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Comparision of profit analysis of a two – Unit cold standby system with instruction and preparation time for repair

Rashmi Gupta and Suresh Kumar Gupta

Abstract
The comparative study of the profit of a two-unit cold standby system with instruction and preparation time for repair is carried out. The repair man (who is an expert) does not come alone but he comes with his assistant. If the expert repair man is busy in repairing a failed unit and second unit fails then the assistant repairman repairs the latter unit after getting instructions from expert repairman. The paper consists of two models. In model-1 expert repairman takes the unit for repair from his assistant if he completes the repair earlier than his assistant whereas in model-2 either of the repairman (expert or his assistant) do not leave the repair till its completion.

Keywords: Unit cold standby system, assistant repairman, expert repairman

Introduction
Many investigations concerning the reliability of two-unit system are being made. Various authors including [1-3] discussed two-unit cold standby systems assuming that whenever the operative unit fails, it goes under repair immediately i.e. preparation time for repair is negligible. Guo Tong De [4] and some others studied two-unit systems with preparation time for repair and with one repairman serving at a time. However, practically there may be situations when on failure of a unit the expert repairman does not come alone but he comes with his assistant. The assistant repairman repairs the failed unit only after getting instructions from expert which are given at the time of system failure so that both the failed unit go under repair of both the repairman – one under expert and the other under his assistant.

So, in this paper, we have investigated a two-unit cold stand by system with instruction and preparation time for repair. Whenever a unit fails, we call an expert repairman immediately to repair the failed unit. He comes with his assistant who does the repair of failed unit perfectly only if instructions are given to him by the expert. The expert repairman first prepare himself for repair i.e. he makes the arrangement of tools and other essential materials which are necessary for repair the failed unit and then he starts repair. If a unit is under repair/preparation for repair of the expert and at that time a second unit fails the repairman leaves the repair/preparation for repair of former unit and starts giving instruction to his assistant. It is assumed that after getting instructions the assistant repairman repairs perfectly. Former unit goes under repair of expert and the later under repair of his assistant with the assumptions that both the repairmen first prepare themselves for repair. The paper consists of two models. In model-1 expert repairman takes the unit for repair from his assistant if he completes the repair earlier than his assistant whereas in model-2 either of the repairman (expert or his assistant) do not leave the repair till its completion.

The system is analysed by making use of semi-Markov-Renewal Processes and regenerative processes and determine the expressions for the various measures of system effectiveness such as mean time to system failure (MTSF), steady-state availability, total fraction of busy time of the repairman, expected number of visits by the expert repairman and expected profit earned by the system. Graphs are plotted and comparison between models is made through graphs.

Notations

o/cs operative/cold-standby

F_{up} unit in F mode and under preparation for repair

F_{re}/F_{ra} unit in F-mode and under repair of expert/assistant.

F_{Re} unit in F-mode and repair is continued by an expert repairman from earlier state.

F_{wri} unit in F-mode and waiting for repair when instructions are being given to assistant.

λ Constant failure rate of an operative unit.

μ Constant repair rate of assistant repairman.

h(t), H(t) pdf and cdf of time to preparation for repair of failed unit.

g(t), G(t) pdf and cdf of time to repair by expert.

i(t), I(t) pdf and cdf of time when expert gives instructions to his assistant.

For rest of the notations see reference [5].

Model - 1

In this model, the expert takes the unit for repair from his assistant if he completes the repair earlier than his assistant. Thus considering the above symbols, possible states of the system and the transitions into the states are shown in Fig. 1. The epoch of entrance into the states 6 from 5 is non regenerative.

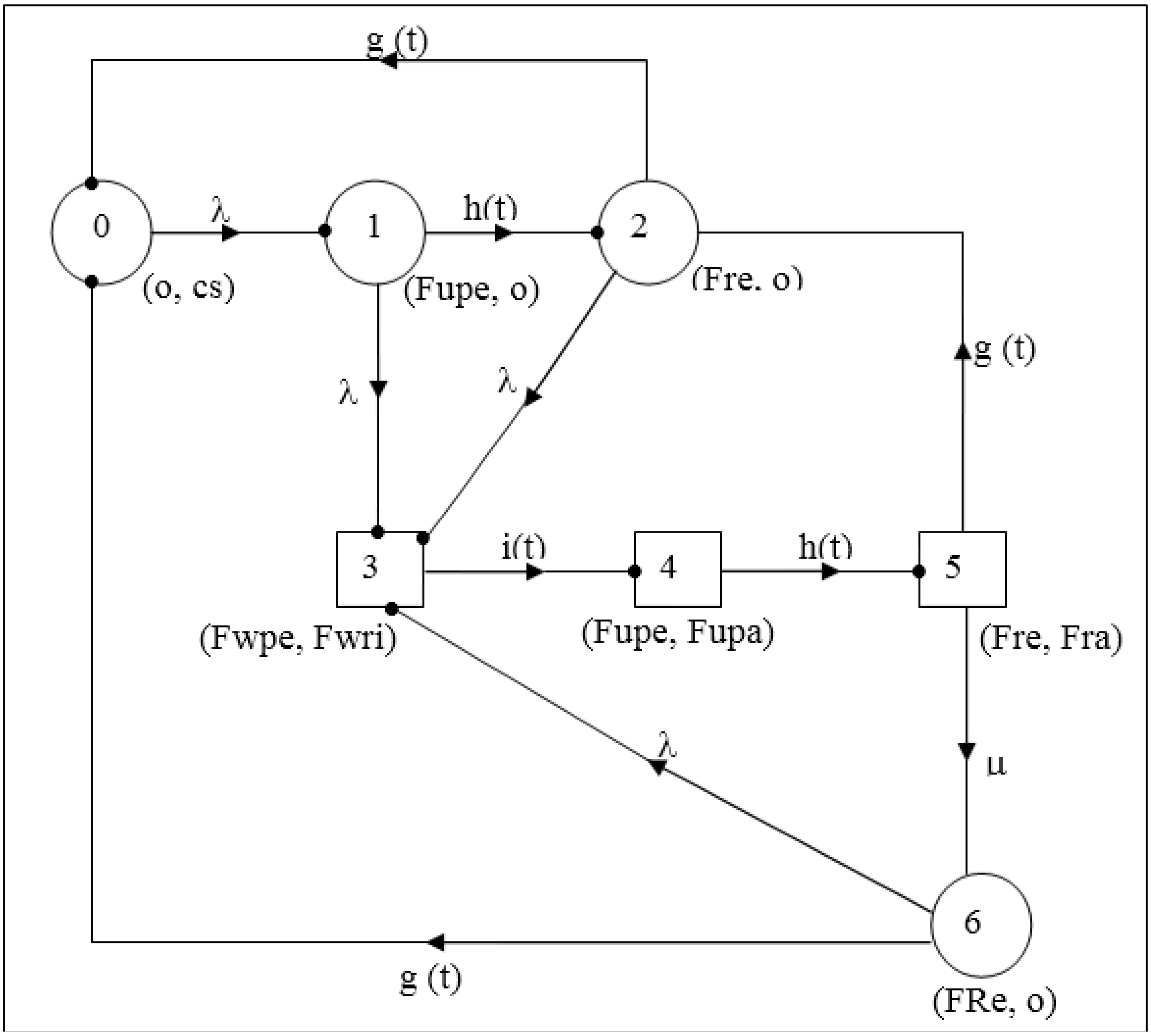
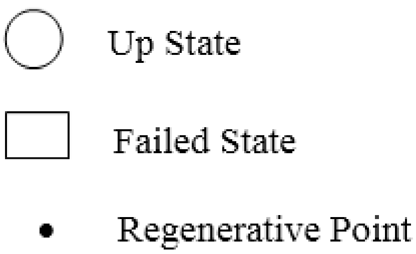


Fig 1: State Transition Diagram



Transition Probabilities and Soujourn Times
The non-zero elements p_{ij} of the transition probability matrix (t.p.m.) for the system are as follows:

$p_{01} = 1$, $p_{12} = h^*(\lambda)$, $p_{13} = 1 - h^*(\lambda)$, $p_{20} = g^*(\lambda)$
$p_{23} = 1 - g^*(\lambda)$, $p_{34} = 1$, $p_{45} = 1$, $p_{52} = g^*(\mu)$,
$p_{50}^{(6)} = \mu[g^*(\mu) - g^*(\lambda)]/(\lambda-\mu)$, $p_{53}^{(6)} = [\mu g^*(\lambda) - \lambda g^*(\mu) + (\lambda-\mu)]/(\lambda-\mu)$

By these transition probabilities, it can be verified that

$p_{01} = p_{34} = p_{45} = 1$, $p_{20} + p_{23} = p_{12} + p_{13} = 1$
$$p_{52} + p_{50}^{(6)} + p_{53}^{(6)} = 1. \tag{1-3}$$

Let T_i be the sojourn time in state S_i and $\mu_i = E(T_i)$, then using the formula
$\mu_i = \int p(T_i > t)dt$
then for $\phi_0^{**}(s)$, we have

$$\phi_0^{**}(s) = \frac{Q_{01}^{**}(s)[Q_{13}^{**}(s) + Q_{12}^{**}(s)Q_{23}^{**}(s)]}{1 - Q_{01}^{**}(s)Q_{12}^{**}(s)Q_{20}^{**}(s)} \tag{4}$$

Now the MTSF, given that the system started at the beginning of state 0 is

$$T_0 = \lim_{s\to 0} \frac{1-\phi_0^{**}(s)}{s} \tag{5}$$

Using L′ Hospital's rule and substituting the value of $\phi_0^{**}(s)$ from equation (4), we have

$$T_0 = \frac{\mu_0 + \mu_1 + \mu_2 p_{12}}{1 - p_{12}p_{20}} \tag{6}$$

Availability Analysis
As defined, $M_i(t)$ denotes the probability that the system starting in up state (regenerative state) is up at time t without passing through any regenerative state. Thus we have

$$M_0(t) = e^{-\lambda t}, \qquad M_1(t) = e^{-\lambda t}\bar{H}(t)$$
$$M_2(t) = e^{-\lambda t}\bar{G}(t), \qquad M_5(t) = \left[\mu e^{-\mu t} © e^{-\lambda t}\right]\bar{G}(t) \tag{7-10}$$

and

$A_0(t) = M_0(t) + q_{01}(t) © A_1(t)$
$A_1(t) = M_1(t) + q_{12}(t) © A_2(t) + q_{13}(t) © A_3(t)$
$A_2(t) = M_2(t) + q_{20}(t) © A_0(t) + q_{23}(t) © A_3(t)$
$A_3(t) = q_{34}(t) © A_4(t)$
$A_4(t) = q_{45}(t) © A_5(t)$
$A_5(t) = M_5 + q_{52}(t) © A_2(t) + q_{50}^{(6)} © A_0(t) + q_{53}^{(6)} © A_3(t)$ (11-16)

Taking the Laplace transform of the above equations and solving them for $AV_0^*(s)$ and then steady-state availability of the system is given by

$$A_0 = \lim_{t\to\infty} A_0(t) = \lim_{s\to 0} A_0^*(s) = \frac{N_1}{D_1} \tag{17}$$

where
$$N_1 = (\mu_0 + \mu_1)(1 - p_{53}^{(6)} - p_{23}p_{52}) + \mu_2(p_{12}p_{50}^{(6)} + p_{52}) + \varepsilon_1(1 - p_{12}p_{20}) \tag{18}$$
where $M_5^*(0) = \varepsilon_1 = \mu(\mu_5 - \mu_2)/(\lambda-\mu)$ (say)

$$D_1 = (\mu_2 + (\mu_0 + \mu_1)p_{20})p_{52} + (\mu_0 + \mu_1 + \mu_2 p_{12})p_{50}^{(6)} + (\mu_3 + \mu_4 + m_1)(1 - p_{12}p_{20}) \tag{19}$$

where $m_1 = m_{52} + m_{50}^{(6)} + m_{53}^{(6)} = (\lambda\mu_5 - \mu\mu_2)/(\lambda - \mu)$

Busy-Period Analysis of an Expert Repairman

$B_0(t) = q_{01}(t) © B_1(t)$
$B_1(t) = W_1(t) + q_{12}(t) © B_2(t) + q_{13}(t) © B_3(t)$
$B_2(t) = W_2(t) + q_{20}(t) © B_0(t) + q_{23}(t) © B_3(t)$
$B_3(t) = W_3(t) + q_{34}(t) © B_4(t)$
$B_4(t) = W_4(t) + q_{45}(t) © B_5(t)$
$B_5(t) = W_5(t) + q_{52}(t) © B_2(t) + q_{50}^{(6)}(t) © B_0(t) + q_{53}^{(6)}(t) © B_3(t)$ (20-25)

where

$$W_1(t) = e^{-\lambda t}\bar{H}(t), \qquad W_2(t) = e^{-\lambda t}\bar{G}(t), \qquad W_3(t) = \bar{I}(t)$$
$$W_4(t) = \bar{H}(t), \qquad W_5(t) = e^{-\mu t}\bar{G}(t) + [\mu e^{-\mu t} © e^{-\lambda t}]\bar{G}(t)$$

Solving equations (20-25) for B*(s) with the help of Laplace-transform and then in a steady-state, the total fraction of the time for which the expert repairman is busy, is given by

$$B_0 = \lim_{s \to 0} sB_0^*(s) = \frac{N_2}{D_1} \tag{26}$$

$$N_2 = \mu_1(1 - p_{53}^{(6)} - p_{23}p_{52}) + \mu_2(p_{52} + p_{12}p_{50}^{(6)})(\mu_3 + \mu_4 + \varepsilon_1)(1 - p_{12}p_{20}) \tag{27}$$

and D_1 is already specified.

Expected Number of Visits by the Expert Repairman

$V_0(t) = Q_{01}(t)$ Ⓢ $[1 + V_1(t)]$
$V_1(t) = Q_{12}(t)$ Ⓢ $V_2(t) + Q_{13}(t)$ Ⓢ $V_3(t)$
$V_2(t) = Q_{20}(t)$ Ⓢ $V_0(t) + Q_{23}(t)$ Ⓢ $V_3(t)$
$V_3(t) = Q_{34}(t)$ Ⓢ $V_4(t)$
$V_4(t) = Q_{45}(t)$ Ⓢ $V_5(t)$
$V_5(t) = Q_{52}(t)$ Ⓢ $V_2(t) + Q_{50}^{(6)}(t)$ Ⓢ $V_0(t) + Q_{53}^{(6)}(t)$ Ⓢ $V_3(t)$ (28-33)

Taking Laplace-Stieltjes transform and solving the above equations for $V_0^{**}(s)$ and in steady-state the number of visits per unit time is given by

$$V_0 = \lim_{t \to \infty}\left[\frac{V_0(t)}{t}\right] = \lim_{s \to 0} s.V_0^{**}(s) = \frac{N_3}{D_1} \tag{34}$$

Where

$$N_3 = p_{20}p_{52} + p_{50}^{(6)} \tag{35}$$

and D_1 is already specified.

Profit Analysis

The expected total profit incurred to the system in a steady-state is

$$P = K_0AV_0 - K_1B_0 - K_2V_0 \tag{36}$$

where K_0 is the revenue per unit up-time of the system, K_1 is the cost per unit time for which the expert repairman is busy, K_2 is the cost per unit visits by the expert repairman.

Particular Case

Assume that the preparation time for repair, repair time of expert repairman and instruction time are exponentially distributed as follows:

$h(t) = \alpha \exp(-\alpha t)$ $\qquad$ $g(t) = \beta \exp(-\beta t)$ $\qquad$ $i(t) = \gamma \exp(-\gamma t)$

Then, we have

$p_{01} = 1$, $p_{12} = \alpha/(\alpha + \lambda)$, $p_{13} = \lambda/(\alpha + \lambda)$, $p_{20} = \beta/(\beta + \lambda)$
$p_{23} = \lambda/(\beta + \lambda)$, $p_{34} = p_{45} = 1$, $p_{52} = \beta/(\beta + \mu)$, $p_{50}^{(6)} = \beta\mu/(\beta+\mu)(\beta+\lambda)$, $p_{53}^{(6)} = \lambda\mu/(\beta+\mu)(\beta+\lambda)$,
$\mu_0 = 1/\lambda, \mu_1 = 1/(\alpha+\lambda)$, $\mu_2 = 1/(\beta+\lambda)$ $\mu_3 = 1/\gamma = (\text{Mean instruction time})$,
$\mu_4 = 1/\alpha = (\text{Mean preparation time})$, $\mu_5 = 1/(\beta+\mu)$

$$\text{MTSF} = \frac{(\alpha+\beta)\left[(\beta+\lambda)(\alpha+\lambda)+\alpha\lambda\right]}{\lambda^2(\alpha+\lambda)(\alpha+\beta+\lambda)}$$

Using the above equations and equations (17), (26), (34) and finally equation (36), we get the expected total profit in study state.

Model 2
This model is discussed with the additional assumption that either of the repairman (expert and his assistant) do not leave the repair of unit till its completion. The transition diagram showing the various rate of transition is given in Fig. 2. The various characteristics are obtained as follows:

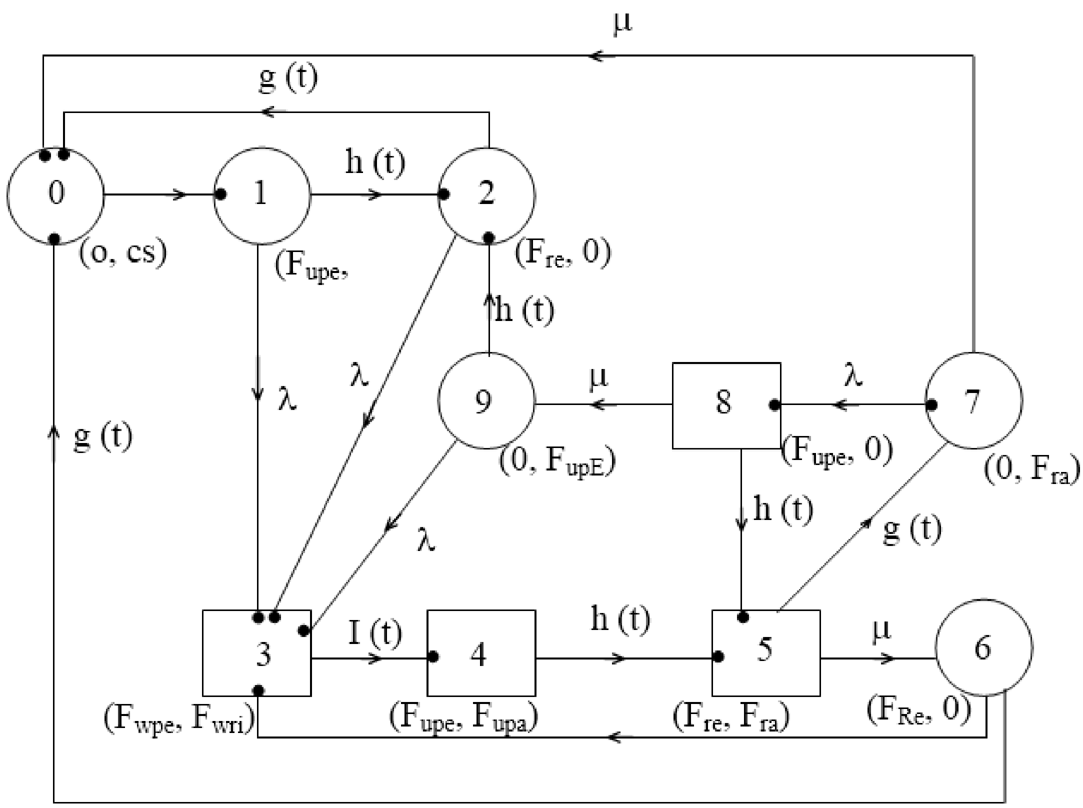


Fig 2: State Transition Diagram for Model – 2

By probabilistic arguments, the non-zero elements p_{ij} are given below:-

$$p_{01} = p_{34} = p_{48} = 1, \quad p_{12} = h^*(\lambda) =, \; p_{13} = (1-h^*(\lambda)), \; p_{20} = g^*(\lambda), \; p_{23} = (1-g^*(\lambda))$$
$$p_{50}^{(*)} = \mu[g^*(\mu)-g^*(\lambda)]/(\lambda-\mu), \; p_{53}^{(*)} = [\mu g^*(\lambda)-\lambda g^*(\mu)+(\lambda-\mu)]/(\lambda-\mu)$$
$$p_{57} = g^*(\mu). \; p_{70} = \mu/(\lambda+\mu), \; p_{78} = \lambda/(\lambda+\mu), \; p_{82}^{(*)} = \mu[h^*(\mu)-h^*(\lambda)]/(\lambda-\mu)$$
$$p_{83}^{(*)} = [\mu h^*(\lambda)-\lambda h^*(\mu)+(\lambda-\mu)]/(\lambda-\mu), \; p_{85} = h^*(\mu) \qquad (37–49)$$

It can be verified that

$$p_{12} + p_{13} = p_{20} + p_{23} = 1, \; p_{50}^{(6)} + p_{53}^{(6)} + p_{57} = 1, \; p_{52}^{(9)} + p_{53}^{(9)} + p_{80} = 1 \qquad (50–53)$$

Also μ_1, the mean sojourn in sate S_1 are

$$\mu_0 = 1/\lambda, \; \mu_1 = [1-h^*(\lambda)]/\lambda, \; \mu_2 = [1-g^*(\lambda)]/\lambda$$

$\mu_2 = \int_0^\infty \bar{I}(t)dt = \int_0^\infty td(t)$ = (Mean instruction time)

$\mu_4 = \int_0^\infty \bar{H}(t)dt = \int_0^\infty t\, d\bar{H}(t)$ = (Mean preparation time)

$\mu_5 = [1-g^*(\mu)]/\mu$, $\mu_7 = 1/(\lambda+\mu)$, $\mu_8 = [1-h^*(\mu)]/\mu$ (54–61)

The unconditional mean time taken by the system to transit for any regenerative state I, when it is counted from the epoch of entrance into that state is, mathematically, stated as

$$m_{ij} = \int tdQ_{ij}(t) = q_{ij}(s)|_S = 0 \quad (62)$$

$$m_{01} = 1/\lambda,\ m_{34} = i^{*\prime}(0),\ m_{46} = -h^{*\prime}(0)$$
$$m_{12} = -h^{*\prime}(\lambda),\ m_{13} = h^{*\prime}(\lambda) + [(1-h^*(\lambda))/\lambda]$$
$$m_{20} = -g^{*\prime}(\lambda),\ m_{23} = g^{*\prime}(\lambda) + [(1-h^*(\lambda))/\lambda]$$
$$m_{50}^{(6)} = \mu[g^{*\prime}(\lambda) - g^{*\prime}(\lambda) + [(1-g^*(\lambda))/\lambda]$$
$$m_{53}^{(6)} = \left[\lambda g^{*\prime}(\mu) - \mu g^{*\prime}(\lambda) + \frac{\lambda(1-g^*(\mu))}{\mu} - \frac{\mu(1-g^*(\lambda))}{\lambda}\right]/(\lambda-\mu)$$
$$m_{57} = g^{*\prime}(\mu),\ m_{70} = \mu/(\lambda+\mu)^2, \quad m_{78} = \lambda/(\lambda+\mu)^2$$
$$m_{82}^{(9)} = \mu[h^{*\prime}(\lambda) - h^{*\prime}(\mu)]/(\lambda-\mu)$$
$$m_{83}^{(9)} = [\lambda h^{*\prime}(\mu) - \mu h^{*\prime}(\lambda) + \frac{\lambda(1-h^*(\mu))}{\mu} - \frac{\mu(1-h^*(\lambda))}{\lambda}](\lambda-\mu),\ m_{85} = -h^{*\prime}(\mu)$$

From these are conclude that

$$m_{01} = \mu_0,\ m_{34} = \mu_3,\ m_{45} = \mu_4,\ m_{12} + m_{13} = \mu_1,\ m_{20} + m_{23} = \mu_2$$
$$m_{50}^{(6)} + m_{53}^{(6)} + m_{57} = [\lambda\mu_5 - \mu\mu_2]/(\lambda-\mu) = m_1(\text{say})$$
$$m'_{70} + m_{75} = \mu_7,\ m_{82}^{(9)} + m_{83}^{(9)} + m_{85} = [\lambda\mu_8 - \mu\mu_1]/(\lambda-\mu) = m_2(\text{say}) \quad (65\text{–}70)$$

Mean Time to System Failure

The expression for MTSF will remain same as explained in previous Model –1

Availability Analysis

$A_0(t) = M_0(t) + q_{01}(t)$ © $A_1(t)$

$A_1(t) = M_1(t) + q_{12}(t)$ © $A_2(t) + q_{13}(t)$ © $A_3(t)$

$A_2(t) = M_2(t) + q_{20}(t)$ © $A_0(t) + q_{22}(t)$ © $A_3(t)$

$A_3(t) = q_{34}(t)$ © $A_4(t)$

$A_4(t) = q_{45}(t)$ © $A_5(t)$

$A_5(t) = M_5(t) + q_{50}^{(6)}(t)$ © $A_0(t) + q_{53}^{(6)}(t)$ © $A_3(t) + q_{57}(t)$ © $A_7(t)$

$A_7(t) = M_7(t) + q_{70}(t)$ © $A_0(t) + q_{78}(t)$ © $A_8(t)$

$A_8(t) = M_8(t) + q_{82}^{(9)}(t)$ © $A_2(t) + q_{83}^{(9)}(t)$ © $A_3(t) + q_{85}(t)$ © $A_5(t)$

(71–78)

Taking Laplace transform of equations and the letting $s \to 0$, we get

$$M_0^*(0) = \mu_0, \quad M_1^*(0) = \mu_1, \quad M_2^*(0) = \mu_2, \quad M_7^*(0) = \mu_7$$
$$M_5^*(0) = \mu[\mu_5 - \mu_2](\lambda-\mu) = \in_1 (\text{say})$$
$$M_8^*(0) = \mu[\mu_8 - \mu_1]/(\lambda-\mu) = \in_2 (\text{say}) \quad (79\text{–}84)$$

The steady–state availability of the system is given by

$$A_0 = \lim_{s\to 0}[sA_0^*(s)] = N_1/D_1 \quad (85)$$

where

$N_1 = [1 - p_{57}\, p_{78}\, p_{85} - (p_{53}^{(6)} + p_{57}p_{78}p_{83}^{(9)})][\mu_0 + \mu_2 p_{12}] + [\in_1 + p_{57}(\mu_7 + \in_2\, p_{78})]$
$\times [p_{12}\, p_{23} + p_{13}] - p_{57}\, p_{78}\; p_{82}^{(9)}\; [\mu_0\, p_{23} + \mu_1\, p_{23} - \mu_2\, p_{13}]$

and

$D_1 = (\mu_0 + \mu_1)\,(p_{20}p_{57}p_{78}\; p_{82}^{(9)} + p_{57}\, p_{70} + p_{50}^{(8)})\_ + \mu_2(p_{57}\, p_{78}\; p_{82}^{(9)} + p_{12}\, p_{57}\, p_{70} + p_{12}\; p_{50}^{(6)}) +$
$(1 - p_{12}p_{20})[(\mu_3 + \mu_4)(1 - p_{57}p_{78}p_{85}) + m_1 + p_{57}(\mu_7 + m_2 p_{78})]$ (86–87)

Busy–Period Analysis For The Expert Repairman

$B_0(t) = q_{01}(t) © B_1(t)$
$B_1(t) = W_1(t) + q_{12}(t) © B_2(t) + q_{13}(t) © B_3(t)$
$B_2(t) = W_2(t) + q_{20}(t) © B_0(t) + q_{23}(t) © B_3(t)$
$B_3(t) = W_3(t) + q_{34}(t) © B_4(t)$
$B_4(t) = W_4(t) + q_{45}(t) © B_5(t)$
$B_5(t) = W_5(t) + q_{50}^{(6)}(t) © B_0(t) + q_{53}^{(6)}(t) © B_3(t) + q_{57}(t) © B_7(t)$
$B_7(t) = W_7(t) + q_{70}(t) © B_0(t) + q_{78}(t) © B_8(t)$
$B_8(t) = W_8(t) + q_{82}^{(9)}(t) © B_2(t) + q_{83}^{(9)}(t) © B_3(t) + q_{85}(t) © B_5(t)$ (88–95)

Taking Laplace transform of equations and letting $s \to 0$, we have

$$W_1^*(0) = \mu_1,\quad W_2^*(t) = \mu_2,\; W_3^*(0) = \mu_3,\; W_4^*(0) = \mu_4,\quad W_5^*(0) = (\mu_5 + \in_1),\quad W_7^*(0) = \mu_7$$
$$W_8^*(0) = (\mu_5 + \in_2) \qquad (96–102)$$

In steady–state the total fraction of time for which the system is under repair is given by

$$B_0^* = \lim_{s\to 0}[sB_0^*(s)] = N_2 / D_1 \qquad (103)$$

Where

$N_2 = (1 - p_{57}\, p_{78}\, p_{85})\,[\mu_1 + p_{12}(\mu_2 + p_{23}(\mu_3 + \mu_4)) + p_{13}(\mu_3 + \mu_4)] - [(p_{53}^{(6)} + p_{57}p_{78}p_{83}^{(9)})$
$\times (\mu_1 + \mu_2\, p_{12}) + (\mu_5 + \in_1 + p_{57}(\mu_7 + (\mu_8 + \in_2)p_{78}))] - p_{57}p_{78}\; p_{82}^{(9)}\,[\mu_1 p_{23} + \mu_2 p_{13} + p_{13}]$ (104)

and D_1 is already specified.

Expected Number Of Visits By The Expert Repairman

$V_0(t) = Q_{01}(t)$ Ⓢ $[1 + V_1(t)]$
$V_1(t) = Q_{12}(t)$ Ⓢ $V_2(t) + Q_{13}(t)$ Ⓢ $V_3(t)$
$V_2(t) = Q_{20}(t)$ Ⓢ $V_0(t) + Q_{23}(t)$ Ⓢ $V_3(t)$
$V_3(t) = Q_{34}(t)$ Ⓢ $V_4(t)$
$V_4(t) = Q_{45}(t)$ Ⓢ $V_5(t)$
$V_5(t) = Q_{50}^{(6)}(t)$ Ⓢ $V_0(t) + V_{53}^{(6)}(t)$ Ⓢ $V_3(t) + Q_{57}(t)\, V_7(t)$
$V_7(t) = Q_{70}(t)$ Ⓢ $V_0(t) + Q_{78}(t)$ Ⓢ $[1 + V_8(t)]$
$V_8(t) = Q_{82}^{(9)}(t)$ Ⓢ $V_2(t) + Q_{53}^{(9)}(t)$ Ⓢ $V_3(t) + Q_{85}(t)$ Ⓢ $V_5(t)$ (105–112)

In steady–state, the expected number of visits per unit time is given by

$$V_0 = \lim_{t\to\infty}\left[\frac{V_0(t)}{t}\right] = \lim_{s\to 0}[sV_0^{**}(s)] = N_3 / D_1 \qquad (113)$$

Where $N_3 = 1 - p_{53}^{(6)} + p_{20}p_{57}p_{78}(p_{82}^{(9)} - p_{12})$ (114)

Profit analysis

The expected total profit incurred to the system in a steady–state is

$P_2 = K_0A_0 - K_1B_0 - K_2V_0$

Where

K_0 = Revenue per unit up time of the system

K_1 = Cost per unit time for which the expert repairman is busy

K_2 = Cost per visit by the expert repairman.

Particular Case

Assume that the preparation time to repair time of expert repairman and instruction tiem are exponentially distributed as follows :

$h(t) = \alpha \exp(-\alpha t)$, $g(t) = \beta \exp(-\beta t)$, $i(t) = \gamma \exp(-\gamma t)$

Then, we have

$p_{01} = 1$, $p_{12} = \alpha/(\alpha + \lambda)$, $p_{13} = \lambda/(\alpha + \lambda)$, $p_{20} = \beta/(\beta + \lambda)$, $p_{23} = \lambda / (\beta + \lambda)$,

$p_{34} = p_{45} = 1$, $p_{50}^{(6)} = \beta\mu / (\beta + \mu)(\beta + \lambda)$, $p_{53}^{(6)} = \lambda\mu / (\beta + \mu)(\beta + \lambda)$,

$p_{57} = \beta / (\beta + \mu)$ $p_{70} = \mu/(\mu + \lambda)$, $p_{78} = \lambda / (\mu + \lambda)$,

$p_{82}^{(9)} = \alpha\mu / (\alpha + \mu)(\alpha + \lambda)$ $p_{83}^{(9)} = \lambda\mu / (\alpha + \mu)(\alpha + \lambda)$, $p_{85} = \lambda / (\alpha + \mu)$.

Also mean sojourn times are :

$\mu_0 = 1 / \lambda$, $\mu_1 = 1(\alpha + \lambda)$, $\mu_2 = 1/(\beta + \lambda)$, $\mu_2 = 1/\gamma$,

$\mu_4 = 1/\alpha$, $\mu_5 = 1/(\beta + \mu)$, $\mu_7 = 1(\lambda + \mu)$, $\mu_8 = 1/(\alpha + \mu)$

$m_1 = (\alpha + \lambda + \mu)/(\alpha + \mu)(\alpha + \lambda)$, $m_2 = (\beta + \lambda + \mu) / (\beta + \mu)(\beta + \lambda)$

$\in_1 = \mu/(\beta + \mu)(\beta + \lambda)$, $\in_2 = \mu/(\alpha + \mu)(\alpha + \lambda)$

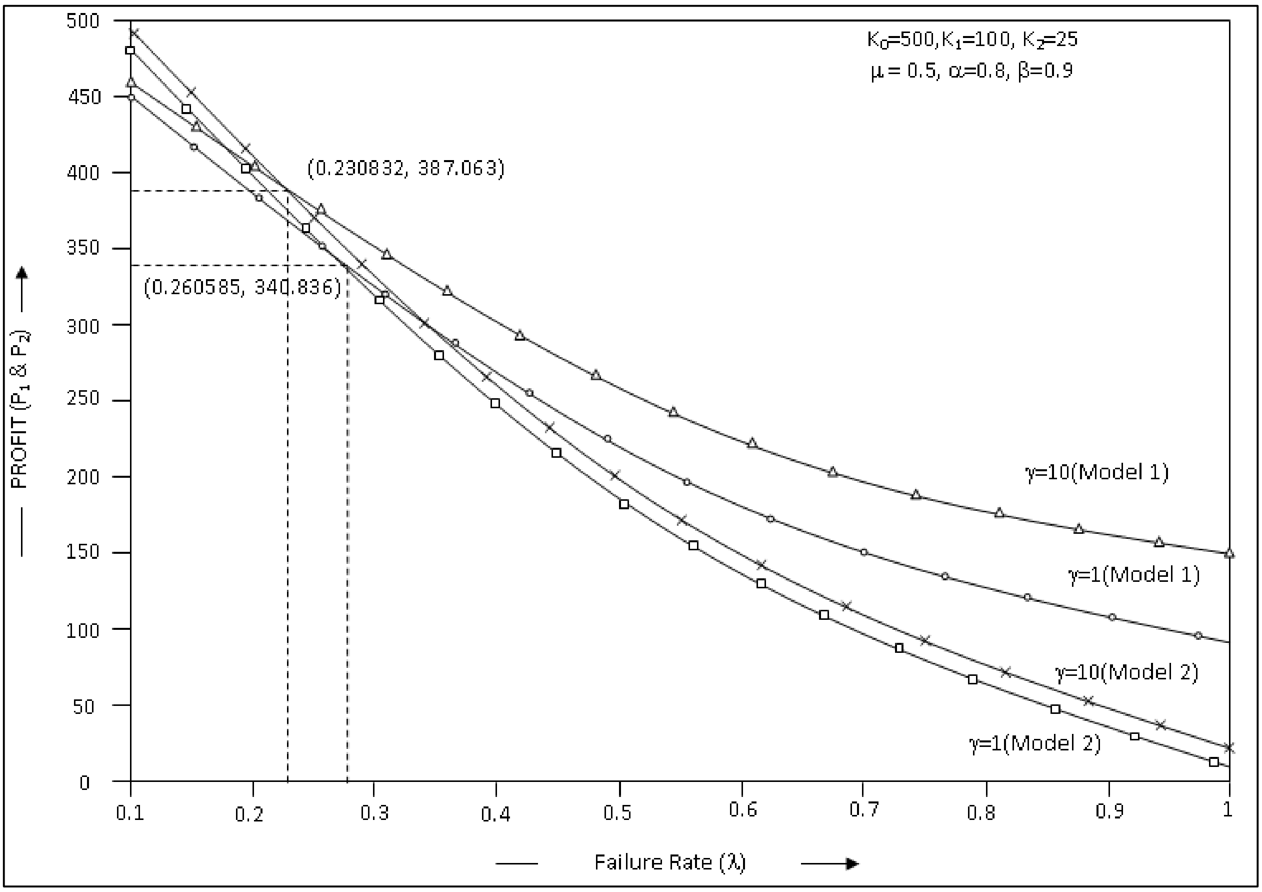
Comparative Study of Profit for Model–1 and Model–2

Fig. 3 shows the behaviour of profits of Model–1 and Model–2 with respect to failure rate (λ) for two values of instructions rate (γ = 1 & 10), keeping other parameters fixed. From the graph, it is interpreted that :

(i) Profit of each model decreases with a increase in failure rate (λ) while it increases with the increases in instructions rate (γ).

(ii) When $\gamma = 1$: If $\lambda = 0.260585$, the profits of two models coincide and equal to 340.836. If $\lambda < 0.260585$ then $P_2 > P_1$ i.e. Model–2 is better than Model–1. If $\lambda > 0.260585$ then $P_1 > P_2$ i.e. Model–1 is better than Model–2.

(iii) When $\gamma = 10$: If $\lambda = 0.230832$, the profits of both the models coincide and equal to 387.063. If $\lambda < 0.230832$ then $P_2 > P_1$ i.e. Model–2 is better than Model–1. If $\lambda > 0.230832$ then $P_1 > P_2$ i.e. Model–1 is better than Model–2.



□ $\lambda(P_2) = 1$ × $\gamma(P_2) = 10$ ○ $\gamma(P_1) = 1$ Δ $\gamma(P_1) = 10$

Fig 3: Comparison between profit of Model 1 and 2.

References

1. Barlow RE, Proschan F. Mathematical Theory of Reliability, John Wiley, New York, 1965.
2. Mokaddis GS, El-said KHM. Two models for two-unit warm standby redundant system with two different kinds of repair and tolerable time, Microelectron. Reliab. 1990; 30:327-374.
3. Tuteja RK, Gulshan Taneja. Proper utilization of a system with two identical repairmen, Int. J. of Management and Systems, 1991; 7:273-281
4. Guo Tong De. Stochastic behaviour of a two-unit cold standby system with preparation time for repair, Microelectron. Reliab. 1989; 29:125-129.
5. Tuteja RK, Gulshan Taneja. Cost-benefit analysis of a two-server, two-unit, Warm standby system with different types of failure, Microelectron. Reliab. 1992; 32:1353-1359.
6. Rander MC, Suresh K Gupta, Ashok Kumar. Cost Analysis of a two dissimilar cold standby system with preventive maintenance and replacement of standby system, Microelectron Reliab. 1994; 34(1):171-174,
7. Ashok Kumar, Suresh K Gupta, Tuteja RK. Cost benefit analysis of a two-unit cold standby system with instruction time, IAPQR Transactions, 1997; 22(2):127-133
8. Ashok Kumar, Suresh K Gupta, Gulshan Taneja. Probabilistic analysis of a two-unit cold standby system with instructions at need, Microelection. Reliab. 1995; 35(5):829-832.
9. Kumar A, Gupta SK, Taneja G. Comparative study of the profit of a two server system including patience time and instruction time. 1996; 36(10):1595-1601.
10. Suresh Kumar Gupta. Profit analysis of a two – unit cold standby system with instruction and preparation time for repair, International Journal of Advances in Engineering Sciences, 2016; 6(4).