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An alternative of converting feasible solution into basic feasible solution of linear programming problem

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Abstract
The simplex algorithm is a procedure that iteratively selects extreme point solutions or basic feasible solutions, but it must start with an extreme point or a basic feasible solution. So if a feasible solution of a linear programming problem (which satisfies the given linear equations along with non-negative constraints) is given, it is more important to have a basic feasible solution. In this paper, an alternative way of converting a feasible solution into a basic feasible solution of linear programming problem is described.

Keywords: Simplex algorithm, extreme point solution, basic feasible solution, linear programming problem

1. Introduction

Linear Programming Technique is used to solve the optimization problems which involve the all relationships between the variables in linear form. It was the first method which was widely used for solving optimization problems using digital computation. Linear programming has a wide range of applications *e.g.* in transportation problem, assignment of activities to different persons or machines, manufacturing, personnel planning, investment management, dietary, public health etc. In general a linear programming problem can involve a large number of variables and equations. Present available software can solve a problem in millions of equations and variables in a reasonable time. The linear programming problems can be solved through a very efficient algorithm known as "simplex" algorithm. This algorithm was developed by George Dantzig in the 1940's, and it remains the standard method for numerical solution of linear programming problems. The simplex algorithm is a procedure that iteratively selects extreme point solutions or basic feasible solutions, but it must start with an extreme point or a basic feasible solution. So if a feasible solution of a linear programming problem (which satisfies the given linear equations along with non-negative constraints) is given, it is more important to have a basic feasible solution.

2. Historical Background

The optimum solutions to different types of problems always attracted human throughout the ages. The great mathematicians of 17^{th} and 18^{th} centuries had developed the powerful techniques for optimization problems. In recent times, a new type of optimization problems has come out of the different organizational structures. The linear programming techniques is an optimization technique for finding maximum or minimum value of a linear function subject to some linear constraints. Though seems simple in its mathematical structure, it has a wide range of applications. The applications of linear programming techniques can be broadly classified into three types:

(i) Military related applications under the project "Scientific computation of optimum programs" (Scoop).

In 1947, George B. Dantzig, Marshall Wood and their associated of U.S department of Air Force formed a group to investigate and apply the Mathematical & related techniques to military related problems. The project scoop was outcome of this group. This group formally developed and applied LP models.

(ii) Inter industry economics based on Leontief input output model.
(iii) Problems involving two person zero-sum games and LP [1]

The linear programming has successful application in fields of agriculture, education, health, environment quality, inventory control, economic analysis, law enforcement, structural design, traffic analysis, assignment problems etc [2].
The General Linear Programming Problem is to find a vector X= $(x_1, x_2 \ldots\ldots\ldots, x_n)$ which optimizes (maximizes or minimizes) the linear function of the type

$c_1x_1 + c_2x_2 + \text{-----} + c_n x_n$

Subject to linear constraints

$a_{11}x_1 + \text{-----} + a_{1n}x_n = b_1$
$a_{21}x_1 + \text{-----} + a_{2n}x_n = b_2$

$a_{m1}x_1 + \text{-----} + a_{mn}x_n = b_m$
and $x_i > 0$ for i=1, 2, ---- n

where a_{ij}, c_i and b_i are known constants and $m<n$ [3].
Linear systems in which number of variables are more than number of equations are called underdetermined [4]. Mathematically, linear programming deals with non-negative solutions to underdetermined system of linear equations [5]. For solving a linear programming problem, there are various criterion to check whether a solution (s) to a LPP exists or not [6].
Definition: A feasible solution (FS) to the LPP is a vector $X=(x_1, x_2,\text{--}, x_n)$ such that it satisfies the linear constraints and each x_i > or =0
Definition: A Basic Feasible solution (BFS) to LPP is a FS in which at most m variables out of n variables $x_1,x_2,\text{--},x_n$ are non-negative and remaining n-m variables are zero, provided the matrix formed by the columns associated with these m variables is non-singular [7]
Another notation: The general LPP can also be written in the following form:
Maximize (or Minimize) Z=CX
Subject to $x_1P_1+x_2P_2+\text{---}+x_nP_n= P_0$ and X > or = 0
where P_j (j=1,2,---,n) is jth column of coefficient matrix and P_0 is R.H.S. of system of linear constraints [8].

3. Construction of BFS from a given FS
Now we shall discuss a procedure of obtaining BFS from a given FS with the help of following numerical examples. Let S be the set of all feasible solutions to LPP. In this discussion, our aim is to obtain a extreme point or corner point of set S using some interior point (FS) of S.
Numerical Example:

(i) Consider the LPP
Maximize $f(X)=x+2y+4z$
Subject to $2x+y+4z=11$
$3x+y+5z=14$
$x,y,z \geq 0$

Let x=2,y=3,z=1 be a FS to above LPP.

This is not BFS as here number of equations is two. So number of non-negative variables in BFS must be two. To reduce this FS into BFS, We use linearly dependence of vectors, P_1, P_2 and P_3 in the given system

$xP_1+yP_2+zP_3=P_0$ -----(i)
i.e. $2P_1+3P_2+1P_3=P_0$ -----(ii)

where $P_1=[2\ 3]^t$, $P_2=[1\ 1]^t$, $P_3=[4\ 5]^t$, $P_0=[11\ 14]^t$
let $P_1=aP_2+bP_3$ be the linear relation between linearly dependent vectors P_1,P_2,P_3
i.e. a + 4b = 2 and a + 5b = 3
on solving for a and b, we get
a = -2 and b = 1

so $P_1= -2P_2+P_3$ ----(iii)

Substituting this value of P_1 in the given system $2P_1+3P_2+1P_3=P_0$, we get

$2(-2P_2+P_3) +3P_2+1P_3=P_0$
or $-P_2+3P_3=P_0$

So we get a new solution of given system as (0, -1, 3) which is infeasible.
Let us again use equation (iii) as

$P_2=(P_3/2)-(P_1/2)$
From (i) we get
$P_1/2 + 5P_3/2 = P_0$

i.e. we get a new BFS as (1/2, 0, 5/2)
again let us use (iii) as $P_1+2P_2=P_3$
so from (i) we have $3P_1+5P_2=P_0$
i.e. we get a new BFS as (3,5,0)
Now f(1/2, 0, 5/2) = 21/2
And f(3,5,0) = 13
So maximum value of f(X)=13 at BFS (3,5,0)

(ii) Consider the following system of linear equations:

$x_1 +2 x_2 + 4 x_3 + x_4 = 7$
$2 x_1 - x_2 + 3 x_3 -2 x_4 = 4$

Let a given feasible solution of above system of linear equations be $x_1 = 1$, $x_2 = 1$, $x_3 = 1$, $x_4 = 0$
Now we shall reduce this feasible solution into two different basic feasible solutions.
Rewriting the given linear equations as

$x_1P_1 + x_2P_2+ x_3P_3 + x_4P_4= P_0$
where $P_1 = [1\ 2]^t$, $P_2 = [2\ -1]^t$, $P_1 = [4\ 3]^t$, $P_4 = [1\ -2]^t$ and $P_0 = [7\ 4]^t$

Here m = 2 and n = 4 so BFS contains at most 2 non negative variables and other two as zero.
To reduce this FS into BFS, We use linearly dependence of vectors, P_1, P_2, P_3 and P_4 in the given system

$x_1P_1+ x_2P_2+ x_3P_3 +x_4P_4=P_0$ ----(i)
i.e. $P_1+ P_2+ P_3 =P_0$ -----(ii)

The three vectors in P_1, P_2, P_3 in (ii) which are in R^2 are linearly dependent vectors and let the relation between these be $P_1=aP_2+bP_3$
i.e. 1 = 2a + 4b and 2 = -a +3b
on solving for a and b we get

a = -1/2 and b = 1/2
so $P_1=(-1/2)P_2+(1/2)P_3$ ----(iii)

which is the linear relation between P_1, P_2 and P_3.
On substituting the value of P_1 from (iii) in (ii) we get

$(-1/2)P_2+(1/2)P_3+P_2+P_3 = P_0$
$(1/2)P_2+(3/2)P_3 = P_0$

So we obtain a BFS as (0,1/2, 3/2)
Again let us find P_2 from (iii) as $P_2= -2P_1+P_3$ and substitute in (ii) we have

$P_1+ (-2P_1+P_3)+P_3=P_0$
Or $-P_1+2P_3=P_0$

So we obtain a BFS as (3, 2, 0)
Hence we have two different BFS as (0, 1/2, 3/2) and (3, 2, 0)

4. Conclusion
The simplex procedure, devised by G.B. Dantzig, is a computation scheme in which once any basic feasible solution (BFS) has been determined, an optimum feasible solution can be obtained in a finite number of steps (generally between m and 2m). This procedure of obtaining BFS from a given FS provides a method, without using artificial variable (or basis). So using the present scheme, although may be laborious for large number of equations, uses the linearly dependence of vectors to FS to get L.I vectors.

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