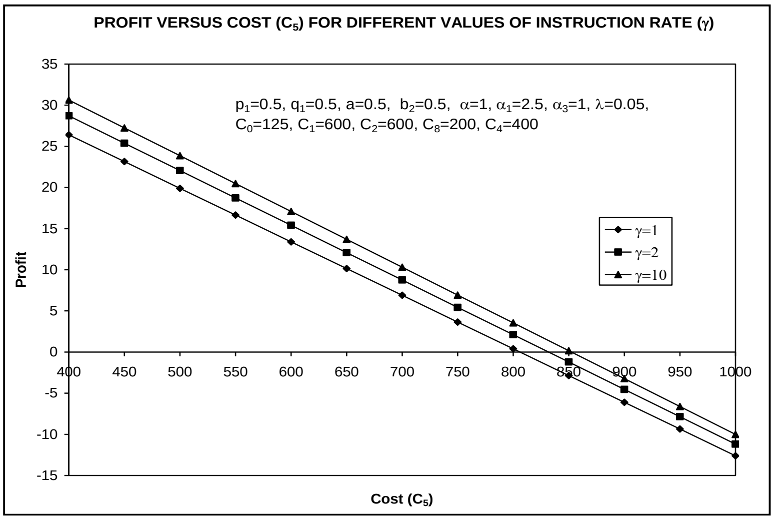

Fig 1.6

Fig. 1.7 reveals the pattern of profit with respect to probability (p_1) for different values of probability (a). The profit increases as p increases and becomes higher for higher values of probability (a). It is also observed that change in the values of probability (a) does not affect the profit significantly as $p_1 \rightarrow 1$. Following is also observed from this figure:

(i) For a = 0.2, the profit is positive or zero or negative according as p_1 > or = or < 0.538.
(ii) For a = 0.4, the profit is positive or zero or negative according as p_1 > or = or < 0.501.
(iii) For a = 0.6, the profit is positive or zero or negative according as p_1 > or = or < 0.455.

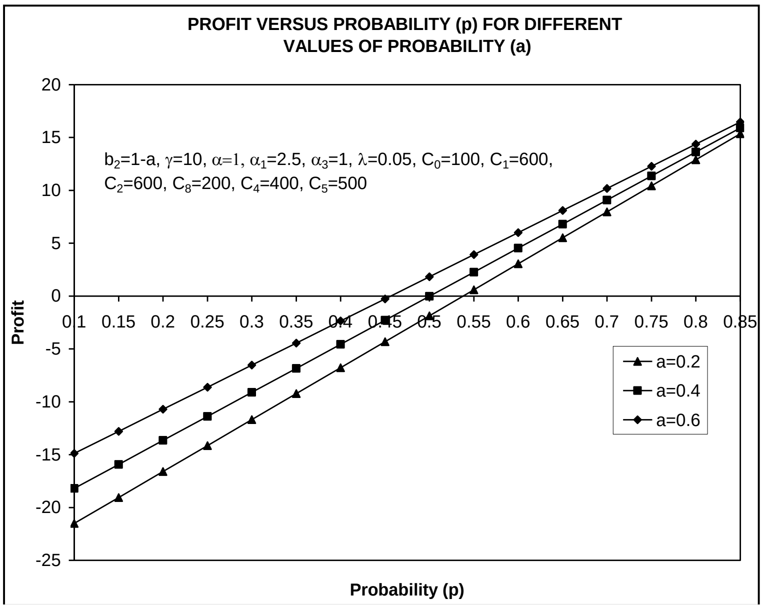

Fig 1.7

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