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An introduction to differential equations

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Abstract
Whole of the physics and engineering depends upon differential equations, their laws dominate electronics, mechanical and civil engineering. Independent use of differential equations makes it perfect tool for the use of applied mathematics. Here we are discussing some aspects of the differential equations their definitions, types of solutions, their applications and uses.

Keywords: Differential equations, applied mathematics, Linear homogeneous

Introduction

Differential Equation

An equation involving independent, dependent variable and derivative of dependent variable with respect to independent variables is called differential equation.
It can be further divided into two categories:
1. O.D.E.
2. P.D.E

ODE Ordinary Differential Equation

A differential equation involving derivative with respect to a single independent variable is called an ordinary differential equation. for example
$\frac{dy}{dx}$ + 4y = sinx and $\frac{dy}{dx}$ + 2xcosxy = xe^{x} are ordinary differential equations.

Ordinary differential equations are further divided into many branches as follows-
a. exact differential equations.
b. orthogonal trajectories.
c. Linear homogeneous differential equation with constant coefficients.
d. Ordinary simultaneous equations.

PDE Partial Differential Equation

A differential equation involving partial derivatives with respect to more than one independent variables is called a partial differential equation.
For example $\left(\frac{\partial y}{\partial x}\right)+\left(\frac{\partial y}{\partial t}\right)$+5cosx=0 is partial differential equation. Here we are providing some basic definitions of ingredients used in differential equations:-

Order of a differential equation

The order of highest order derivative involved in a differential equation is called the order of the differential equation. for example order of

dy =(x+sinx)dx is 1
$\frac{d^3x}{dt^3} + \frac{d^2x}{dt^2} + (\frac{dy}{dx})^5 = e^t$ is 3.

Degree of a differential equation

The degree of a differential equation is the degree of highest derivative which occurs in it, after the dependent has been made free from radicals and fractions as far as the derivatives are concerned .for example degree of $\frac{d^3x}{dt^3}+\frac{d^2x}{dt^2}+(\frac{dy}{dx})^5=e^t$ is 1 as it is the degree of highest order derivative occurring in the equation.

Linear and nonlinear differential equations

A differential equation is said to be linear if:-

1. Every dependent variable and every derivative involved occur in the first derivative only.
2. No products of dependent variables and/or derivative occur.

For example dy = (x + sinx) dx is the linear differential equation.

A differential equation which is not linear is called nonlinear differential equation. For example $\frac{d^3x}{dt^3}+\frac{d^2x}{dt^2}+(\frac{dy}{dx})^5=e^t$ is non linear differential equation.

Solution of a differential equation

Any relation between the dependent variables and independent variables, when substituted in the differential equation, reduces it to an identity is called a solution or integral of the differential equation it should be noted that a solution of a differential equation does not involves the derivatives of the dependent variable.

For example y=ce^{2x} is the solution of $\frac{dy}{dx}$= 2x because by substituting y = c$e^{2x}$and $\frac{dy}{dx}$=2 ce^{2x} the given differential equation reduces to the identity 2 ce^{2x}=2 ce^{2x}.

Particular solution and singular solution

LET F(x, y, $y_1, y_2 \dots, y_n$) = 0 … (1)

be the nth order ordinary differential equation.

General Solution

A solution of (1) containing n independents arbitrary constants is called a general solution of the differential equation.

- Singular Solution**:** A solution of (1) obtained from a general solution of (1) by giving particular values to one or more of the n independent arbitrary constants is called the singular solution of (1).
- A delay differential equation (DDE) is a type of equation which contains function of a single variable, usually called time, in which the derivative of the function at a certain time is given in terms of the values of the function at earlier times.
- A stochastic differential equation (SDE) is a type of equation in which the unknown quantity is a stochastic process and the equation involves some known stochastic processes, for example, the Wiener process in the case of diffusion equations.
- A differential algebraic equation (DAE) is a differential equation comprising differential and algebraic terms, given in implicit form.

Applications

Physics

- Euler–Lagrange equation in classical mechanics.
- Hamilton's equations in classical mechanics.
- Radioactive decay in nuclear physics.
- Newton's law of cooling in thermodynamics.
- The wave equation.
- The heat equation in thermodynamics.
- Laplace's equation, which defines harmonic functions.
- Poisson's equation.
- The geodesic equation.
- The Navier–Stokes equations in fluid dynamics.
- The Diffusion equation in stochastic processes.
- The Convection–diffusion equation in fluid dynamics.
- The Cauchy–Riemann equations in complex analysis.
- The Poisson–Boltzmann equation in molecular dynamics.
- The shallow water equations.
- Universal differential equation.
- The Lorenz equations whose solutions exhibit chaotic flow.
- Ground water equations.
- Current voltage relationship.
- Signal systems.
- Maxwell's equations.

Classical mechanics

All we know that, Newton's second law is sufficient to describe the motion of a particle. Once independent relations for each force acting on a particle are available, they can be substituted into Newton's second law to obtain an ordinary differential equation, and various other relations which are called the *equation of motion*.

Electrodynamics

Maxwell's equations forms a set of partial differential equations that, along with the Lorentz force law, form the base of classical electrodynamics, classical optics, and electric circuits. These fields are the base for modern electrical and communications technologies. Maxwell's equations describe how electric and magnetic fields are generated and altered by each other and by charges and currents. They are named after the Scottish physicist and mathematician James Clerk Maxwell, who published an early form of those equations between 1861 and 1862.

General relativity

Quantum mechanics

In quantum mechanics, the analogue of Newton's law is Schrödinger's equation (a partial differential equation) for a quantum system (usually atoms, molecules, and subatomic particles whether free, bound, or localized). It is not a simple algebraic equation, but in general a linear partial differential equation, describing the time-evolution of the system's wave function (also called a "state function").

Biology

- Verhulst equation – biological population growth
- von Bertalanffy model – biological individual growth
- Replicator dynamics – found in theoretical biology
- Hodgkin–Huxley model – neural action potentials

Predator-prey equations

The Lotka–Volterra equations, which are also known as the predator–prey equations, are a pair of first-order, non-linear, differential equations frequently used to describe the population dynamics of two species that interact, one as a predator and the other as prey.

Chemistry

The rate law or rate equation for a chemical reaction is a differential equation that links the reaction rate with concentrations or pressures of reactants and constant parameters (normally rate coefficients and partial reaction orders). To determine the rate equation for a particular system one combines the reaction rate with a mass balance for the system. The concept of rate of reaction can be applied to the tumour clots in which we can minimize the rate of growth of tumour with respect to time and we can have more time to cure that by proper medicine.

Schrodinger equation could not be possible without the use of differential equation for the concept of electron density function. Partial derivatives of the function Ψ with respect to the variables x, y and z gives the required equation and are known as Schrodinger equation

Economics

Major role of the differential equation and mathematics can be seen in the following concepts

- The key equation of the Solow–Swan model.
- The Black–Scholes PDE.
- Malthusian growth model.
- The Vidale–Wolfe advertising model.
- Elasticity of demands.
- Graphs between the various changing variables.

Conclusion

The study of differential equations is a wide field because of their adaptability in pure and applied mathematics, physics, and engineering. All of these branches are concerned with the properties of differential equations of various types.. Differential equations play a vital role in modelling virtually every physical, technical, or biological process, from critical problems, to bridge design, to interactions between neurons. Differential equations such as those used to solve real-life problems may not necessarily be directly solvable, i.e. do not have closed form solutions. Instead, solutions can be approximated using numerical methods and grid generation methods various fundamental laws of physics and chemistry can be obtained from differential equations. In biology and economics, differential equations are used to model the behaviour of complex systems. Theory of differential equations in mathematical form is first developed together with the sciences where the equations had originated and where the results found application. However, diverse problems, sometimes originating in quite separate scientific fields, which give rise to identical differential equations. Whenever this happens, mathematical theory behind the equations can be viewed as a unifying principle behind diverse phenomena. As an example, consider propagation of light and sound in the atmosphere and of waves on the surface of a pond. All of them may be described by the same second-order partial differential equation, the wave equation, which allows us to think of light and sound as forms of waves, much like familiar waves in the water. Conduction of heat, the theory of which was developed by Joseph Fourier, is governed by another second-order partial differential equation, the heat equation. It turns out that many diffusion processes, while seemingly different, are described by the same equation; the Black–Scholes equation in finance is, for instance, related to the heat equation.

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