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Stability of the pexiderized cauchy functional equation in (n, β)-Hilbert space

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Abstract
The concept of pexiderized Cauchy functional equation we obtain the general solution for the new additive functional equation. We prove some results of pexiderized Cauchy functional equation. Let f be the mapping from a linear space x into a complete random normed spacey. The Cauchy functional equation and the Cauchy-pexiderized functional equation, Hilbert space are generalized and their solution are determined.

Keywords: Pexiderized cauchy functional equation, (n, β)-Hilbert space

Introduction
f(x)=ax+ b for some a, b.
The area of the shaded region with base from y to x y is f(xy)−f(y). This follows immediately from the fact that the region under the hyperbola from y to x y is exactly that which is obtained by removing the region from 1 to y from the region from 1 to xy. Thus, using Saint-Vincent's scaling argument, we have f(x)=f(xy)−f(y) or equivalently that f(xy)=f(x)+f(y).

Cauchy functional equations
Let us being by restarting and solving Cauchy's functional equation.
Let $f: R \to R$ be a continuous function satisfying

$$f(x + y) = f(x) + f(y) \text{ for all real} \quad (1.2.1)$$

We show that there exists a real number that $f(x) = ax$ for all $x \in R$ it is straight forward to show by mathematical induction (1.2.1) implies

$$f(x_1 + x_2 + \cdots \ldots . + x_n) = f(x_1) + f(x_2) + \cdots \ldots . + f(x_n) \text{ for all } x_1, x_2, \ldots . . x_n \in R \quad (1.2.2)$$

A special case of this is found by setting $x_1 = x_2 = \cdots \ldots . . = x_n$ say then (1.3.2) becomes

$$f(nx) = f(mt) \text{ For all positive integer n and for all real x} \quad (1.2.3)$$

Let $x = \left(\frac{m}{n}\right) t$ where m and n are positive integers then $nx = mt$ so,

$$f(nx) = f(mt)$$
$$nf(x) = mf(t)$$
$$nf\left(\frac{m}{n} t\right) = mf(t)$$

But this can be written as

$$f\left(\frac{m}{n}\, t\right) = \frac{m}{n}\, f(t) \text{for all } t \in R \quad (1.2.4)$$

Then we have proved that

$$f(qt) = qf(t) \text{ For all real value of t and all rational of q} \quad (1.2.5)$$

We can extended (1.2.5) to include $q = 0$ in the following way returning to (1.2.1) we see that

$$f(y) = f(y + 0)$$
$$= f(y) + f(0)$$

So $f(0) = 0$ from this we immediately have $f(0t) = 0f(t)$ therefore (1.2.5) is true for all non-negative rational q.
Once again, returning to (1.2.1) we obtain

$$0 = f(0)$$
$$= f[q + (-q)]$$
$$= f(q) + f(-q)$$

Therefore $f(-q) = -f(q)$ from this for we get for $q < 0$

$$f(qt) = f[-(-q)t]$$
$$= -f[(-q)t]$$
$$= -(-q)f(t)$$
$$= qf(t)$$

Therefore $f(qt) = qf(t)$ for all real value of t and all rational of q (1.2.6)
Now suppose we substitute $t = 1$ into (1.2.6) letting $f(1) = a$ we deduce that $f(a) = aq$ for all rational number q
We summarization what has been discovered so far in the following proposition.

Pexider's Equations
The Additive Pexider Functional Equation

$$f(x + y) = h(x) + g(y)\ (x, y) \in k \tag{1.4.1}$$

It is well known (1.4.1) that the continuous solutions of this functional equation on the domain K are of the form

$$f(t) = c.t + a + b \tag{1.4.2}$$
$$h(t) = c.t + b \tag{1.4.3}$$
$$g(t) = c.t + a \tag{1.4.4}$$
Where $a = g(0), b = h(0)$

We want to get the solutions for this equation on the curve $\Gamma_1 \cup \Gamma_2$. By placing (x, y) that are in the domain $\Gamma_1 \cup \Gamma_3$ in (1.4.1) we get seven equations

$$f(t) = g(\beta(t)) + h(\alpha(t)) \tag{1.4.5}$$
$$f(1) = g(-\beta(t)) + h(\alpha(t)) \tag{1.4.6}$$
$$f(t) = g(\alpha(t)) + h(\beta(t)) \tag{1.4.7}$$
$$f(-1) = g(\beta(t)) + h(-\alpha(t)) \tag{1.4.8}$$
$$f(t) = g(t) + h(0) \tag{1.4.9}$$
$$f(t) = g(0) + h(t) \tag{1.4.10}$$
$$f(0) = g(0) + h(0) \tag{1.4.11}$$
Where $\alpha(t) = \frac{t+1}{2}$, $\beta(t) = \frac{t-1}{2}$

By placing (1.4.9), (1.4.10) and (1.4.11) in the first four equations we obtain four new equations

$$f(t) = f(\beta(t)) + f(\alpha(t)) - f(0)\ (by(1.4.5))$$
$$f(1) = f(-\beta(t)) + f(\alpha(t)) - f(0)\ (by(1.4.6))$$
$$f(t) = f(\alpha(t)) + f(\beta(t)) - f(0)\ (by(1.4.7))$$
$$f(-1) = f(\beta(t)) + f(-\alpha(t)) - f(0)\ (by(1.4.8))$$

Now we define a new function

$$m(t) = f(t) - f(0) \Rightarrow f(t) = f(t) + f(o)$$

By placing $f(t) = f(t) + f(o)$ in (3) we get

$$m(t) = m(\alpha(t)) + m(\beta(t)) \tag{1.4.12}$$
$$m(1) = m(-\beta(t)) + m(\alpha(t)) \tag{1.4.13}$$
$$m(-1) = m(\beta(t)) + m(-\alpha(t)) \tag{1.4.14}$$

This is Cauchy's additive functional equation, and its continuous solutions are of the form $m(t) = c.t$
That gives us these solutions of the equation (1.4.1) on the domain $\Gamma_1 \cup \Gamma_3$

$f(t) = c.t + a + b$
$h(t) = c.t + b$
$g(t) = c.t + a$
Where $b = h(0)$ $a = g(0)$

So we can conclude that there are no new continuous solutions on the curve. $\Gamma_1 \cup \Gamma_3$
We seek the solutions for this equation on the curve Γ_2. To this end, place $y = x$ in the equation (1.4.1) to obtain

$$f(2x) = h(x) + g(x), x \in [-\tfrac{1}{2}, \tfrac{1}{2}]$$

For this equation we didn't find the general solutions but there are new solutions, for example

* $f(t) = 1, h(t) = \cos^2 t, g(t) = \sin^2 t$
* $f(t) = \cos(2t), h(t) = \cos^2 t, g(t) = -\sin^2 t$
* $f(t) = \sin(2t), h(t) = g(t) = \cos(t).\sin(t)$

Note that these solutions are c^{∞}, so no smoothness condition will preserve the set of solutions (1.4.2). However in Cauchy's additive equation we do not obtain new solutions when we add the condition that the function will be c^1 or differentiable at 0.
In this section, we investigate the stability of the pexiderized Cauchy functional equation in (n, β)-Hilbert spaces.

Theorem
Let X be a vector space and Y be a complete (n, β)-Hilbert space with 0<β≤1. Let φ: X2→ [0, ∞) be a function satisfying
$\Phi(x) = \sum_{i=1}^{\infty} 2 - i\beta\big(\Phi(2i-1x, 0) + \Phi(0, 2i-1x) + \Phi(2i-1x, 2i-1x)\big) < \infty$ (4.1.1)
And

$$\lim_{m\to\infty} 2 - m\beta\Phi(2mx, 2my) = 0 \qquad (4.1.2)$$

For all x, y ∈X. ψ: $\underbrace{Y\times Y\times\cdots\times Y}_{n-1} \to [0, \infty)$ is a function. If mappings f, g, h: X→Y satisfy the inequality

$$\|f(x+y) - g(x) - h(y), z_1 \ldots z_{n-1}\|\beta \le \varphi(x, y)\, \psi(z_1,\ldots,z_{n-1}) \qquad (4.1.3)$$

For all x, y∈X and $z_1 \ldots z_{n-1}$∈Y, then there exists a unique additive mapping A: X→Y satisfying

$$\|f(x)-A(x),z_1,\ldots,z_{n-1}\|\beta \le \Phi(x)\,\psi(z_1,\ldots,z_{n-1})+\|h(0),z_1,\ldots,z_{n-1}\|\beta+\|g(0),z_1,\ldots,z_{n-1}\|\beta, \qquad (4.1.4)$$
$$\|g(x)-A(x),z_1,\ldots,z_{n-1}\|\beta\le\Phi(x)\psi(z_1,\ldots,z_{n-1})+\|g(0),z_1,\ldots,z_{n-1}\|\beta+2\|h(0),z_1,\ldots,z_{n-1}\|\beta+\varphi(x,0)\,\Psi(z_1,\ldots,z_{n-1}), \qquad (4.1.5)$$
$$\|h(x)-A(x),z1,\ldots,zn-1\|\beta\le\Phi(x)\psi(z_1,\ldots,z_{n-1})+\|h(0),z_1,\ldots,z_{n-1}\|\beta+2\|g(0),z_1,\ldots,z_{n-1}\|\beta+\varphi(0,x)\,\Psi(z_1,\ldots,z_{n-1}) \qquad (4.1.6)$$
for all x∈X and z1,…,zn−1∈Y.

Proof
Putting y=x in inequality (4.1.3), we get

$$\|f(2x)-g(x)-h(x),z1,\ldots,zn-1\|\beta\le\varphi(x, x)\psi(z1,\ldots,zn-1) \qquad (4.2.7)$$

for all x∈X and z1,…,zn−1∈Y. Putting y=0 in inequality (4.1.3), we get

$$\|f(x)-g(x)-h(0),z1,\ldots,zn-1\|\beta\le\varphi(x,0)\psi(z1,\ldots,zn-1) \qquad (4.2.8)$$

For all x∈X and $z_1,\ldots, z_{n-1}$∈Y. It then follows from (4.1.8) that

$$\|f(x)-g(x),z_1,\ldots,z_{n-1}\|\beta\le\varphi(x,0)\psi(z_1,\ldots,z_{n-1})+\|h(0),z_1,\ldots,z_{n-1}\|\beta \qquad (4.1.9)$$

for all x∈X and z1,…,zn−1∈Y. Putting x=0 in inequality (4.1.3), we get $\|f(y)-g(0)-h(y),z_1,\ldots,z_{n-1}\|\beta\le\varphi(0,y)\psi(z_1,\ldots,z_{n-1})$
For all y∈X and z1,…,zn−1∈Y. Thus, we obtain

$$\|f(x)-h(x),z1,\ldots,zn-1\|\beta\le\varphi(0,x)\psi(z1,\ldots,zn-1)+\|g(0),z1,\ldots,zn-1\|\beta \qquad (4.1.10)$$

for all x∈X and $z_1,\ldots,z_{n-1}$∈Y.
Let us define
$u(x,z1,\ldots,zn-1)=\|g(0),z_1,\ldots,z_{n-1}\|\beta+\|h(0),z_1,\ldots,z_{n-1}\|\beta+\varphi(x,x)\psi(z_1,\ldots,z_{n-1})+\varphi(x,0)\,\psi(z_1,..,z_{n-1})+\varphi(0,x)\psi(z_1,\ldots,z_{n-1})$.
Using (4.1.7), (4.1.9) and (4.1.10), we have

$\|f(2x)-2f(x),z_1,\ldots,z_{n-1}\|\beta\le\|f(2x)-g(x)-h(x),z_1,\ldots,z_{n-1}\|\beta+\|g(x)-f(x),z_1,\ldots,z_{n-1}\|\beta+\|h(x)-f(x),$
$z_1,\ldots,z_{n-1}\|\beta\le\|g(0),z_1,\ldots,z_{n-1}\|\beta+\|h(0),z_1,\ldots,z_{n-1}\|\beta+\varphi(x,0)\psi(z_1,\ldots,z_{n-1})+\varphi(0,x)\psi(z_1,\ldots,z_{n-1})+\varphi(x,x)\psi(z_1,\ldots,z_{n-1})=u(x,z_1,\ldots,z_{n-1})$ (4.1.11)
for all x∈X and z1,…,zn−1∈Y. Replacing x with 2x in (4.1.11), we get

$\|f(22x)-2f(2x),z_1,\ldots,z_{n-1}\|\beta\le u(2x,z_1,\ldots,z_{n-1})$ (4.1.12)

for all x∈X and $z_1,\ldots,z_{n-1}\in Y$. It then follows from (4.1.11) and (4.1.12) that
$\|f(22x)-22f(x),z_1,\ldots,z_{n-1}\|\beta\le\|f(22x)-2f(2x),z_1,\ldots,z_{n-1}\|\beta+2\beta\|f(2x)-2f(x),z_1,\ldots,z_{n-1}\|\beta\le u(2x, z_1,\ldots,z_{n-1})+2\beta u(x,z_1,\ldots,z_{n-1})$
for all x∈X and $z_1,\ldots,z_{n-1}\in Y$.
Applying an induction argument on m, we will prove that

$\|f(2mx)-2mf(x),z1,\ldots,zn-1\|\beta \le \sum_{i=1}^{m} 2(i-1)\beta\ u(2m-ix,z_1,\ldots.z_{n-1})$ (4.1.13)

for all x∈X, z1,…,zn−1∈Y and m∈N. In view of (4.1.11), inequality (4.1.13) is true for m=1.
Assume that (4.1.13) is true for some m>1. Substituting 2x for x in (4.1.13), we obtain
$\|f(2m+1x)-2mf(2x),z_1,\ldots,z_{n-1}\|\beta\le \sum_{i=1}^{m} 2(i-1)\beta\ u(2m+1-ix,z_1,\ldots.z_{n-1})$
for all x∈X and $z_1,\ldots,z_{n-1}\in Y$.
Hence, it follows from (4.1.11) that
$\|f(2m+1x)-2m+1f(x),z_1,\ldots,z_{n-1}\|\beta\le\|f(2m+1x)-2mf(x),z_1,\ldots,z_{n-1}\|\beta+2n\beta\|f(2x)-2f(x),z_1,\ldots,z_{n-1}\|\beta \quad \le\sum_{i=1}^{m} 2(i-1)\beta\ u(2m+1-ix,z_1,\ldots.z_{n-1})+2m\beta u(x,z_1,\ldots.z_{n-1})$
$=\sum_{i=1}^{m+1} 2(i-1)\beta\ u(2m+1-ix,z_1,\ldots.z_{n-1})$
for all x∈X and z1,…,zn−1∈Y, which proves inequality (4.1.13). By (4.1.13), we have

$\|2-mf(2mx)-f(x), z_1\ldots z_{n-1}\|\beta \le \sum_{i=1}^{\infty} 2(i-1-m)\beta\ u(2m-ix,z_1,\ldots.z_{n-1})$ (4.1.14)

for all x∈X, z1,…,zn−1∈Y and m∈N. Moreover, if m, k∈N with m<k, then it follows from (4.1.11) that
$\|2-kf(2kx)-2-mf(2mx),z_1,\ldots,z_{n-1}\|\beta\le k-1\sum i=m\|2-if(2ix)-2-(i+1)f(2i+1x),z_1,\ldots,z_{n-1}\|\beta$
$\le k-1\sum i=m2-(i+1)\beta\|2f(2ix)-f(2i+1x),z_1,\ldots,z_{n-1}\|\beta$
$=k-1\sum i=m2-(i+1)\beta u(2ix,z_1,\ldots,z_{n-1})$
$=k-1\sum i=m2-(i+1)\beta[\varphi(2ix,0)\psi(z_1,\ldots,z_{n-1})+\varphi(0,2ix)\psi(z_1,\ldots,z_{n-1})+\varphi(2ix,2ix)\psi(z_1,\ldots,z_{n-1})+\|h(0), z_1,\ldots,z_{n-1}\|\beta+\|g(0),z_1,\ldots,z_{n-1}\|\beta]$
$\le k-1\sum i=m2-(i+1)\beta[\varphi(2ix,0)+\varphi(0,2ix)+\varphi(2ix,2ix)]\psi(z_1,\ldots,z_{n-1})+2-m(\|h(0),z1,\ldots,zn-1\|\beta+\|g(0), z_1,\ldots,z_{n-1}\|\beta)$
for all x∈X and $z_1,\ldots,z_{n-1}\in Y$. Taking the limit as m, k→∞ and considering (4.1.1), we get
$\lim_{m,k\to\infty} \|2-kf(2kx-2-mf(2mx),z_1,\ldots.z_{n-1}\|\beta=0$
for all x∈X and $z_1,\ldots,z_{n-1}\in Y$. According to Definition 1.7, we know that {2−mf(2mx)} is a Cauchy sequence for every x∈X and $z_1,\ldots,z_{n-1}\in Y$. Since Y is a complete (n, β)-normed space, we can define a function A: X→Y by
$A(x)=\lim_{m\to\infty} 2-mf(2mx)$
Replacing x, y by 2mx, 2my in (4.1.3) and dividing both sides by 2mβ, we get
$2-m\beta\|f(2mx+2my)-g(2mx)-h(2my),z_1,\ldots,z_{n-1}\|\beta\le2-m\beta\varphi(2mx,2my)\psi(z_1,\ldots,z_{n-1})$
for all x∈X and z1,…,zn−1∈Y. It follows from (4.1.9) that

$\|2-mf(2mx)-2-mg(2mx),z_1,\ldots,z_{n-1}\|\beta\le2-m\beta[\|h(0),z_1,\ldots,z_{n-1}\|\beta+\varphi(2mx,0)\psi(z1,\ldots,zn-1)]$ (4.1.15)

for all x∈X and $z_1,\ldots,z_{n-1}\in Y$. Considering (4.1.1), we get
$2-m\beta\varphi(2mx,0)\psi(z1,\ldots,zn-1)\le2\beta\infty\sum i=m2-(i+1)\beta[\varphi(2ix,0)\psi(z1,\ldots,zn-1)+\varphi(0,2ix)\psi(z1,\ldots,zn-1)+\varphi(2ix,2ix)\psi(z1,\ldots,zn-1)]\to0$ as m→∞.
It follows from (4.1.15) that

limm→∞2−mg(2mx)=limm→∞2−mf(2mx)=A(x) (4.1.16)

for all x∈X. Also, by (4.1.10), we have

$\|2-mh(2mx)-2-mf(2mx),z_1,\ldots,z_{n-1}\|\beta\le2-m\beta[\|g(0),z_1,\ldots,z_{n-1}\|\beta+\varphi(0,2mx)\psi(z_1,\ldots,z_{n-1})]$ (4.1.17)

for all x∈X and $z_1,\ldots,z_{n-1}\in Y$. Similarly, it follows from (4.1.17) that

limm→∞2−mh(2mx)=limm→∞2−mf(2mx)=A(x) (4.1.18)

for all x∈X and $z_1,\ldots,z_{n-1}\in Y$. Thus, by (4.1.2), (4.1.16), (4.1.18) and Lemma, we get
$\|A(x+y)-A(x)-A(y),z_1,\ldots,z_{n-1}\|\beta=\text{limm}\to\infty\|2-mf(2mx+2my)-2-mg(2mx)-2-mh(2my),$
$z_1,\ldots,z_{n-1}\|\beta\le\text{limm}\to\infty2-m\beta\varphi(2mx,2my)\psi(z1,\ldots,zn-1)=0$
for all x∈X and $z_1,\ldots,z_{n-1}\in Y$.
Hence A(x+y)−A(x)−A(y)=0.
Taking the limit as m→∞ in (4.1.14), we get

∥A(x)−f(x),z1,…,zn−1∥β≤limm→∞m∑i=12(i−1−m)βu(2m−ix,z1,…,zn−1)=limm→∞(1−2−mβ) (∥g(0),z_1,…,z_{n-1}∥β+∥h(0),z_1,…,z_{n-1}∥β)+limm→∞m∑i=12(i−m−1)β(φ(2m−ix,0)ψ(z_1,…,z_{n-1})+ φ(0,2m−ix)ψ(z_1,…,z_{n-1})+φ(2m−ix,2m−ix)ψ(z_1,…,z_{n-1}))=∥h(0),z_1,…,z_{n-1}∥β+∥g(0),z_1,…,z_{n-1}∥β+Φ(x)ψ(z_1,…,z_{n-1})
∥A(x)−f(x),z_1,…,z_{n-1}∥β=∥h(0),z_1,…,z_{n-1}∥β+∥g(0),z_1,…,z_{n-1}∥β+Φ(x)ψ(z_1,…,z_{n-1})
By lemma

A(x)−f(x)=h(0)+g(0)+Φ(x)ψ(z1,…,zn−1) (4.1.19)

By using Riesz representation theorem "If H is a Hilbert space and if $f_y \in H^*$ then there exist a unit vector $y\ in\ H\ such\ that\ f_y(x) = < x, y > for\ every\ x\ in\ H$ "
$< x, y > - < x, z_1, \ldots . z_{n-1} > = < x, 0 > + < 0, y > + < z_{1,\ldots\ldots} z_{n-1}, y >$
$= < x, y > - <$ y, $z_{1,\ldots\ldots\ldots} z_{n-1} > = < x, y - z_{1,\ldots\ldots\ldots} z_{n-1} > ie)\ < x, y - z_{1,\ldots\ldots\ldots} z_{n-1} > = < x, y > - < x, z_1, \ldots . z_{n-1} >$

For all x∈X and z_1,…, z_{n-1}∈Y,.
Let $x = \alpha x$ in (4.1.19)
A(αx)−f(αx)=h(0)+g(0)+Φ(αx)ψ(z1,…,zn−1)

By using Riesz representation theorem **"**If H is a Hilbert space and if $f_y \in H^*$ then there exist a unit vector $y\ in\ H\ such\ that\ f_y$(x) $= < x, y > for\ every\ x\ in\ H$ " $< \alpha x, y > - < \alpha x, \mathrm{z}_1, \ldots . \mathrm{z}_{n-1} > = < 0, x > + < 0, y > + < \mathrm{z}_{1,\ldots\ldots} \mathrm{z}_{n-1}, \mathrm{y} >$ ie) $\alpha < x, y - \mathrm{z}_{1,\ldots\ldots\ldots} \mathrm{z}_{n-1} > = \alpha < x, y > -\alpha < x, \mathrm{z}_1, \ldots . \mathrm{z}_{n-1} >$

for all x∈X and z1,…,zn−1∈Y,
It remains to prove the uniqueness of A.
Assume that A′:X→Y is another additive mapping which satisfies (4.1.4). Then we have

∥A(x)−A′(x),z_1,…,z_{n-1}∥β≤2−mβ∥A(2mx)−f(2mx),z_1,…,z_{n-1}∥β+2−mβ∥f(2mx)−A′(2mx),z_1,…,z_{n-1}∥β≤2−mβ+1(∥g(0),z_1,…,z_{n-1}∥β+∥ h(0),z_1,…,z_{n-1}∥β+Φ(2mx)ψ(z_1,…,z_{n-1}))=2−mβ+1(∥g(0),z_1,…,z_{n-1}∥β+∥h(0),z_1,…,z_{n-1}∥β)+2∞∑i=m+12−iβ(φ(2i−1x,0)+φ(0,2i−1x)+ φ(2i−1x,2i−1x))ψ(z_1,…,z_{n-1})→0 as m→∞
for all x∈X and z_1,…,z_{n-1}∈Y,
∥A(x)−A′(x),z1,…,zn−1∥β =0
A(x) −A′(x), z1,…, zn−1=0
A(x) −A′(x) =0
A(x) =A′(x)

which together with Lemma implies that A(x)=A′(x) for all x∈X. Using (4.1.4) and (4.1.9), we can get (4.1.5), and also using (4.1.4) and (4.1.10), we can get (4.1.6).

Conclusion
In this paper, we proved some results of pexiderized Cauchy functional equation. Finally we investigate the stability of pexiderized Cauchy functional equation in (n, β) is normed space the converted to (n, β) is Hilbert space.

References

1. Baker J. The stability of the cosine equation. Proc. Amer. Math. Soc. 1980; 80:411-416.
2. Cholewa PW. Remarks on the stability of functional equations. Aequationes Math. 1984; 27:76-86
3. Czerwik S. On the stability of the quadratic mapping in normed spaces. Abh. Math. Sem. Univ. Hamburg. 1992; 62:59-64.
4. Forti GL. Comments on the core of the direct method for proving Hyers–Ulam stability of functional equations. J. Math. Anal. Appl. 2004; 295:127-133
5. Grabiec A. The generalized Hyers–Ulam stability of a class of functional equations. Publ. Math. Debrecen. 1996; 48:217-235.
6. Hyers DH. On the stability of the linear functional equation. Proceedings of the national academy of sciences of the U.SA. 1941; 27:222-224.
7. Hyers DH, Isac G, Rassias ThM. Stability of Functional Equations in Several Variables. Birkh¨auser, Basel, 1998.
8. Hyers DH, Isac G, Rassias ThM. On the asymptoticity aspect of Hyers–Ulam stability of mappings. Proc. Amer. Math. Soc. 1998; 126:425-430.
9. Hyers DH, Rassias ThM. Approximate homomorphisms. A equations Math. 1992; 44:125-153.
10. Jun KW, Kim HM. The generalized Hyers–Ulam–Rassias stability of a cubic functional equation. J. Math. Anal. Appl. 2002; 274:867-878.
11. Jun KW, Lee YH. On the Hyers–Ulam–Rassias stability of a pexiderized quadratic inequality. Math. Ineq. Appl. 2001; 4:93-118.
12. Parimala PM. The generalized Hyers-ulam-rassias stability of quadratic functional equations with two variables. ISSN:2455-2631,international journal of science & engineering development research, 2017; 2(4).