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Existence of solutions and controllability of impulsive nonlinear mixed Volterra-Fredholm integrodifferential equation with nonlocal condition

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Abstract
The present paper investigates the existence of mild solutions of a impulsive nonlinear mixed Volterra-Fredholm integrodifferential equation with nonlocal condition in Banach spaces. Further sufficient condition for the controllability of impulsive integrodifferential equation is established. The approach used is the Schauder fixed point theorem with the theory of resolvent operators. An example is given to illustrate the results.

Keywords: Cauchy problem, Impulsive Volterra-Fredholm integrodifferential equation, controllability, mild solution, fixed point

1. Introduction

Let X be a Banach space with the norm $\|.\|$. Let $Z = C(J, X)$ be the Banach space of all piecewise continuous functions from J into X endowed with the supremum norm

$\|x\|Z = \sup\{\|x(t)\| : t \in J\}$

and $B(X)$ denotes the Banach space of bounded linear operators from X into itself. Motivated by the work of [28], in this paper we consider the following impulsive nonlinear mixed Volterra-Fredholm integrodifferential equation of the form:

$$
\begin{aligned}
x'(t) = {} & A(t)\left(x(t) + \int_0^t Q(t,s)x(s)ds\right) \\
& + f\left(t, x(t), \int_0^t k(t,s,x(s))ds, \int_0^b h(t,s,x(s))ds\right), t \neq t_k,\ k = 1,2,\dots, m \\
& \Delta x(t)|_{t=t_k} = I_k(x(t_k)),\ t = t_k,\ k = 1, 2,\dots, m, \\
& x(0) + g(x) = x_0, \qquad (1.1)
\end{aligned}
$$

where $t \in J = [0, b]$, the unknown $x(\cdot)$ takes values in the Banach space X, and x_0 is a given element of X. Here $A(t)$ is a closed linear operator on X with dense domain $D(A)$, which is independent of t. $Q(t, s)$, $t, s \in J$, is a bounded operator in X, and $t \in J$, $x(t_k) = x(t_k^+) - x(t_k^-)$ where $x(t_k^+)$, and $x(t_k^-)$ are right and left limits of $x(t_k)$ at t_k, $k = 1, 2,\dots, m$. The nonlinear functions $f : J \times X \times X \times X \rightarrow X$, $g : Z \rightarrow X$, $k, h : J \times J \times X \rightarrow X$ are continuous functions. We define the following sets

$B_r = \{x \in X : \|x\| \leq r\}$ and $E_r = \{z \in Z : \|x\|z \leq r\}$,

The nonlocal condition, which is a generalization of the classical initial condition, was motivated by physical problems. The problem of existence of solutions of evolution equation with nonlocal conditions in Banach space was first studied between others by [5] and he investigated the existence and uniqueness of mild, strong and classical solutions of the nonlocal Cauchy problem. As indicated in [5, 10] and the references therein, the nonlocal condition $y(0) + g(y) = y_0$ can be applied in physics with better effect than the classical condition $y(0) = y_0$. For example, in [10, 13], the author used

$g(y) = \sum_{i=1}^{p} c_i y(t_i)$,

where c_i, $i = 1, 2, \cdots, p$ and $0 < t_1 < t_2 < \cdots b$, to describe the diffusion phenomenon of a small amount of gas in a transparent tube. In this case, the above explanation allows the additional measurements at t_i, $i = 1, 2, \cdots, p$.

The study of differential and integrodifferential equations in abstract spaces with nonlocal conditions have received much attention in recent years. We refer to the papers for example to [6, 7, 16, 19].

The theory of neutral differential equations has been studied by several authors in Banach spaces [5, 15, 16, 17]. Hernandez and Henriquez [18] studied the existence problem for neutral functional differential equations in Banach spaces. Various evolutionary processes from fields as diverse as physics, population dynamics, economics and engineering are characterized by the fact that they undergo abrupt changes of state at certain moments of time between intervals of continuous evolution. Because the duration of these changes are of ten negligible compared to the total duration of the process, such changes can be reasonably well approximated as being instantaneous changes of state or in the form of impulses. These processes are modeled by impulsive differential equations which allow for discontinuity in the evolution of the state. For more details on this theory and on its applications, we refer to Samoilenko and Perestyuk [23] for the case of ordinary impulsive system and to [15, 18, 19] for partial functional differential equations with impulses.

The objective of the present paper is to generalize the results reported in [13, 14, 16, 17], also must remark that our approach to the conditions on functions are different. The papers reported in [3, 20] are also special cases of the problem (1.1) when the function $\sigma(t) = t$. We first investigate the existence of mild solutions of the problem (1.1). The main tool employed in our analysis is based on the Schauder fixed point theorem and the theory of resolvent operators. We also study the nonlocal controllability problem for the above equation.

The paper is organized as follows. In section 2, we present the preliminaries and the hypotheses. Section 3 deals with the main result. Section 4 concerns with the controllability of integrodierential equation. In section 5, we give an example to illustrate the applications of our results.

2. Preliminaries and Main Results

Before proceeding to our results, we shall set forth some preliminaries and hypotheses that will be used in our subsequent discussion.

Let X be a Banach space. Let $PC([0, b],X)$ consist of functions u that are a map from $[0, b]$ into X, such that $x(t)$ is continuous at $t \neq t_i$ and left continuous at
$t = t_i$, and the right limit $u(t_i^+)$ exists for $i = 1, 2, 3,... p$. Evidently $PC([0, b],X)$ is a Banach space with the norm

$$\|x\|PC = \sup \|x(t)\|. \ t \in [0, b] \tag{2.1}$$

Definition 2.1. A resolvent operator for (1.1) is a bounded operator-valued function $R(t, s) \in B(X)$, $0 \leq s \leq t \leq b$, having the following properties:

(a) $R(t, s)$ is strongly continuous in s and t, $R(s, s) = I$, the identity operator X, $0 \leq s \leq t \leq b$, and $\|R(t,s)\| \leq Me^{\beta(t-s)}$ for some constants M and β.

(b) $R(t, s)Y \subset Y$, $R(t, s)$ is strongly continuous in s and t on Y, and Y is the banach space formed from $D(A)$, the domain $A(t)$, endowed with the graph norm.

(c) For each $y \in Y$, $R(t, s)y$ is continuously differential in $s \in J$ and
$\frac{\partial}{\partial s}R(t, s)y = -R(t, s)A(s)y - \int_s^t R(t,\tau)Q(\tau,s)A(s)y d\tau$.

(d) For each $y \in Y$, and $s \in J$, $R(t, s)y$ is continuously differential in $t \in J$ and $\frac{\partial}{\partial t}R(t, s)y = R(t, s)A(s)y + \int_s^t R(t,\tau)Q(\tau,s)A(s)y d\tau$.
with $\frac{\partial}{\partial s}R(t, s)y$ and $\frac{\partial}{\partial t}R(t, s)y$ are strongly continuous on $0 \leq s \leq t \leq b$. Here,
$R(t, s)$ can be deduced from the evolution operator of the generator $A(t)$.

Definition 2.2. A continuous solution $x(\cdot) : J \rightarrow X$ is said to be a mild solution of problem (1.1) on J if for $x_0 \in X$, it satisfies the following integral equation

$$x(t)=R(t)[x_0-g(x)]+ \int_0^t R(t-s)f\left(s,x(s),\int_0^s k(s,\tau,x(\tau))d\tau,\int_0^s h(s,\tau,x(\tau))d\tau\right)ds + \sum_{0<t_k<t} R(t-t_k)I_k\left(x(t_k^-)\right).$$

We need the following theorem (Schauder fixed point theorem [26], p-37) for further discussion:

Theorem 2.1. Let s be a bounded, closed and convex subset of a Banach space X. If $f \in C(S, S)$-set of all compact maps from S into S, then f has at least one fixed point.

We list the following hypotheses for our convenience.

(H1) The resolvent operator $R(t-s)$ is compact when $t-s > 0$ and there exists a positive constant M_1 such that
$\|R(t,s)\| \leq M_1$.

(H2) There are constants L_1, K_1 and H_1 such that
$L_1=\max_{t\in J}\|f(t,0,0,0)\|$, $K_1=\max_{0\leq s\leq t\leq b}\|k(t,s,0)\|$,

$H_1 = \max_{0\le s\le t\le b} \|h(t,s,0)\|$

(H3) There exists a constant $G_1 > 0$ such that

$\|g(x)\| G_1$, for $x \in E_r, g(\lambda x_1 + (1-\lambda)x_2) = \lambda g(x_1) + (1-\lambda) g(x_2)$,

for $x_i \in E_r$, (i = 1, 2), $\lambda(0,1)$

(H4) $I_k \in C(X,X)$, $k = 1, 2, \cdots, m$ are all bounded, that is there exist constant d_k, $k = 1, 2, \cdots, m$ such that

$\|I_k(x)\| \le d_k, x \in X$.

(H5) The set $\{x(0) : x \in E_r, x(0) + g(x) = x_0\}$

where

$$M_1\left[\|x_0\| + G_1 + Lrb + LKrb^2 + Lk_1b^2 + LHrb^2 + LH_1b^2 + L_1b + \sum_{k=1}^{m} d_k\right] \le r,$$

with $[M_1Lb + M_1LKb^2 + M_1LHb^2] < 1$, is precompact in X,

3. Existence Results

Now we shall prove the following result of existence of mild solution.

Theorem 3.1. Assume that

(i) Hypotheses *(H1) − (H5) hold,*

(ii) $f \in PC(J \times X \times X \times X; X)$ and there exists a constant $L > 0$ such that

$\| f(t,x_1,y_1,z_1) - f(t,x_2,y_2,z_2) \| \le L (\| x_1 - x_2 \| + \|y_1 - y_2\| + \| z_1 - z_2\|)$,

for $x_i, y_i, z_i \in B_r$, $i = 1, 2$ *and* $t \in J$.

(iii) $k, h \in PC(J \times J \times X; X)$ and there exist constants $K,H > 0$ such that

$\|k(t, s, x_1) - k(t, s, x_2)\| \le K \| x_1 - x_2\|$

and

$\| h(t, s, x_1) - h(t, s, x_2)\| \le H \| x_1 - x_2\|$

for $x_i, y_i \in B_r$, $i = 1, 2$ *and* $t, s \in J$. Then problem (1.1) has a mild solution

Proof. We define the set E by $E = \{x \in Z : x \in E_r, x(0) + g(x) = x_0\}$. It is easy to see that E is a bounded closed convex subset of Z. Define a mapping $F : E \to E$ by

$$(Fx)(t) = R(t)[x_0 - g(x)] + \int_0^t R(t-s) f\left(s, x(s), \int_0^s k(s,\tau,x(\tau))dT, \int_0^b h(s,\tau,x(\tau))dr\right) ds$$

$$+ \sum_{0<t_k<t} R(t-t_k) I_k(x(t_k^-)), t \in J. \quad (3.1)$$

Since all the functions involved in the definition of the operator are continuous, the operator F is continuous. For $x \in E$, $t \in J$ and using hypotheses *(H1) − (H5)* and assumptions

(ii) − (iii)

$$\| (Fx)(t)\| \le M_1(\|x_0\| + G_1)$$

$$+M_1 \int_0^t \left[\left\| f\left(s, x(s), \int_0^s k(s,\tau,s(\tau))d\tau, \int_0^b h(s,\tau,x(\tau))d\tau, f(s,0,0,0)\right)\right\| + \|f((s,0,0,0)\|\right] ds + M_1 \sum_{k=1}^m d_k$$

$$\le M_1(\|x_0\| + G_1$$

$$+ M_1 L \int_0^t \Bigg[r$$

$$+ \int_0^s \left\|k(s,\tau,s(\tau)) - k(s,t,0) + k(s,t,0)\right\| d\tau + \int_0^b \left\|h(s,\tau,x(\tau)) - h(s,\tau,\tau,0) + h(s,\tau,0)\right\| dr \Bigg] ds$$

$$+L_1M_1b + M_1\sum_{k=1}^m d_k$$

$$\le M_1(\|x_0\| + G_1) + M_1 \int_0^t [Lr + Lb(Kr + K_1) + Lb(Hr + H_1)] ds + L_1M_1b + M_1 \sum_{k=1}^m d_k$$

$$\le M_1 [\|x_0\| + G_1 + Lrb + LKrb^2 + LK_1b^2 + LHrb^2 + LH_1b^2 + L_1b] + M_1 \sum_{k=1}^m d_k$$

$$\le r \quad (3.2)$$

Thus, F maps E into itself and consequently $F \in C(E;E)$, Now, we prove that F maps E into a precompact subset $F(E)$ of E. For this purpose, we first show that the set $E(t) = \{(Fx)(t) : x \in E\}$, $t \in J$ is precompact in X. Observe that

$E(0) = \{(Fx)(0) : x \in E\} = \{x_0 - g(x) : x \in E_r, x(0) + g(x) = x_0\}$,

Therefore, according to hypothesis ($H5$), $E(0)$ is precompact in X. Let $t > 0$ be fixed. For an arbitrary $0 < \epsilon < t$, we define a mapping

$$(F_\epsilon x)(t) = R(t)[x_0 - g(x)] + \int_0^{t-\epsilon} R(t-s) f\left(s, x(s), \int_0^s k(s,\tau,x(\tau))dT, \int_0^b h(s,\tau,x(\tau))d\tau\right) ds,$$

$$+ \sum_{0<t_k<t} R(t-t_k) I_k(x(t_k^-)), t \in J. \quad (3.3)$$

Since $R(t-s)$ is compact operator for every t, s ≥ 0, then the set $E_\varepsilon(t) = \{(Fx)(t) : x \in E\}$ is precompact in X for every ≥ 0. By using the equations (3.1), (3.3) and the hypotheses (*H1*) - (*H5*), we obtain

$$\| (Fx)(t) - (F\varepsilon x)(t) \| \leq M_1 \int_{t-\epsilon}^{t} [\| f(s, x(s), \int_o^s k(s,\tau,x(t))d\tau, \int_o^b h(s,\tau,x(\tau))d\tau)$$
$$- f(s,0,0,0)\| + L1]ds$$
$$\leq M_1 \int_{t-\epsilon}^{t} [Lr + Lb(Kr + K_1) + Lb(Hr + H_1) + L_1]ds$$
$$\leq M_1 [Lrb + LKrb^2 + LK_1 b^2 + LHrb^2 + LH_1 b^2 + L_1 b] \epsilon. \quad (3.4)$$

This implies that there exist precompact sets arbitrary close to the set $E(t) = \{(Fx)(t) : x \in E\}$. Hence, the set $\{(Fx)(t) : x \in E\}$ is precompact in X. Next, we show that $F(E)$ is an uniformly equicontinuous family of fuctions. Let 0 < s < t. By using hypotheses (*H2*), (*H3*), (*H4*) and (*ii*), (*iii*), we have

$$\| (Fx)(t) - (Fx)(s) \| \leq \| R(t) - R(s) \| (\| x_0 \| + \| g(x) \|) + \int_0^s \| R(t,\tau) - R(s,\tau) \|$$
$$X \| f(\tau, x(\tau), \int_O^\tau k(\tau,\sigma,x(\sigma))d\sigma, \int_o^b h(\tau,\sigma,x(\sigma))d\sigma) \| d\tau$$
$$+ \int_s^t \| R(t,\tau) \| \| \| f(\tau, x(\tau), \int_O^\tau k(\tau,\sigma,x(\sigma))d\sigma, \int_o^b h(\tau,\sigma,x(\sigma))d\sigma) \| d\tau$$
$$+ \sum_{0 \leq t_k \leq s} \|R(t-t_k) - R(s-t_k)\| \, \|I_k(x(t_k^-))\|$$
$$+ \sum_{s \leq t_k \leq t} \|R(t-t_k)\| \, \|I_k(x(t_k^-))\|$$
$$\leq \|R(t) - R(s)\|(\|x_0\| + G_1) + M_1[L_r + L(Kr + K_1 + Hr + H_1)b + L_1](t-s)$$
$$+ \int_0^s \|R(t,\tau) - R(s,\tau)\|[L_r + L(Kr + K_1 + Hr + H_1)b + L_1]dr$$
$$+ \sum_{0<t_k<s} \|R(t-t_k) - R(s-t_k)\| d_k + \sum_{s \leq t_k \leq t} M_1 d_k \quad (3.5)$$

Here we have proceeded as in the result (3.4). The right hand side of (3.5) is independent of $x \in E$ and tends to zero as $t \to s$ as a consequence of the continunity of $R(t-s)$ in the uniform operator topology for $t > 0$, which follows from the compactness of $R(t-s)$, $t-s > 0$. Therefore, *F(E)* is equicontinuous family of functions. Thus by Arzela Ascoli's theorem, *F(E)* is precompact. Hence by the Schauder fixed point theorem, *F* has a fixed point in *E* and any fixed point of *F* is a mild solution of (1.1) on *J*.

4. Controllability Result

Controllability is one of the fundamental concepts in the mathematical control theory and plays an important role in both deterministic and stochastic control systems. It is well known that the controllability of deterministic systems are widely used in many fields of science and technology. The controllability of nonlinear deterministic systems represented by equations in abstract spaces, whereas the stochastic control theory is a stochastic generalization of classical control theory. Such problems have been studied by several authors, see [1, 2, 4, 8, 9, 10] and the references cited therein. Now we will establish a set of sufficient conditions for the controllability of nonlinear mixed impulsive Volterra-Fredholm integrodifferential equation with control parameter of the form:

$$x'(t) = A(t)\left(x(t) + \int_0^t Q(t,s)x(s)ds\right)$$
$$+ f\left(t, x(t), \int_0^t k(t,s,x(s))ds, \int_0^b h(t,s,x(s))ds\right) + (Bu)(t) + t \neq tk,$$
$k = 1, 2, \dots, m$,
$\Delta x(t)|_{t=t_k} = I_k(x(t_k)), t = t_k, k = 1, 2, \dots, m$,
$$x(0) + g(x) = x_0, \quad (4.1)$$

where the state x(·) takes values in the Banach space X and the control function $u(\cdot)$ is given in $L^2(J,U)$, a Banach space of admissible control functions with U as a Banach spaces. Here B is a bounded linear operator from U into X. Then, for equations (4.1), there exists a mild solution of the following form

$$x(t) = R(t)[x_0 - g(x)] + \int_0^t R(t-s)[f(s, x(s), \int_0^s k(s,\tau,x(\tau))d\tau, \int_0^b h(s,\tau,x(\tau))d\tau)$$
$$+ (Bu)(s)]ds + \sum_{0<t_k<t} R(t-t_k) I_k (x((_k^-)),$$

where the resolvent operator $R(t - s) \in B(X)$ for $t - s > 0$ and the functions *f*, *g*, *k* and *h* satisfy the conditions stated in Section 3.

Definition 4.1. The system (4.1) is said to be nonlocally controllable on the interval *J* if, for every $x_1 \in X$, there exists a control *u* $\in L^2(J,U)$ such that the mild solution $x(\cdot)$ of the problem (4.1) *satisfies* $x(b) + g(x) = x_1$:
To establish the result, we need the following additional hypothesis.
(H6) The operator *W* from $L^2(J,U)$ into *X*, defined by
$W_u = \int_0^b R(b-s)(Bu)(s)ds,$
has an induced inverse W^{-1} which takes values in $L^2(J,U)/\ker W$, and there exist positive constants M_2, M_3 such that
$\|B\| \leq M_2$, $\|W^{-1}\| \leq M_3$.

Theorem 4.1. If the hypotheses (*H1*)−(*H6*) and conditions (ii), (iii) of Theorem 3.1 are satisfied, then the (4.1) is nonlocally controllable on *J*.
Proof. Using hypothesis (*H6*), for an arbitrary $x(\cdot)$, define the control

$$u(t) = W-1\{x1 - g(s) - R(b)(x0 - g(x)) - \int_0^b R(b,s)\left[f\left(s, x(s), \int_0^s k(s,\tau,x(\tau))dT, \int_0^b h(s,\tau,x(\tau))dt\right)\right]ds - \sum_{k=1}^m R(b-t_k)I_k(x(t_k^-))\}(t). \quad (4.2)$$

Let

$Z_0 = \{x \in Z : x(0) + g(x), \|x\| \leq r_1, \text{ for } t \in J\}$,
where the positive constant r1 is given by

$$r1 \geq M_1 [\|x_0\| + G_1 + Lr_1b + LKr_1b^2 + LK_1b^2 + LHr_1b^2 + LH_1b^2 + L_1b + M_1\sum_{k=1}^m d_k](1 + M_1M_2M_3b) + M_1M_2M_3(\|x_1\| + G_1)b,$$

with $(1+M_1M_2M_3b)[M_1Lb+M_1Lb^2+M_1LHb^2] < 1$.Then Z_0 is clearly a bounded, closed and convex subset of *Z*. Define a mapping $\Phi : Z_0 \to Z_0$ by

$$(\Phi x)(t) = R(t)[x_0 - g(x) + \int_0^t R(t \to s)\left[f\left(s, x(s), \int_0^s k(s,\tau,x(\tau))d\tau, \int_0^b h(s,\tau,x(\tau))dt\right) + (Bu)(s)\right]ds + \sum_{0<t_k<t} R(t - t_k)I_k(x((_k^-)). \quad (4.3)$$

Now, we shall show that, when using control u, the operator Φ has a fixed point. This fixed point is then a mild solution of the system (4.1). Clearly, $x1-g(x) = (\Phi x)(b)$, which means that the control *u* steers the mixed integrodifferential system from the initial state x_0 to x_1 in time $_b$ provided we can obtain a fixed point of the nonlinear operator Φ. Using the definition of the control *u*, we get

$$(\Phi x)(t) = R(t)[x_0 - g(x)]\int_0^t R(t \to s)\left[f\left(s, x(s), \int_0^s k(s,T,x(T))dT, \int_0^b h(s,T,x(T))dt\right)\right]ds$$
$$+ \sum_{0<t_k<t} R(t-t_k)I_k(x((t_k^-))$$
$$+ \int_0^t R(t-s)BW^{-1}\left[x_1 - g(x) - R(b)(x_0 - g(x)) - \right.$$
$$\left.\int_0^b R(b,\theta)f(\theta, x(\theta), \int_0^\theta k(\theta,T,x(T))dT, \int_0^b h(\theta,T,x(T))dT)d\theta + \sum_{0<t_k<t} R(t-t_k)I_k(x((t_k^-))\right](s)ds. \quad (4.4)$$

Since all the functions involved in the definition of the operator are continuous, the operator Φ is continuous. For $x \in Z_0$, $t \in J$ and following steps as in the proof of Theorem 3.1 in
equation (3.2), from hypotheses (*H1*) − (*H6*) and assumptions (ii), (iii), we have

$$\|(\Phi x)(t)\| \leq M_1[\|x_0\| + G_1 + Lr_1b + LKr_1b^2 + LK_1b^2 + LHr_1b^2 + LH_1b^2 + L_1b + M_1\sum_{k=1}^m d_k](1 + M_1M_2M_3b) + M_1M_2M_3(\|x_1\| + G_1)b = r_1 \quad (4.5)$$

Thus, Φ maps Z_0 into itself and consequently $\Phi \in C(Z_0;Z_0)$.
Now, we prove the Φ into a precompact subset $\Phi(Z_0)$ of Z_0. For this purpose, we first show that for every fixed $t \in J$, the set $Z_0(t) = \{(\Phi x)(t) : x \in Z_0\}$, is precompact in *X*. This is clear for $t = 0$, since $Z_0(0)$ is precompact by hypothesis (*H5*). Let $t > 0$ be fixed. For an arbitrary $0 < \epsilon < t$, we define a mapping

$$(\Phi_\epsilon x)(t) = R(t)(x_0 - g(x)) + \int_0^{t-\epsilon} R(t-s)[f(s,x(s), \int_0^s k(s,\tau,x(\tau))d\tau, \int_0^b h(s,\tau,x(\tau))d\tau)]ds$$
$$+ \sum_{0<t_k<t} R(t-t_k)I_k(x(t_k^-))$$

$+\int_0^{t-\epsilon} R(t-s)BW^{-1}\Big[x_1 - g(x) - R(b,0)\big(x_0 - g(x)\big) -$
$\int_0^b R(b,\theta)f(\theta,x(\theta),\int_0^\theta k\big(\theta,\tau,x(\tau)\big)d\tau,\int_0^b h\big(\theta,\tau,x(\tau)\big)d\tau)d\theta + \sum_{k=1}^m R(b-t_k)I_k(x(t_k^-))\Big](s)ds$ (4.6)

Since $R(t-s)$ is compact operator for every $t,s \geq 0$, then the set $Z\epsilon(t)=\{ (\Phi_\epsilon x)(t): x \in Z_0\}$ is precompact in X for every $\epsilon > 0$. By using the equations (4.5), (4.7) and the hypotheses (*H1*) − (*H5*), and (*ii*), (*iii*), we obtain

$\|(\Phi x)(t) - (\Phi_\epsilon x)(t)\| \leq M[Lr_1 + LK_{r1}b + LKb + LHr_1 b + LH_1 b + L_1 + d_k] \in$
$+M_1M_2M_3[||x_1|| + G1 + M_1(||x_0|| + G1 + Lr_1 b + LKr_1 b^2 + LK_1 b^2 + LHr_1 b^2 + LH_1 b^2 + L_1 b + dk] \in,$ (4.7)

which implies that $Z_0(t)$ is totally bounded, that is, precompact in X,
Next, we show that $\Phi(Z_0)$ is an uniformly equicontinuous family of functions. Let 0 < s < t. Following steps as in the proof of Theorem 3.1 in equation (3.5), from hypotheses (*H2*), (*H3*)

and (*ii*), (*iii*), we have

$\|(\Phi x)(t) - (\Phi x)(s)\| \leq \|R(t) - R(s)\| x_0|| + G_1) + M_1\Big[L_r + L(k_r + k_1 + H_r + H_1)b + L_1 + d_k^{'}\Big](t-s) + \int_0^s\|R(t,T) - R(s,T)\|[L_r + L(k_r + k_1 + H_1 + H_r)b + L_1]d_r + \int_0^s\|R(t,T) - R(s,T)\|M_2M_3[\|x_1\| + G_1 + (L_{r1} + k_{r1}b + Lk_1 b + LH_{r1}b + L_1 + d_k)b)]dT + M_1M_2M_3[\|x_1\| + G_1 + M_1(\|x_0\| + G_1 + (Lr_1 + LKr_1 b + LK_1 b + LHr_1 b + L_1 + d_k)b)](t-s).$ (4.8)

Here we have proceeded as in the result (4.7). The right hand side of (4.8) is independent of $x \in Z_0$ and tends to zero as $s \to t$ as a consequence of the continuity of $R(t-s)$ in the uniform operator topology for $t > 0$, which follows from the compactness of $R(t-s)$, $t-s > 0$. Therefore, $\Phi(Z_0)$ is equicontinuous family of functions. Thus by Arzela-Ascoli's theorem, $\Phi(Z_0)$ is precompact. Hence by the Schauder fixed point theorem, Φ has a fixed point in Z_0 and any fixed point of Φ is a mild solution of (4.1) on *J*. Therefore, the system (4.1) is nonlocally controllable on *J*.

5. Example

In this section, we give an example to illustrate the usefulness of our main result. Let us consider the following impulsive partial integrodierential equation of the form:

$$\frac{\partial}{\partial t}\omega(t,x) = a_0(t)\frac{\partial^2}{\partial x^2}\left[\omega(t,x) + \int_0^t \frac{1}{(1+t^2)(1+s^2)}\,\omega(s,x)ds\right] + \mu(t,x)$$
$$+\omega(t,x) + \int_0^t \frac{1}{(1+t^2)(1+s^2)}\,\omega(s,x)ds$$
$$+\int_0^t \frac{1}{(1+t^2)(1+s^2)}\,[\omega^2(s,x) + \sin(\omega^2(s,x))]\,ds$$
$$\omega(t,0) = \omega(t,\pi) = 0$$
$$\Delta y\backslash t = t_k = I_k\big(y(t_k^-)\big), k = 1,2, ...$$
$$\omega(0,x) + \int_0^1 \frac{1}{2}\omega(s,x)ds = \omega_0(x), 0 \leq t \leq 1, 0 \leq x \leq \pi,$$

where $\omega_0(x) \epsilon X = L^2([0;\pi]), \omega_0(0) = \omega_0(\pi) = 0$ and the functions a_0 and $\mu : [0,1] \times (0,\pi) \times (0,\pi)$ are continuous on $0 \leq t \leq 1$. Let $X = L^2([0,\pi])$ and the operators A(t) be
defined by
$A(t)z = a_0(t)z''$.
With the domain $D(A) = \{z \epsilon X : z, z''$ are absolutely contunuous, $z'' \epsilon X$, $z(0) = z(1) = 0\}$, then A(t) generates an evolution system and R(t, s) can be deduced from the evolution systems [11, 12, 15] such that $R(t, s)$ is compact and $\|R(t, s)\| \leq M_1 e^{(t-s)}$ for some constants M_1 and β. On comparison of functions f, g and I_k with the problem (5.1), then by assumptions, we have

$$\| f\big(t,x_1,y_1,z_1\big) - f\big(t,x_2,y_2,z_2\big) \leq \{1 + \frac{log2}{(1+t^2)}[1 + 2 \| x_1 + x_2 \|\} \| x_1 - x_2 \|$$

and There exists a constant $d_k > 0$, $k = 1, 2..., m$,
$$\| I_k \| \leq d_k.$$

Also for $x_1, x_2 \epsilon C[0, 1]$, the function

$$g(\lambda x_1 + (1-\lambda)x_2) = \int_0^1 \frac{1}{2}(\lambda x_1 + (1-\lambda)x_2)(s)ds$$
$$= \int_0^1 \frac{1}{2}\lambda x_1(s)ds + \int_0^1 \frac{1}{2}(1-\lambda)x_2(s)ds = \lambda g(x_1) + 1(1-\lambda)g(x_2)$$

is convex. Let Bu : [0, 1] × X be defined by
$(Bu)(t)$x = $\mu(t, x)$, $x \in (0,\pi)$.

With the choice of $A(t)$, B and f, the equations (5.1) take the abstract form as (1.1). Now, the linear operator W is given by
$(Wu)x = \int_0^1 R(1,s)\mu\,(s,x), x\epsilon(0,\pi)$

Assume that this operator has a bounded invertible operator W^{-1} in $L^2([0, 1],U)/\ker W$. Furthere, all the other conditions stated in Theorem 3.1 and Theorem 4.1 are satisfied. Hence, the problem (5.1) has a mild solution on [0, 1] and the system (5.1) is controllable on [0, 1].

6. Conclusion
In this paper, we study the existence of mild solutions of a impulsive nonlinear mixed Volterra-Fredholm integrodifferential equation (1.1) with nonlocal condition in Banach spaces and controllability of impulsive integrodifferential equation (4.1) is also established by the Schauder fixed point theorem with the theory of resolvent operators. As applications, example is presented to illustrate the main results.

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8. References

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