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A Sugumaran
Department of Mathematics, Government Arts College, Tiruvannamalai, Tamil Nadu, India

P Vishnu Prakash
Department of Mathematics, Government Arts College, Tiruvannamalai, Tamil Nadu, India

Correspondence
A Sugumaran
Department of Mathematics, Government Arts College, Tiruvannamalai, Tamil Nadu, India

Some new families of 3-equitable prime cordial graphs

A Sugumaran and P Vishnu Prakash

Abstract
Let $G = (V(G), E(G))$ be a graph. A 3-equitable prime cordial labeling of a graph G is a bijection f from $V(G)$ to $\{1,2, \ldots |V(G)|\}$ such that if an edge uv is assigned the label 1 if $\gcd(f(u), f(v)) = 1$ and $\gcd(f(u)+f(v), f(u)-f(v)) = 1$, the lable 2 if $\gcd(f(u), f(v)) = 1$ and $\gcd(f(u)+f(v), f(u)-f(v)) = 1$, and the label 0 otherwise, then the number of edges labeled with i and the number of edges labeled with j differ by at most 1 for $0 \le i \le 2$ *and* $0 \le j \le 2$. If a graph has a 3-equitable prime cordial labeling, then it is called a 3-equitable prime cordial graph. In this paper, we have prove that the bistar $B_{n,n}(n \ge 2)$, comb $P_n^+(n \ge 2)$, ladder $P_2 \times P_n$, kite K(3,n) and slanting ladder SL_n admit 3-equitable prime cordial labeling.

Keywords: Cordial labeling, prime cordial labeling, 3-equitable prime cordial labeling

Introduction
Graph labeling is an active area of research in graph theory which has rigorous applications in coding theory, communication networks, optimal circuits layouts and crystallography. In this paper, we consider only finite, simple undirected graphs. We consider a graph $G = (V(G), E(G))$ and we let $|V(G)| = p$ and $|E(G)| = q$. For graph theoretic notations and terminology we follow Harary [4] and for number theory we follow Burton [1]. A labeling of a graph G is a mapping that carries vertices and/or edges into a set of numbers, usually integers. An excellent survey of graph labeling and various types of graph labeling can be found in Gallian [3]. We provide the brief summary of results which will be useful for the present investigations.

Definition 1.1: *Let* G *be a graph. A mapping* $f : V(G) \to \{0,1\}$ *is called binary vertex labeling of* G *and* $f(v)$ *is called the label of the vertex* v *of* G *under* f.

For an edge $e = uv$, the induced edge labeling $f^* : E(G) \to \{0,1\}$ is given by $f^*(e) = |f(u) - f(v)|$. We denote $v_f(i)$ is the number of vertices of G having label i and $e_f(i)$ is the number of edges of G having label i under f^*.

Definition 1.2: *A binary vertex labeling* f *of a graph* G *is called a cordial labeling if* $|v_f(0) - v_f(1)| \le 1$ *and* $|e_f(0) - e_f(1)| \le 1$. *A graph* G *is called cordial if it admits cordial labeling.*
The concept of cordial labeling was introduced by Cahit [2].

Definition 1.3: A prime cordial labeling of a graph G with vertex set $V(G)$ is a bijection $f : V(G) \to \{1,2,3, \ldots, |V(G)|\}$ defined by

$$f(e=uv)=\begin{cases}1 & if \quad gcd\ (f(u), f(v))=1;\\ 0 & otherwise\end{cases}.$$

further $|e_f(0) - e_f(1)| \le 1$.

A graph which admits prime cordial labeling is called a *prime cordial graph*. The concept of prime cordial labeling was introduced by Sundaram *et al* [6].

Definition 1.4: A 3-equitabile prime cordial labeling of a graph G with vertex set $V(G)$ is a bijection $f: V(G) \rightarrow \{1,2,3, \ldots, |V(G)|\}$ defined by

$$f(e=uv)=\begin{cases}1 & if \ \gcd(f(u), f(v))=1 \ and \ \gcd(f(u)+f(v), f(u)-f(v))=1\\ 2 & if \ \gcd(f(u), f(v))=1 \ and \ \gcd(f(u)+f(v), f(u)-f(v))=2\\ 0 & otherwise\end{cases}$$

further $|e_f(i) - e_f(j)| \le 1$ for all $0 \le i, j \le 2$.

A graph which admits 3-equitable prime cordial labeling is called a 3-equitable prime cordial graph. The concept of 3-equitable prime cordial labeling was introduced by Murugesan et al. [5, 7], the following graphs are proved to have 3-equitable prime cordial labeling: path P_n, cycle C_n ($n \ge 4$), star graph K(1,n) if n ≡ 1(mod 3), complete graph K_n ($n \le 2$), cycle with one chord, cycle with twin cord and split graph.
In this paper, we prove that the following graphs are 3-equitable prime cordial graphs.

Definition 1.5: The comb P_n^+ is the graph obtained from a path P_n by attaching a pendent edge at each vertex of the path.

Definition 1.6: The ladder L_n ($n\ge 2$) is the product graph $P_2 \times P_n$ which contains 2n vertices and 3n − 2 edges.

Definition 1.7: The slanting ladder SL_n obtained from two parallel paths with vertices u_1, u_2, u_n and $v_1, v_2, \ldots v_n$ respectively, by joining each u_i with v_{i+1}, $1 \le i \le n-1$.

Definition 1.8: The kite K (3,n) is the graph obtained from a cycle C_3 with one vertex is attached by a path of length n.

Theorem 1.9 The bistar $B_{n,n}$ is a 3-equitable prime cordial graph for $n \ge 2$.

Proof: Let $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_n$ be the pendent vertices with *u, v* are the apex vertices of $B_{n,n}$. Let *G* be the graph $B_{n,n}$.
Then $|V(G)| = 2n + 2$ and $|E(G)| = 2n + 1$.
We define $f: V(G) \rightarrow 1, 2, 3, \ldots 2n + 2$ as $f(u) = 1$, $f(v) = 2$,

$$f(u_i)=\begin{cases}2i+2, & for \ i \equiv 0 (\text{mod } 3)\\ 2i+1, & otherwise\end{cases} \quad and \quad f(v_i)=\begin{cases}f(u_i)-1, & for \ i \equiv 0 (\text{mod } 3)\\ f(u_i)+1, & otherwise\end{cases}$$

From the definition of *f*, we have

$$e_f(0)=\frac{2n}{3}, e_f(1)=\frac{2n+3}{3}, e_f(2)=\frac{2n}{3} \ when \quad n \equiv 0 (\text{mod } 3)$$

$$e_f(0)=\frac{2n+3}{3}, e_f(1)=\frac{2n+1}{3}, e_f(0)=\frac{2n+1}{3} \ when \quad n \equiv 1 (\text{mod } 3)$$

$$e_f(0)=\frac{2n+2}{3}, e_f(0)=\frac{2n-1}{3}, e_f(0)=\frac{2n+2}{3} \ when \quad n \equiv 2 (\text{mod } 3)$$

From the above cases,
We conclude that $|e_f(i) - e_f(j)| \le 1$ for all $0 \le i, j \le 2$.

Hence $B_{n,n}$ is 3-equitable prime cordial graph for $n \ge 2$.

Example 1.10 The 3-equitable prime cordial labeling of graphs B(4,4) and B(9,9) are shown in Figure 1 and Figure 2 respectively.

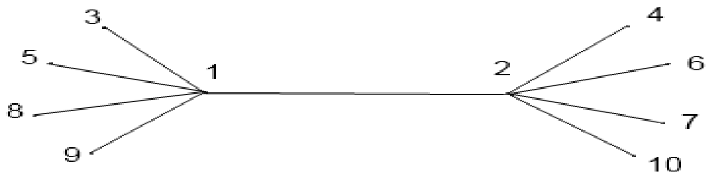

Fig 1: *B* (4, 4)

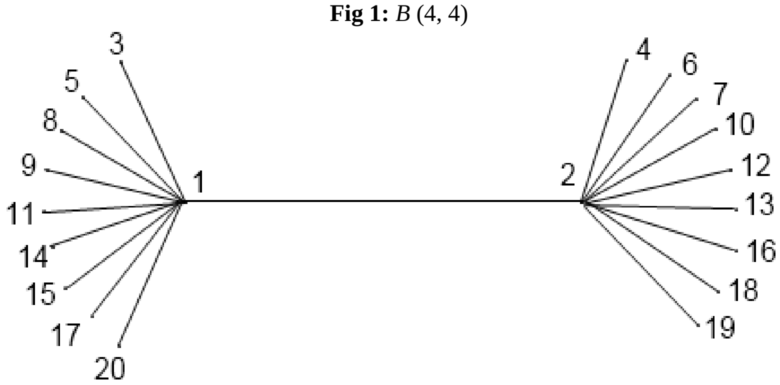

Fig 2: *B*(9,9)

Theorem 1.11 The comb P_n^+ is 3-equitable prime cordial graph for $n \geq 2$.

Proof: Let $V(P_n^+) = \{v_i, u_i : 1 \leq i \leq n\}$, where $v_1, v_2, ..., v_n$ are the vertices of path P_n and $u_1, u_2, , v_n$ are the pendent vertices in P_n^+ .Then $|V(P_n^+)| = 2n$ and $|E(P_n^+)| = 2n-1$.

To define f : $V(P_n^+) \rightarrow \{1, 2, 3..., 2n\}$, we consider two cases.

Case 1: when *n* is an even integer.
We define $f(u_1) = 1, f(u_2) = 2, f(v_1) = 3, f(v_2) = 4$.
For $3 \leq i \leq n$, define

$$f(u_i) = \begin{cases} 2i-1, & for \quad i \equiv 0 (\text{mod} \quad 3) \\ 2i-2, & for \quad i \equiv 4 (\text{mod} \quad 6) \\ 2i+2, & for \quad i \equiv 5 (\text{mod} \quad 6) \\ 2i+1, & for \quad i \equiv 1 (\text{mod} \quad 6) \\ 2i, & for \quad i \equiv 0 (\text{mod} \quad 3) \end{cases}$$

and

$$f(u_i + 2), \quad for \quad i \equiv 3,4 (\text{mod} \quad 6)$$
$$f(u_i - 2), \quad otherwise$$

From the above labeling pattern of *f*,we obtain $|e_f(i) - e_f(j)| \leq 1$ for all $0 \leq i, j \leq 2.$. Evidently *f* is a 3-equitable prime cordial labeling of P_n^+ for *n* is an even integer.

Case 2: When *n* is an odd integer.
Sub case 2.1. $n \equiv 0 (mod 3)$
We define $f(u_i)$ and $f(v_i)$ for $1 \leq i \leq n$ as same as P_{n-1}^+.
Further, we define $f(u_n) = 2n - 1$ and $f(v_n) = 2n$.

Sub case 2.2. $n \equiv 1 (mod 3)$
We define $f(u_i)$ and $f(v_i)$ for $1 \leq i \leq n - 3$ as same as P_{n-3}^+ .
Further, we define $f(u_{n-2}) = 2(n - 1)$ and $f(v_{n-2}) = 2(n - 2)$
$f(u_{n-1}) = 2n$ and $f(v_{n-1}) = 2n - 1$
$f(u_n) = 2n - 3$ and $f(v_n) = 2n - 5$

Sub case 2.3. $n \equiv 2 (mod 3)$
We define $f(u_i)$ and $f(v_i)$ for $1 \leq i \leq n - 3$ as same as P_{n-3}^+.
Further, we define $f(u_{n-2}) = 2(n - 2)$ and $f(v_{n-2}) = 2n - 2$
$f(u_{n-1}) = 2n - 5$ and $f(v_{n-1}) = 2n - 3$
$f(u_n) = 2n$ and $f(v_n) = 2n - 1$

From all the above cases, we have $| e_f(i) - e_f(j) | \leq 1$ for all $0 \leq i, j \leq 2$.

Evidently *f* is a 3-equitable prime cordial labeling of P_n^+ for *n* is an odd integer.

Example 1.12 A 3-equitable prime cordial labeling of graphs P_{10}^+ and P_{11}^+ are shown in the Figure 3 and Figure 4 respectively.

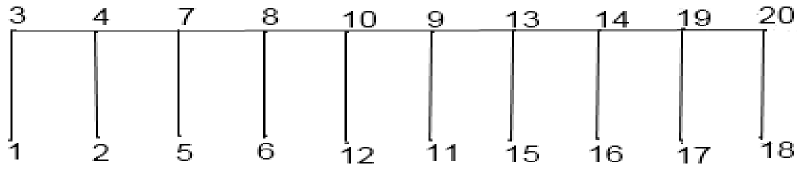

Fig 3: (P_{10}^+)

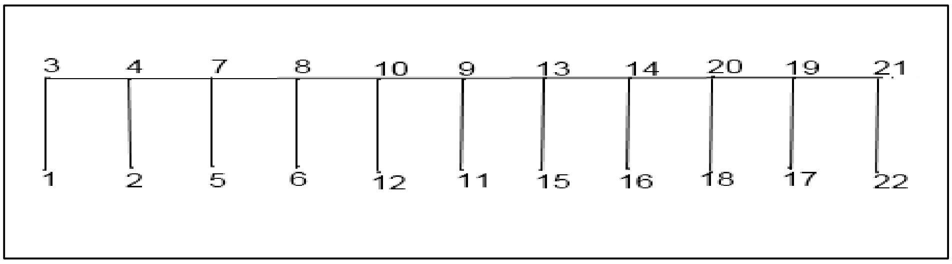

Fig 4: (P_{11}^+)

Theorem 1.13 The ladder $P_2 \times P_n$ *is a* 3-equitable prime cordial labeling *for* $n \geq 3$.
Proof: Let $u_1, u_2, \ldots u_n$ be the vertices of the lower path and $v_1, v_2, \ldots v_n$ are the vertices of of the upper path of the ladder $P_2 \times P_n$. Let G be the graph $P_2 \times P_n$. Then $| V(G) | = 2n$ and $| E(G) | = 3n\text{-}2$.
The vertex labeling of $P_2 \times P_n$ for $3 \leq n \leq 4$ is shown in the Table 1.

Table 1

n	$f(u_1)$	$f(u_2)$	$f(u_3)$	$f(u_4)$	$f(v_1)$	$f(v_2)$	$f(v_3)$	$f(v_4)$
3	6	3	1	-	2	4	5	-
4	6	3	1	5	2	4	8	7

We define the vertex labeling $f : V(G) \rightarrow \{1, 2, \ldots 2n\}$ as follows.
$f(u_1) = 6$ $f(v_1) = 2$
$f(u_2) = 3$ $f(v_2) = 4$
$f(u_3) = 1$ $f(v_3) = 8$
$f(u_4) = 5$ $f(v_4) = 7$
$f(u_i) = 2i$ for $i = 5, 6, \ldots, n$.
$f(v_j) = 2j - 1$ for $j = 5, 6, \ldots, n$.
Then $| e_f(i) - e_f(j) | \leq 1$ for all $0 \leq i, j \leq 2$.
Hence $P_2 \times P_n$ is a 3-equitable prime cordial graph.

Example 1.14 The 3-equitable prime cordial labeling of graph $P_2 \times P_9$ is shown in Fig. 5.

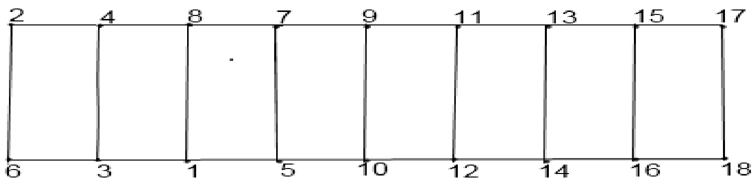

Fig 5: $(P_2 \times P_9)$

Theorem 1.15 The slanting ladder SL_n *is a* 3-equitable prime cordial labeling for $n > 2$.

Proof: Let $u_1, u_2, \ldots u_n$ be the vertices of the lower path and $v_1, v_2, \ldots v_n$ be the vertices of the upper path of slanting ladder SL_n. Let G be the graph SL_n. Then $| V(G) | = 2n$ and $| E(G) | = 3(n - 1)$.
Define $f : V(G) \rightarrow \{1, 2, \ldots 2n\}$ as follows.
$f(u_i) = 2i - 1$, $1 \leq i \leq n$ and $f(v_j) = 2j$, $1 \leq j \leq n$.

It is evident that $| e_f(i) - e_f(j) | \leq 1$ for all $0 \leq i, j \leq 2$.

Hence SL_n is a 3-equitable prime cordial graph.

Remark 1.16 *The ladder P_2 is not a* 3-*equitable prime cordial labeling.*

Example 1.14 The 3-equitable prime cordial labeling of graph SL_7 is shown in Figure 6.

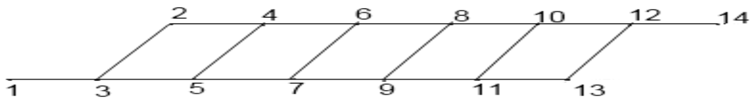

Fig 6: (SL_7)

Theorem 1.15 The Dragon (or) kite *K(3,n) is a* 3-equitable prime cordial labeling for n≥2.
Proof: Let $u_1, u_2, \ldots, u_n$ be the vertices of cycle C_3 and let $v_1, v_2, \ldots, v_n$ are the vertices of path in P_n with $v_1 = u_3$. Let G be the graph *K(3, n)*.
Then $| V(G) | = n + 2$ and $| E(G) | = n + 2$.
The vertex labeling of $f : V(G) \rightarrow \{1, 2, ..., n + 2\}$ is defined in Table 2 for $n = 1, 2, 3$.

Table 2

n	$f(u_1)$	$f(u_2)$	$f(u_3)$	$f(v_2)$	$f(v_3)$	$f(v_4)$
1	4	2	3	1	-	-
2	4	2	3	1	5	-
3	1	6	3	5	4	2

We define the vertex labeling of $K(3, n)$ for $n > 3$ is divided into three cases.

Case 1: $n \equiv 1 (\text{mod}3)$
Define $f(v_i)$ is as same as $K(3, n - 1)$ for $1 \leq i \leq n$.
Further $f(v_{n+1}) = n + 3$

Case 2: $n \equiv 2 (\text{mod}3)$
Define $f(v_i)$ is as same as $K(3, n-1)$ for $1 \leq i \leq n-1$.
Further $f(v_n) = n + 3$, $f(v_{n+1}) = n + 2$

Case 3: $n \equiv 0 (\text{mod}3)$
Define $f(v_i)$ is as same as $K(3, n - 1)$ for $1 \leq i \leq n$.
Further $f(v_{n+1}) = n + 3$
If we apply the cases 1,2 and 3 successively for $n = 4, 5, 6$, we obtain the labeling of $f(u_i)$, $f(v_j)$, where $1 \leq i \leq 3$, $1 \leq j \leq 7$.
Continue in the same manner, we obtain labeling $K(3,n)$ for any integer n.
From the above three cases, we obtain $| e_f(i) - e_f(j) | \leq 1$ for all $0 \leq i, j \leq 2$.
Hence $K(3, n)$ is a 3-equitable prime cordial graph.

2. Concluding Remarks

In the present paper we determine the 3-equitable prime cordial labeling of the graphs such as bistar, comb, ladder, kite and slanting ladder. It is an interesting open area of research to find out the graphs which admit 3-equitable prime cordial labeling.

3. References

1. David M Burton. Elementary Number theory, Second Edition, Wm. C. Brown Company Publishers, 1980.
2. Cahit I. On Cordial and 3-equitable Labelings of Graphs, Utilitas Math, 1990; 37:189-198.
3. Gallian JA. A Dyanamic Survey of Graph Labeling, Electronic Journal of Combinatorics, 2016; 16(6).
4. Harary F. Graph Theory, Addition-Wesley, Reading Mass, 1972.
5. Murugesan S. 3-equitable Prime cordial Labeling of Graphs, International Journal of Applied Information Systems. 2013; 5:1-4.
6. Sundaram M, Ponraj R, Somasundaram S. Prime cordial labeling of Graphs, Journal of Indian Academy of Mathematics, 2005; 27:373-390.
7. Dr. Sweta Srivastav, Dr. Sangeeta Gupta. 3-equitable Prime cordial Labeling of Some Graphs, International Journal of Engineering Research. 2015; 4(3):115-117.