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Complex numbers: Its operations and properties

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Abstract
Complex number is a combination of real and imaginary parts. It is denoted by the symbol 'i'. In this paper, different operations on complex numbers are discussed. There is a detailed description of properties of these operations. It also tells about how to find the modulus and inverse of a complex number. Different representations of the complex number are also highlighted in this paper. Its applications are also mentioned.

Keywords: Complex number, operations

Introduction

Complex numbers: Combination of real and imaginary parts of a number is called a Complex number. Real part can have any value like 0, 1, 2,…, n. Whereas the imaginary part looks same as the real part but it consists of iota which is denoted by i. Iota is considered to be the solution of the quadratic equation $x^2 = -1$
2+3i is an example of complex number.
Complex numbers help us to find solutions of the problem which do not have a real solution.
The iota follows same exponential rules as other numbers.
Powers of iota:
$i = -1^{1/2}$
$i^2 = -1$
$i^3 = -i$
$i^4 = 1$

Purely real complex number: A complex number is called purely real complex number if its imaginary part is 0.

Purely imaginary complex number: A complex number is called purely imaginary complex number if its real part is 0.

Operations on complex numbers

Addition: The addition in complex numbers follows only one rule. Real part is added to the real part and imaginary part is added to the imaginary part only. The sum of two complex numbers is always a complex number. This is known as the closure law for addition.

Example: (3 + 2i) + (5 + 6i) = (8 + 8i)

Properties for addition of complex numbers

- The addition of complex numbers is commutative, i.e., $z_1 + z_2 = z_2 + z_1$
- The addition of complex numbers is associative, i.e., $(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)$
- There always exist an additive identity I such that z + I = z

Subtraction: The subtraction in complex numbers follows only one rule. Real part is subtracted from the real part and imaginary part is subtracted from the imaginary part only. The difference of two complex numbers is always a complex number. This is known as the closure law for subtraction.

Example: (5 + 6i) - (2 + 3i) = (3 + 3i).

Properties for subtraction of complex numbers

- The addition of complex numbers can never be commutative
- There always exist an subtractive identity I such that z - I = z

Multiplication: Multiplication in complex numbers follows the simple algebraic procedure. Real part is multiplied with both real as well as imaginary parts. Same is the rule for multiplication of imaginary part. The product of two complex numbers is always a complex number. This is known as the closure law for multiplication.

Example: (2 + 3i) * (3 + 2i)
= $6 + 4i + 9i + 6i^2$
= 6 + 13i - 6
⇨ (2 + 3i) * (3 + 2i) = 13i

Properties for multiplication of complex numbers

- The multiplication of complex numbers is commutative, i.e., $z_1.z_2 = z_2.z_1$
- The multiplication of complex numbers is associative, i.e., $(z_1.z_2).z_3 = z_1.(z_2.z_3)$
- There always exist an multiplication identity I such that z.I = z
- There always exist an multiplicative inverse z^{-1} such that $z.z^{-1} = 1$
- The distribution law for multiplication of complex number is followed.

$z_1 (z_2 + z_3) = z_1z_2 + z_1z_3$ (left distribution law)
$(z_2 + z_3) z_1 = z_2z_1 + z_3z_1$ (right distribution law)

Division: Division of complex number is a 2 step process:
Step 1: Find conjugate of denominator: The sign of the imaginary part in the denominator is changed to obtain the conjugate.

Example: Conjugate of 2 + 3i is 2 - 3i.

Step 2: Divide and multiply the equation with the conjugate of denominator and simplify.

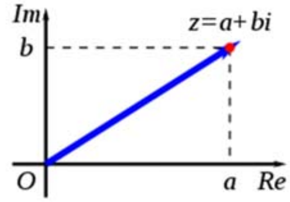
Example: $(2 + 3i) * (3 - 4i) = 6 - 8i + 9i - 12i^2 = 6 + 12 + i = 18 + i$
$(3 + 4i) (3 - 4i) 9 - 12i + 12i - 16i^2 9 + 16 25$

Modulus of a complex number: The modulus of a complex number z = x + iy can be defined as:
$|z| = (x^2 + y^2)^{1/2}$.
The modulus of a complex number can never be negative. It is also known as absolute value.
Reciprocal/ Multiplicative inverse of a complex number:
Let z= a+ib be a complex number, then
$z^{-1=}$ (1/a+ib)*(a-ib/a-ib)
⇨ $z^{-1=}a/(a^2+b^2) + i(-b)/(a^2+b^2)$

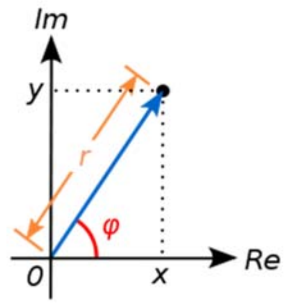
Representation of complex number

- Cartesian complex plane: The horizontal (x axis) represents the real part of the complex number while the vertical axis (y axis) represents the imaginary part of the complex number.

A purely real number is represented on the x axis and a purely imaginary number is represented on the y axis.

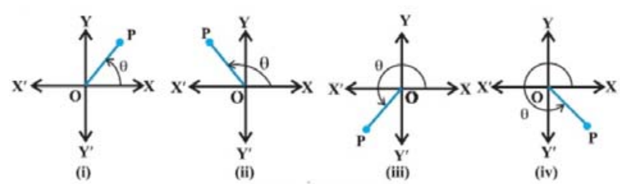


- Polar complex plane: The polar coordinate system is a two dimensional system in which distance of every point on a plane can be calculated by using a reference point and its angle with the reference direction.

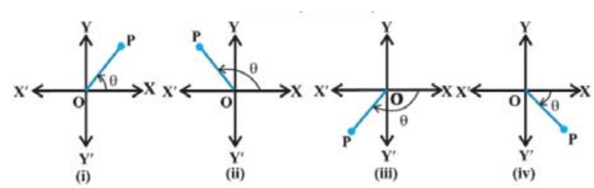


z = x + yi

It can be represented as
z = r (cos Φ + i sin Φ)
$R = |z| = (x^2 + y^2)^{1/2}$
Φ is called argument or the amplitude
For any non-zero complex number the value of Φ lies in the range 0<=Φ<=2π



0<= Φ<=2 π



- π< Φ <π

Operations in polar form

- Multiplication: Let z_1 and z_2 be two complex numbers, $z_1 = r_1(\cos \Phi_1 + i \sin \Phi_1)$,

$z_2 = r_2(\cos \Phi_2 + i \sin \Phi_2)$
$z_1.z_2 = r_1.r_2 (\cos (\Phi_1+\Phi_2) + i \sin (\Phi_1+\Phi_2)$

- Division: Let z_1 and z_2 be two complex numbers, $z_1 = r_1 (\cos \Phi_1 + i \sin \Phi_1)$,

$z_2 = r_2(\cos \Phi_2 + i \sin \Phi_2)$
$z_1 = r_1$ ($\cos (\Phi_1-\Phi_2) + i \sin (\Phi_1-\Phi_2)$)
z_2 r_2

- Inverse of a complex number:

z = cos Φ + i sin Φ
1/z or $z^{-1} = (\cos \Phi + i \sin \Phi)^{-1}$
= cos(-Φ) + i sin(-Φ)

⇨ $z^{-1} = \cos \Phi - i \sin \Phi$

Euler's formula: For any real number x, $e^{ix} = \cos x + i \sin x$, where e is the base for natural logarithm

De Moivre's theorem: For any complex number z and any integer n, De moivre's theorem can help to calculate powers of a complex number.

$(r (\cos \Phi + i \sin \Phi))^{n} = r^{n} (\cos(n\Phi)+ i \sin (n\Phi))$

It is necessary to convert a complex number into its polar form before applying de Moivre's theorem.

Applications of De Moivre's theorem:

- By using De Moivre's theorem, various trigonometric identities can be derived

Expressions like sin nΦ, cos nΦ, tan nΦ can be expressed in terms of sin Φ, cos Φ, tan Φ

- To find n^{th} root of a unity for a non-zero complex number z, which is the solution for equation $z^{n} = 1$

The cube roots of unity are $1, \omega, \omega^2$ and where $\omega^{3r} = 1$

The cube roots of -1 are $-1, -\omega, -\omega^2$

- To express $\cos^{n} \Phi$ and $\sin^{n} \Phi$ in terms of angles in multiple.

Complex number and coordinate geometry

- Straight lines: the general equations of straight lines is $a^{|}z + az^{|} + b = 0$, where a is a complex number and b is a real number.
- Triangle: ABC is a triangle with points A (z_1), B (z_2), C (z_3)

The centroid is given by $z = 1/3 (z_1 + z_2 + z_3)$

- Circle: Equation of a circle is $zz^{|} - az^{|} - a^{|}z + b = 0$, where a is a complex number and b is a real number and center of circle is –a.

Conclusion

Complex number is a combination of real and imaginary parts. It is denoted by the symbol 'i'. Its value is √(-1). Different operations on complex numbers and their properties are discussed. It also tells how to find the modulus and inverse of a complex number. Different representations of the complex number are the Cartesian form and the polar form. These forms are explained briefly. Its applications are also mentioned.

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