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A mathematical model for the slip condition of the blood flow through a stenosed artery using casson-fluid flow

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Abstract
The mathematical model is developed by considering blood as Casson fluid through the asymmetric artery with the stenosis. The non-Newtonian fluid behaviour is characterized by using Casson-fluid flow model. The analytic expression for flow rate, resistance to flow and skin friction are derived by using this model. We observed that the flow rate decreases when the yield stress and viscosity coefficient increases, the skin-friction increases with the increase of yield stress in presence of slip velocity for velocity coefficient parameter. The results are derived in detail with the help of graphs for the variation of different flow parameter. Flow rate, resistance to flow, and skin-friction and impedance analysis of the casson fluid have been shown graphically by varying the yield stress.

Keywords: Stenosis, artery blood flow, flow rate, blood flow and casson fluids model

Introduction

Arteriosclerosis is often referred to as "hardening of the arteries." The word comes from the Greek words athero (meaning paste or gruel) and sclerosis (hardness). It's the process in which deposits of fatty substances, cellular waste, cholesterol, calcium and other substances build up in the inner lining of an artery. This buildup is called plaque. Plaques can block blood flow or break off. It causes a heart attack, stroke and another severe disease. There are many pieces of evidence that mathematical modeling in fluid dynamics plays a significant role in the development of blood vessel structure. Stenosis (narrowing the artery) is one of the serious cardiovascular diseases. Some researchers derived that Herschel-Bulkley fluid model and Casson fluid model are the fluid models with the yield stress, flow rate, and wall shear stress. Merrill *et al*. [16] developed a Casson-fluid model holds satisfactorily for blood flowing in tubes of diameter 130-1300, whereas Herschel-Bulkley fluid model may be employed in tubes of diameter 20-100. Several researchers have been studying on this diseases like Younge [29], Biswas, *et al* [2], chaturani *et al* [6], Shukla *et al*. [23] and many more.

An analysis on the effect of stenosis on the non-Newtonian flow of the blood in an artery by Shukla *et al* [23]. A mathematical model for the study of blood flow through a channel with permeable walls is given by Mishra *et al* [17]. The steady flow of blood through a stenosed artery is developed by Biswas *et al* [2]. The effect of time-dependent stenosis on flow through a tube is investigated by Young [29]. A mathematical model for the blood flow through narrow vessels with the mild stenosis is developed by Jain *et al* [11]. Two-layered poiseuille flow model for blood flow through arteries of small diameter and arterioles is developed by Chaturani *et al* [7]. The study of the effects of couple stresses on the blood flow through the thin artery with mild stenosis has been developed by Sinha and Singh [25]. A mathematical model for different shapes of stenosis and slip velocity at the wall through mild stenosis artery studies by Kumar *et al* [15]. The two-phase model of blood flow through stenosed tubes in the presence of a peripheral layer: Applications is derived by Srivastava [27]. The effect of peripheral layer viscosity on blood flow through an artery mild stenosis is studied by Shukla *et al* [24]. The effect of the Magnetic field on the peristaltic transport of couple stress fluid in a channel with wall properties is proposed by Sankad *et al* [22].

Effect of stenosis on the non-Newtonian flow of the blood in an artery is derived by Rao [21]. The effect of elastic property of the wall on flow characteristics through an arterial stenosis is developed by Moayeri *et al.* [18]. A nonlinear Mathematical model of blood flow in a constricted artery experiencing body acceleration is derived by Chakravarty *et al.* [5]. The steady flow of blood treated as a Bingham plastic fluid in a circular cylindrical tube studied by Fung [10]. A technique for solving the approximate equations that the steady flow in tubes having axisymmetric mild stenosis is studied by MacDonald [15]. He derived an observation that the approximate equations are also valid if the stenosis is severe. The study of blood flow is very useful to cure atherosclerosis problem.

Mathematical Analysis

Let us consider the steady blood flow through an axially symmetric but radially non-symmetric constricted artery.
The geometry of equation of stenosis is derived by:

$$\frac{R(z)}{R_0} = 1 - \frac{\delta_s}{R_0} e^{\left[-\frac{c^2 l^2 z^2}{R_0^2}\right]} \tag{1}$$

If $R(z)$ is the radius of the artery with stenosis, R_0 is the radius of the artery without stenosis, δ_s denotes the maximum height of stenosis, c is a parametric constant l is the relative length of stenosis, defined as the ratio of the radius to the half-length of stenosis. If L and L_0 are the half-lengths of arterial segment and the stenosed segment respectively, then $l = R_0 / L_0$

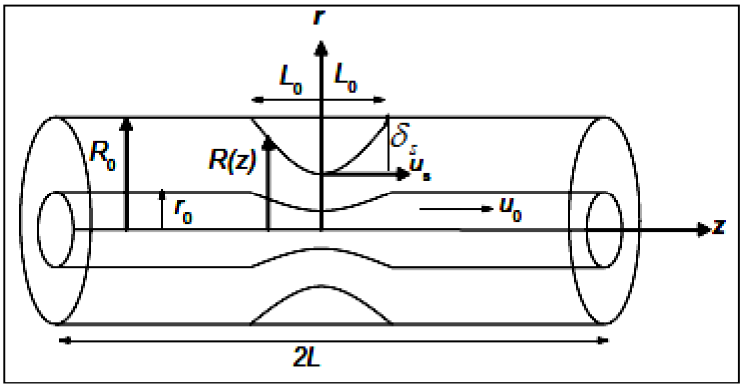

Fig 1: Geometry of arterial segment with stenosis

The geometry of equation (1) can be written as:

$$R(z) = R_0\left(1 - A e^{-Bz^2}\right) \tag{2}$$

where $A = \frac{\delta_s}{R_0}$ and $B = \frac{c^2 l^2}{R_0^2}$

The constitutive equation of Herschel-Bulkley equation is given by:

$$\left(-\frac{du}{dr}\right) = \begin{cases} \frac{(\tau - \tau')}{g}; & when \quad \tau \geq \tau' \\ 0; & when \quad \tau < \tau' \end{cases} \tag{3}$$

where u is the axial velocity of blood, g is the velocity coefficient of Casson-fluid, τ is the shear stress and τ' is the yield stress.
We consider the flow equation in the form:

$$\left(-\frac{dp}{dz}\right) = \frac{1}{r}\frac{d}{dr}(r\tau) \tag{4}$$

here p is the pressure at any point and τ is the shear stress of blood considered as Casson fluid.

Boundary Conditions: The boundary conditions are:
The velocity boundary condition on the arterial wall is the slip condition, given by:

$u = u_s$ at $r = R(z)$ (5)

The regularity condition is given by:

τ is finite at $r = 0$ (6)

using the equation. (4) into (6), we have the skin friction:

$$\tau = -\frac{r}{2}\frac{dp}{dz}$$

(7) from the equation (7) the skin friction at $R = R(z)$ can be written as:

$$\tau_s = -\frac{R}{2}\frac{dp}{dz} \tag{8}$$

from equation (7) and (8), we have:

$$\frac{\tau}{\tau_s} = \frac{r}{R}$$

which can be written as:

$$r = \frac{\tau R}{\tau_s} \tag{9}$$

and,

$$dr = \frac{R}{\tau_s} d\tau \tag{10}$$

Flow Rate: The volumetric flow rate is given by:

$$Q = \int_0^R 2\pi ru \ dr \tag{11}$$

Integration (11) and using equation (5), we have:

$$Q = \pi R^2 u_s + \pi \int_0^R \left(-\frac{du}{dr} \right) r^2 dr$$

using equation (4), we have:

$$Q = \pi R^2 u_s + \pi \int_0^r \frac{(\tau - \tau')}{g} r^2 dr$$

using equation(9) and (10), we get:

$$Q = \pi R^2 u_s + \pi \int_0^{\tau_s} \frac{(\tau - \tau')}{g} \left(\frac{\tau R}{\tau_s} \right)^2 \left(\frac{R}{\tau_s} d\tau \right)$$

$$Q = \pi R^2 u_s + \frac{\pi R^3}{g \tau_s^3} \int_0^{\tau_s} (\tau - \tau')\tau^2 dr$$

$$Q = \pi R^2 u_s + \frac{\pi R^3}{g}\left[\frac{\tau_s}{4} - \frac{\tau'}{3}\right]$$

$$Q = \pi R^2 u_s + \frac{\pi r^3 \tau_s}{4g} - \frac{\pi R^3 \tau'}{3g} \tag{12}$$

from equation (12), we can get:

$$\tau_s = \frac{4}{3}\tau' + \frac{4gQ}{\pi R^3} - \frac{4gu_s}{R} \tag{13}$$

using equation (8), we get:

$$\left(-\frac{R}{2}\frac{dp}{dz}\right) = \frac{4}{3}\tau' + \frac{4gQ}{\pi R^3} - \frac{4gu_s}{R}$$

$$\left(\frac{dp}{dz}\right) = -\frac{8}{3R}\tau' - \frac{8gQ}{\pi R^4} - \frac{8gu_s}{R^2} \tag{14}$$

Integrating equation (14) with conditions $p = p_0$ at $z = -L$ and $p = p_1$ at $z = L$, we get:

$$p_1 - p_0 = -\frac{8}{3R}\tau'\int_{-L}^{L}\left(\frac{R}{R_0}\right)^{-1} dz - \frac{8gQ}{\pi R^4}\int_{-L}^{L}\left(\frac{R}{R_0}\right)^{-4} dz - \frac{8gu_s}{R^2}\int_{-L}^{L}\left(\frac{R}{R_0}\right)^{-2} dz \tag{15}$$

Resistance Parameter: The resistance to flow λ is given by:

$$\lambda = \frac{p_1 - p_0}{Q}$$

$$\lambda = -\frac{16}{3}\left(\frac{\tau'_{,}}{R_0 Q}\right)\left[(L - L_0) + + \int_0^{L_0}\left(\frac{R}{R_0}\right)^{-1} dz\right] + -16\frac{g}{\pi R_0^4}\left[(L - L_0) + \int_0^{L_0}\left(\frac{R}{R_0}\right)^{-4} dz\right] +$$

$$16\frac{gu_s}{R_0^2}\sqrt{\frac{\tau'}{R_0^5 \pi Q}}\left[(L - L_0) + \int_0^{L_0}\left(\frac{R}{R_0}\right)^{-2} dz\right] \tag{16}$$

The resistance to flow in the dimensionless form is given by:

$$\bar{\lambda} = \frac{\lambda}{\lambda_0} \tag{17}$$

where λ_0 is resistance to flow in the absence of stenosis, using eq.(16), we get:

$$\lambda_0 = L\left[\left(-\frac{16}{3}\right)\left(\frac{\tau'}{R_0 Q}\right) - 16\left(\frac{g}{R_0^4 \pi}\right) + 16\left(\frac{gu_s}{R_0^2 Q}\right)\right] \tag{18}$$

using eq. (16) and (18) into (17), we get,

$$\bar{\lambda} = 1 - \frac{L_0}{L} + \frac{1}{L}\left[\frac{h_1\int_0^{L_0}\left(\frac{R}{R_0}\right)^{-1}dz + h_2\int_0^{L_0}\left(\frac{R}{R_0}\right)^{-4}dz + h_3\int_0^{L_0}\left(\frac{R}{R_0}\right)^{-2}dz}{(h_1+h_2+h_3)}\right] \quad (19)$$

where,

$h_1 = \left(-\frac{16}{3}\frac{\tau'}{R_0 Q}\right)$, $h_2 = \left(-\frac{16\,g}{\pi R_0^4}\right)$ and $h_3 = \left(-16\frac{g u_s}{R_0^2 Q}\right)$

using equation (2), the Equation (19) can reduced to:

$$\bar{\lambda} = 1 - \frac{L_0}{L} + \frac{1}{L}\left[\frac{h_1\int_0^{L_0}\frac{dz}{\left(1 - Ae^{-Bz^2}\right)} + h_2\int_0^{L_0}\frac{dz}{\left(1 - Ae^{-Bz^2}\right)^4} + h_3\int_0^{L_0}\frac{dz}{\left(1 - Ae^{-Bz^2}\right)^2}}{(h_1+h_2+h_3)}\right] \quad (20)$$

Results and Discussion

The analytic expressions have been derived for the resistance to flow, yield stress and wall shear stress. The expression for the resistance to flow is given by equation (20) and for the yield stress is given by equation (13).These results are plotted with using MATLAB software. In fig.2 the variation of flow rate Q is plotted with the yield stress and slip velocity, with the variation of δ_s/R_0, this shows the flow rate Q decreases with the increase of δ_s/R_0 and yield stress, but the reverse effect happens when slip velocity increases. The variation of flow rate Q is plotted with δ_s/R_0 in fig.3. The effect of yield stress and slip velocity on resistance to flow with δ_s/R_0 is represented by figs.4 and 5 It is observed that fixed value of g, resistance to flow increases with the increases with the increases of δ_s/R_0. It is also increased when both yield stress and slip velocity increase. In figs. 6 and 7 show the variation of skin-friction for different values of yield stress and slip velocity. It is noticed in these graphs that skin friction increases with the increases of δ_s/R_0. Skin-friction increases with the increase of yield stress and slip velocity for fixed values of g.

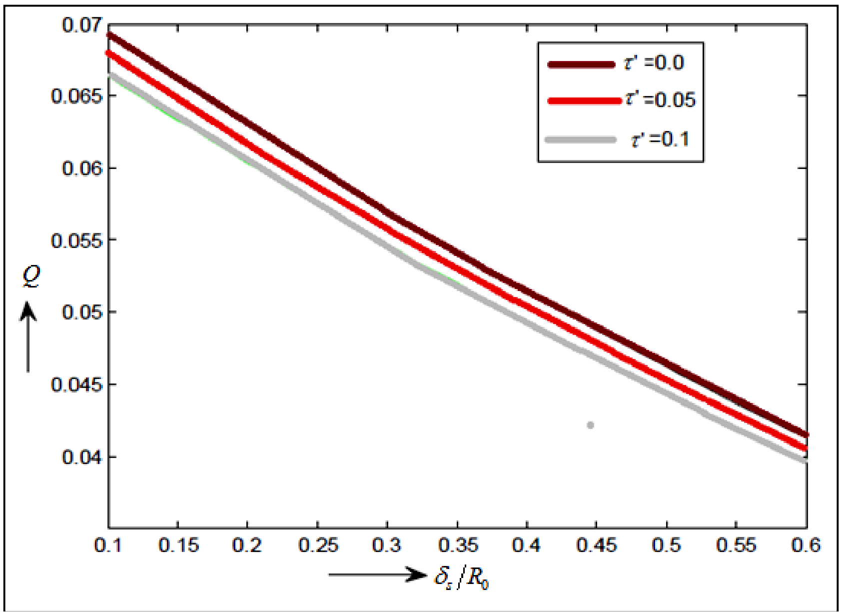

Fig 2: variation of flow rate Q with δ_s/R_0

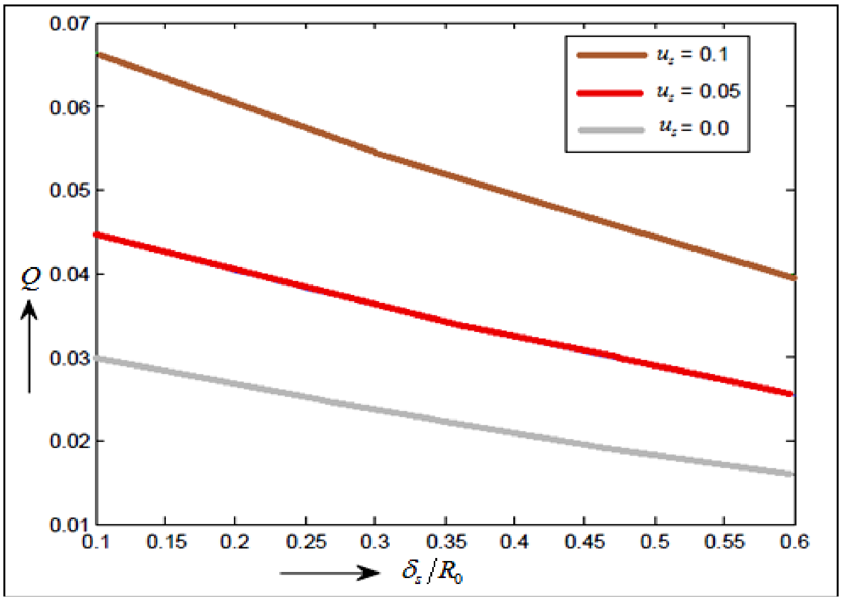

Fig 3: variation of flow rate Q with δ_s/R_0

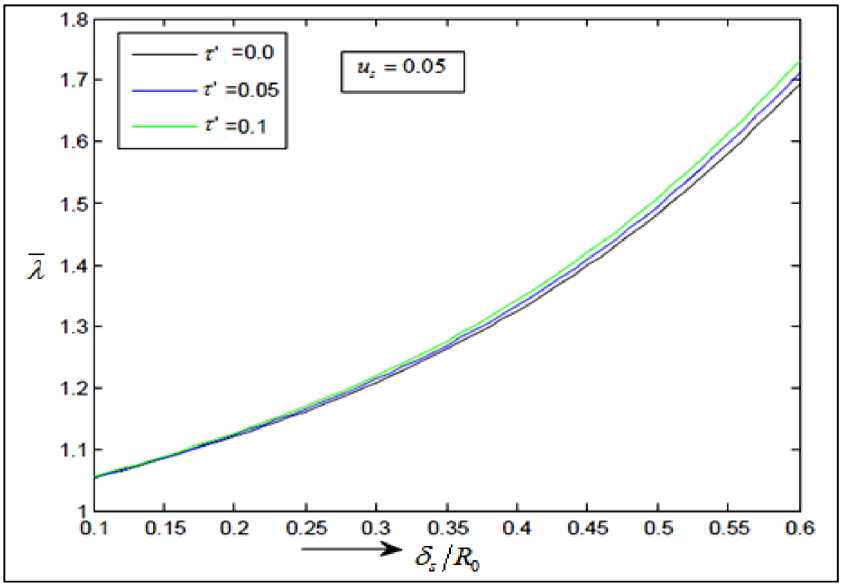

Fig 4: variation of resistance to Q with δ_s/R_0

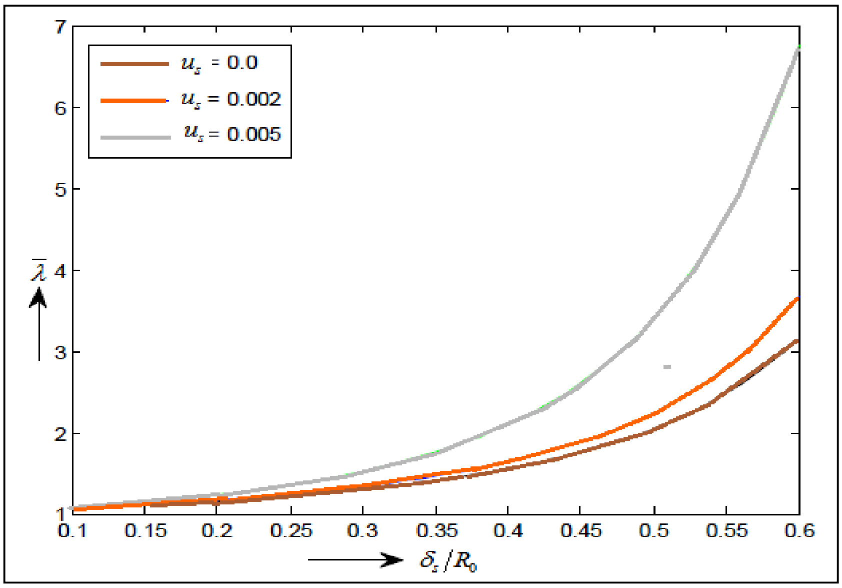

Fig 5: variation of resistance to Q with δ_s/R_0

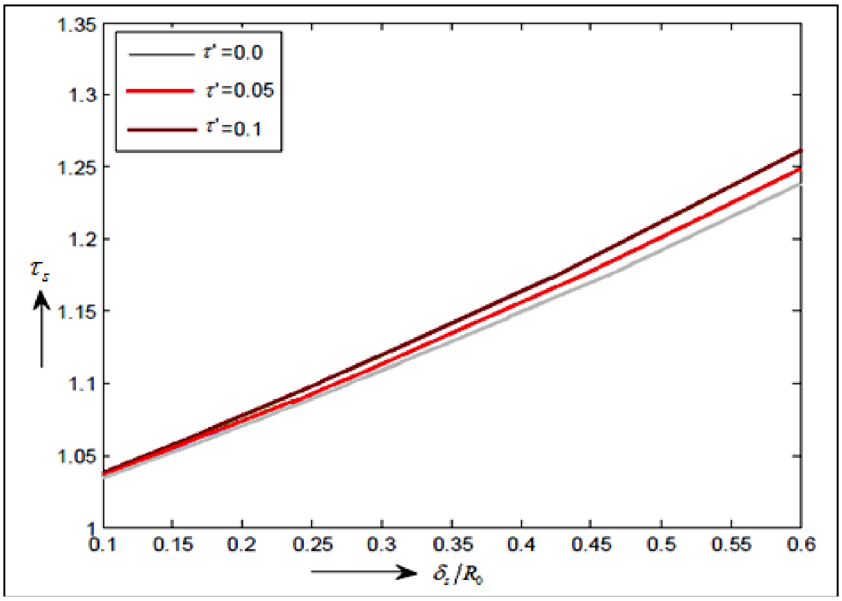

Fig 6: variation of skin-friction for different values of yield stress

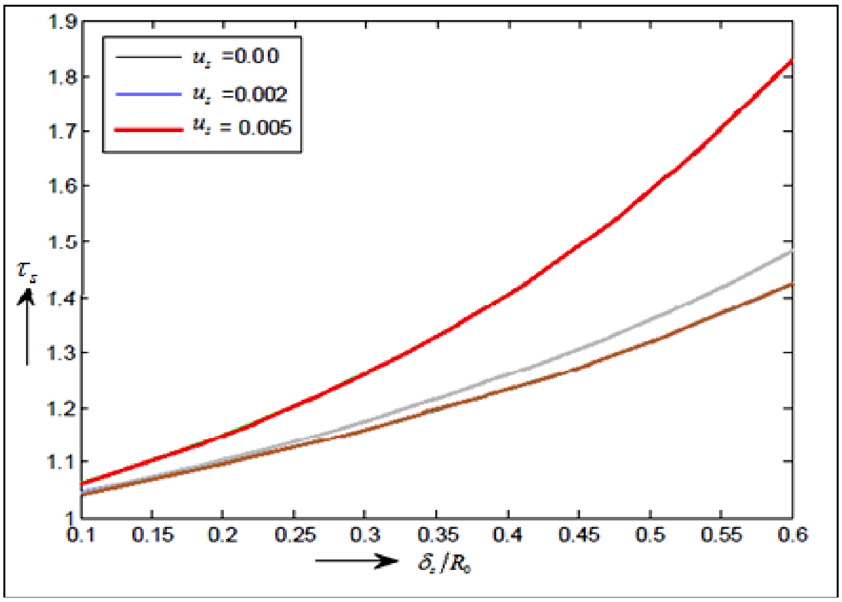

Fig 7: variation of skin-friction for different values of slip velocity

Conclusion

In this paper, we developed the flow characteristics of a Casson fluid model for blood flow with velocity slip conditions in presence of the magnetic effect in the stenosed arteries. In this paper, we studied of the mathematical modeling of blood flow in narrow arteries stenosis. We observed that the flow rate decreases when the yield stress and viscosity coefficient increases, the skin-friction increases with the increase of yield stress and slip velocity for velocity coefficient parameter g . The results are compared with the results of Chaturani *et al*. [7] for blood flow in cosine curve–shaped stenosed arteries treating blood as Hershel-Bulkley fluid and Misra *et al*. [17]. This type of model can be developed for the deformation of stenosis with delivery models in human artery. This study will be useful in prediction of flow characteristics in stenosed arteries.

References

1. Bhatt BS, sachet NC. On the analogy in slip flows. Indian Journal of Pure and Applied Mathematics. 1979; 10:303-306.
2. Biswas D, Chakraborty US. Steady flow through a stenosed artery: A non-Newtonian fluid model", Assam University journal of Science and Tech. 2010; 7(11):144-153.
3. Bulugliarello G, Hyden JW. Detailed characteristics of the flow of blood in vitro", Trans. Soc. Rheol. 1963; 7.
4. Moayeri MS, Zendehboodi GR. Effect of elastic property of the wall on flow characteristics through arterial stenosis", Journal of Biomechanics. 2003; 36:525-535.
5. Chakravarty S, Sannigrahi. A nonlinear Mathematical model of blood flow in a constricted artery experiencing body acceleration, Mathematical and Computer Modelling. 1999; 29:9-25.
6. Chakravarty S, Datta A, Mandal AK. Effect of stenosis on the flow behaviour of blood in an artery", IJES. 1987; 25:1003-1018.
7. Chaturani P, Kaloni PN. Two-layered poiseuille flow model for blood flow through arteries of small diameter and arterioles", Biorheology. 1976; 13:242-250.

8. Chaturani P, Upadhyay. Two phase model for blood flow through small diameter tubes, Biorheology. 1979; 16:109-118.
9. Enderle J, Susan B, Bronzino B. Introduction to Biomedical Enginnering, Academic Press, Londaon, 2000.
10. Fung YC, Lee JS. Flow in Locally constricted tubes at low Reynolds numbers, J Appl. Mech., Trans. ASME. 1970; 37:9.
11. Jain M, Sharma GC, Kumar S. A mathematical model for the blood flow through narrow vessels with mild stenosis, IJE. 2009; 1:99-106.
12. Kapur JN. Mathematical Models in Biology and Medicine", Affiliated East-Wes press, New Delhi, 1985.
13. Kumar H, Chandel RS, Kumar S, Kumar S. A mathematical model for different shapes of stenosis and slip velocity at the wall through mild stenosis artery", Advances in Applied Mathematical Biosciences. 2014; 5(1):9-18.
14. Mac Donald DA. Blood flow in Arteries, Edward Arnold, Londan, 1974.
15. MacDonald DA. On steady flow through modeled vascular stenosis. J Biomechanics. 1979; 12:13-30.
16. Merrial EW, Benis AM, Gilliland ER, Sherwood TK, salzman EW. Pressure-flow relations of human blood in hollow fibers at low flow rates. Journal of Applied Physiology. 1965; 20(5):954-967.
17. Mishra JC, Ghosh SK. A mathematical model for the study of blood flow through a channel with permeable walls", Acta. Mechanica. 1997; 12:137-153.
18. Moayeri MS, Zendehbudi. The effect of elastic property of the wall on flow characteristics through an arterial stenosis", Journal of Biomechanics. 2003; 36:525-535.
19. Naeem RK, Younus S, Dania. Inverse solutions for unsteady in-compressible couple fluid flow, IJAMM. 2010; 6:1-17.
20. Radhakrishnamacharya G, Shrinivasa Rao P. Flow of a magnetic fluid through a non-uniform wavy tube, PNASI. 2007; 76:241-245.
21. Rao BRP. Effect of stenosis on non-Newtonian flow of the blood in an artery, Bull. Math. 1980; 42:283-294.
22. Sankad GC, Radhakrishnamacharya G. Effect of the Magnetic field on the peristaltic transport of couple stress fluid in a channel with wall properties. Int. J Biomath. 2011; 4:365-378
23. Shukla JB, Parihar RS, Rao BRP. Effects of stenosis on non-Newtonian flow of the blood in an artery", Bulletin of Mathematical Biology. 1980b; 42:283-294.
24. Shukla JB, Parihar RS, Rao BRP. Effect of peripheral layer viscosity on blood flow through an artery mild stenosis", Bull Math. Bio. 1980; 17:403-410.
25. Sinha P, Singh C. Effect of couple stresses on the blood flow through an artery with mild stenosis, Biorheology. 1984; 15:3-4.
26. Srinivasacharya D, Shrikanth D. Effect of couple stresses on the pulsatile flow through a constricted annulus", Comtes Rendus Mecanique. 2008; 336:820-827.
27. Srivastava VP. Two-phase model of blood flow through stenosed tubes in the presence of a peripheral layer: Applications", Journal of Biomechanics. 1996; 29(10):1377-1382.
28. Stokes VK. Couple stresses in fluids, Fluid. 1966; 9:1710.
29. Younge DF. Effects of time-dependent stenosis on flow through a tube. J Engrg. Ind. Trans. ASME. 1968; 90:248-254.
30. Zendehbudi GR, Moayeri MS. Comparison of physiological and simple pulsatile flow through stenosed arteries. Journal of Biomechanical. 1999; 32:959-965.