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## Techniques for solving maximal flow problem

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### Abstract

There are several methods accessible for the solution of Maximal flow network problems. Marking technique is an option strategy for Maximal flow network problems. The fundamental thing in the marking methodology is to efficiently append names to the hubs of a network until the point that optimum solution is acquired. Labeling techniques can be utilized to fathom distinctive sorts of network problems. For example, most brief way issues, maximal flow problems, general negligible cost stream problems etc. what's more, minimal spanning tree problems. It is the motivation behind this paper to represent the general idea of the labeling algorithms by depicting a marking strategy for the Maximal-flow problem

**Keywords:** Techniques, Solving, Maximal Flow Problem, solution, network problems, labeling, procedure, network, techniques

### Introduction

This problem involves a coordinated network with circular segments conveying stream. The main applicable parameter is the upper bound on circular segment stream, called bend limit. The problem is to discover the Maximal flow that can be sent through the bends of the network from some predefined hub  $s$ , called the source, to a moment determined hub  $t$ , called the sink. Applications of this problem incorporate finding the Maximal flow of requests through an occupation shop, the Maximal flow of water through a tempest sewer framework, and the Maximal flow of item through an item dispersion framework, among others. A particular case of the problem with source hub  $s = a_n$  and sink hub  $t = F$  is indicated in Fig. 1 with its solution. Specifically, the solution is the task of streams to curves. For practicality, protection of stream is required at every hub aside from the source and sink, and each bend stream must be not exactly or equivalent to its ability. The solution demonstrates that the Maximal flow from  $A$  to  $F$ . A cut is an arrangement of curves whose expulsion will hinder all paths from the source to the sink. The limit of a cut is the entirety of the bend limits of the set. The minimal cut is the cut with the littlest limit. Given an answer for the Maximal flow problem, one can simply establish that no less than one minimal cut, as delineated in Fig. 2. The minimal cut is an arrangement of circular segments that point of confinement the estimation of the Maximal flow<sup>[1]</sup>. Not adventitiously, the case demonstrates that the aggregate limit of the curves in the minimal cut equals the estimation of the Maximal flow (this outcome is called the max-flow min-cut theorem). The algorithm described in this segment fathoms both the Maximal flow and minimal cut problems<sup>[2]</sup>.

Network flow problems can be settled by a few strategies. This paper continues the examination of network problems by portraying the utilization of the naming calculation to the Maximal flow network problem<sup>[3]</sup> Labeling methods give an option way to deal with explaining network problems. The fundamental thought behind the labeling procedure is to deliberately append marks to the hubs of a network until the ideal arrangement is reached<sup>[4]</sup>. Labeling techniques can be utilized to unravel a wide assortment of network problems, for example, most limited way issues, maximal flow problems, general minimal cost network flow problems, and minimal spanning tree issues<sup>[5]</sup>. It is the motivation behind this outlines the general idea of the marking calculations by depicting a naming strategy for the Maximal flow problem.

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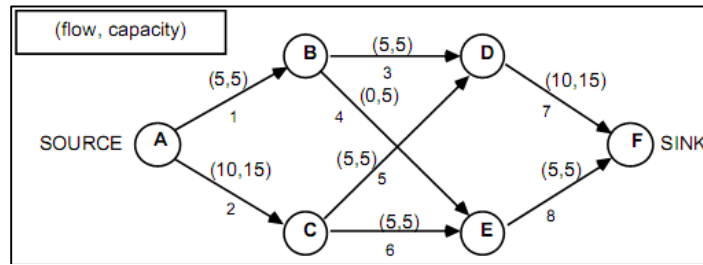


Fig 1: Example Maximal flow problem with solution

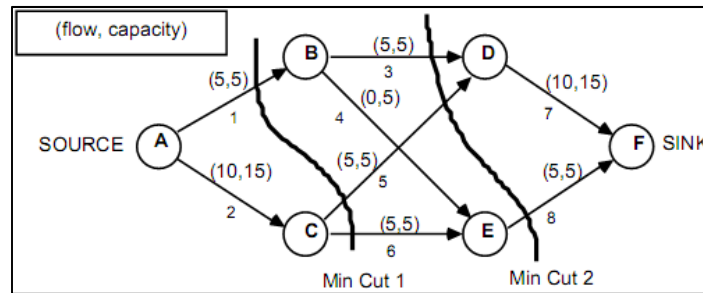


Fig 2: Minimal cuts determined by the Maximal flow

**Review of Literature**

The maximal-flow problem was introduced. If  $v$  denotes the amount of material sent from node  $s$ , called the source, to node  $t$ , called the sink, the problem can be formulated as follows:

Maximize  $v$ ,

$$\sum_j x_{ij} - \sum_k x_{ki} = \begin{cases} v & \text{if } i = s, \\ -v & \text{if } i = t, \\ 0 & \text{otherwise,} \end{cases}$$

$$0 \leq x_{ij} \leq u_{ij}, \quad (i = 1, 2, \dots, n; \quad j = 1, 2, \dots, n).$$

We assume that there is no arc from  $t$  to  $s$ . Also,  $u_{ij} = +\infty$  if arc  $i-j$  has unlimited capacity. The interpretation is that units are supplied at  $s$  and consumed at  $t$ .

Letting  $x_{ts}$  denote the variable  $v$  and rearranging, we see that the problem assumes the following special form of the general network problem:

Maximize  $x_{ts}$

$$\sum_j x_{ij} - \sum_k x_{ki} = 0 \quad (i = 1, 2, \dots, n)$$

$$0 \leq x_{ij} \leq u_{ij} \quad (i = 1, 2, \dots, n; \quad j = 1, 2, \dots, n).$$

Here arc  $t-s$  has been introduced into the network with  $u_{ts}$  defined to be  $+\infty$ ,  $x_{ts}$  simply returns the  $v$  units from node  $t$  back to node  $s$ , so that there is no formal external supply of material. Let us recall the example (see Fig. 3). The numbers above the arcs indicate flow capacity and the bold-faced numbers below the arcs specify a tentative flow plan.

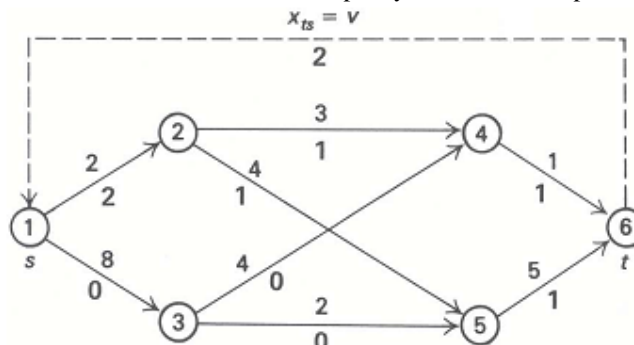
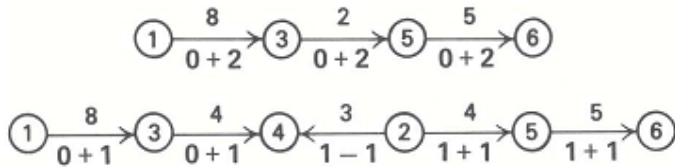


Fig 3

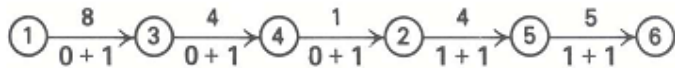
The algorithm for finding maximal flow rests on observing two ways to improve the flow in this example. The following two “paths” appear in Fig. 3



In the first case, the directed path 1–3–6 has the capacity to carry 2 additional units from the source to the sink, as given by the capacity of its weakest link, arc 3–5. Note that adding this flow gives a feasible flow pattern, since 2 units are added as input as well as output to both of nodes 3 and 5.

The second case is not a directed path from the source to the sink since arc 2–4 appears with the wrong orientation. Note, however, that adding one unit of flow to the “forward arcs” from 1 to 6 and subtracting one unit from the “reverse arc” 2–4 provides a feasible solution, with increased source-to-sink flow [6]. Mass balance is maintained at node 4, since the one more unit sent from node 3 cancels with the one less unit sent from node 2. Similarly, at node 2 the one additional unit sent to node 5 cancels with the one less unit sent to node 4.

The second case is conceptually equivalent to the first if we view decreasing the flow along arc 2–4 as sending flow from node 4 back to node 2 along the reverse arc 4–2. That is, the unit of flow from 2 to 4 increases the *effective capacity* of the “return” arc 4–2 from 0 to 1 in the original network to 1. At the same time, it decreases the usable or effective capacity along arc 2–4 from 3 to 2. With this view, the second case becomes:



Now both instances have determined a directed *flow-carrying path* from source to sink, that is, a directed path with the capacity to carry additional flow.

The maximal-flow algorithm inserts return, arcs, such as 4–2 here, and searches for flow-carrying paths. It utilizes a procedure common to network algorithms by “fanning out” from the source node, constructing flow-carrying paths to other nodes, until the sink node is reached.

**Labeling Algorithm:** This is used to find a flow augmenting path from source to the sink. From any node  $i$ , the node  $j$  can be labeled if one of the following conditions is satisfied:

1. That is the flow in the arc  $(i,j)$  is less than its capacity.
2. The flow in the arc  $(j,i)$  is greater than zero.

This process is continued till sink node is labeled and a flow augmenting path is obtained.

**Maximal-Flow Algorithm**

1. First source node  $s$  and the sink node  $t$  have been identified.
2. An initial feasible flow solution has been assumed to be equal to 0.
3. Then a flow augmenting path defined by the sequence of arcs has been determined.
4. Then Maximal flow increase in the path has been determined.

**Verifying the Algorithm— Maximal-Flow/Min-Cut:** The final solution to the sample problem illustrates an important conceptual feature of the Maximal-flow problem [7, 8]. The heavy dashed line in that figure “cuts” the network in two, in the sense that it divides the nodes into two groups, one group containing the source node and one group containing the sink node. Note that the most that can ever be sent across this cut to the sink is two units from node 1 to node 2, one unit from 4 to 6, and two units from 3 to 5, for a total of 5 units (arc 2–5 connects nodes that are both to the right of the cut and is not counted). Since no flow pattern can ever send more than these 5 units and the final solution achieves this value, it must be optimal.

Similarly, the cut separating labeled from unlabeled nodes when the algorithm terminates (see Fig. 4) shows that the final solution will be optimal.

By conservation of mass,  $v$  equals the net flow from left to right across this cut, so that  $v$  can be no greater than the total capacity of all arcs  $i - j$  pictured. In fact, this observation applies to any cut separating the source node  $s$  from the sink node  $t$ .

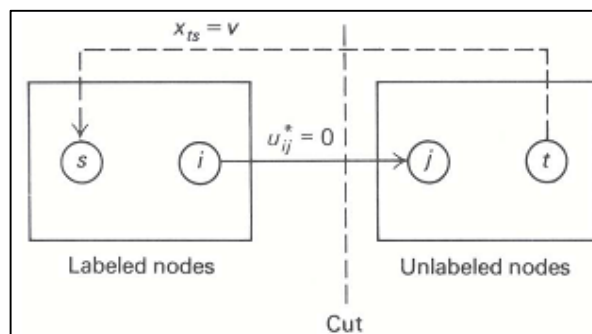


Fig 4.

By virtue of the labeling scheme, however, every arc  $i - j$  directed from left to right across the cut separating the labeled and unlabeled nodes must have  $u_{ij}^* = 0$ ; otherwise  $j$  would have been labeled from  $i$ . This observation implies that

$$x_{ij} = u_{ij} \quad \text{and} \quad x_{ij} = 0,$$

Since if either  $x_{ij} < u_{ij}$  or  $x_{ji} > 0$ , then  $u_{ij}^* = (u_{ij} - x_{ij}) + x_{ji}$  would be positive. Consequently, the net flow across the cut, and thus  $v$ , equals the cut capacity. Since no flow can do any better, this flow pattern is optimal.

### Conclusion

There are a few techniques accessible for the arrangement of Maximal flow network problems. Marking strategy is an option technique for Maximal flow network problems. Marking methods can be utilized to unravel a wide assortment of network problems, for example, most brief way issues, maximal flow problems, general insignificant cost organize flow issues, and minimal spanning tree issues. It is the reason for this represents the general idea of the naming calculations by depicting a marking strategy for the Maximal flow problem. System flow issues can be unraveled by a few techniques. This paper proceeds with the investigation of system issues by portraying the application of the naming calculation to the Maximal flow network problem. Utilizing this hypothesis maximal flow in a network can be found by finding the limit of the considerable number of cuts, and picking the base limit. In spite of the fact that this gives maximal value of  $f$ , it doesn't demonstrate how this stream is sent through different circular segments. It is a result of this thing that an alternate methodology known as maximal flow algorithm has been produced which is likewise based on max-flow min-cut theorem.

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