International Journal of Statistics and Applied Mathematics 2018; 3(1): 136-142

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2018; 3(1): 136-142

www.mathsjournal.com
Received: 17-11-2017
Accepted: 23-12-2017

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Bulk service queue with server breakdown and repairs

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Abstract
A single server Erlangian bulk service queue with certain concepts like vacation, break down and repair has been analysed. The units arrive to the service point according to Poisson process with various arrival rates which depend on the position of the server. The units are served in batches under general bulk service rule in k service phases. When the system becomes empty, the server leaves the system and takes vacation for a random period of time. The server may breakdown which leads to repairs. In this paper, the expressions for the state probabilities and the expected number of units in the system under different positions of the server are derived.

Keywords: Bulk service, vacation, breakdown, repair, state probabilities, mean queue size

1. Introduction

In an era of technological development, performance modeling is one of the major concerns that impact the design, development, configuration and modification of any real time system. Queueing modeling is used in a wide variety of congestion problems encountered in everyday and in industrial scenario including computer systems, call centers, web services and communication networks, waiting lines at airports, banks, public offices, etc,. Countless queueing models have been developed for nearly a century to examine the performance of many systems and recommendations have been made to suggest that how to deal with congestion situations. In several queueing models, the server may take vacation from the primary job when there is no job present in the system and provides the service for the secondary job; such vacation is known as working vacation. Queueing models with working vacations can be described and defined easily by taking an example of 'an international company' wherein the main task of the receptionist is to answer and divert the calls. Rest of the time, the receptionist may call the potential customers for promoting the company's products as an additional job.

Queueing models with vacation have been the subject of interest and interrogation for many years for the queue researchers. Servi and Finn (2002) [23] examined a classical vacation queueing model in which a single server works with a different rate rather than completely stopping service during the vacation period. Gupta and Sikdar (2004) [12] have analysed a single server finite-buffer bulk-service queueing model with single vacation in which the inter-arrival and service times are considered to be exponentially and arbitrarily distributed, respectively. Madan and Al-Rawwash (2005) [18] have analysed a single server queue with batch arrivals and general service time distribution. Wu and Takagi (2006) [28] have studied an M/G/1 queue with multiple vacations and exhaustive service discipline such that the server works with different rate rather than completely stopping the service during vacation. A vacation queueing model has been examined by Arumuganathan and Jeyakumar (2005) [2], Palaniammal *et al*. (2006) [22] and Zadesh and Shahkar (2008) [29], in different frame works. Ke (2008) [16] has investigated an $M^X/G/1$ queueing system under a variant vacation policy, where the server takes at most j vacations. Omey and Gulck (2008) [21] have studied maximum entropy of the $M^X/M/1$ queueing model with multiple vacations and server breakdowns. Ke and Chang (2009) [15] have analysed the $M^X/(G_1, G_2)/1$ retrial queue with general retrial times, where the server provides two phases of heterogeneous service to all customers under Bernoulli vacation schedules. Goswami and Selvaraju (2010) [11] have considered a discrete-time single-server queueing model with multiple working vacations.

Baba (2012) [4] has studied a batch arrival $M^X/M/1$ queue with multiple working vacation and obtained probability generating function and stochastic decomposition structure of the system and some performance indices, mean system length and the mean waiting time. Vijaya Laxmi *et al* (2013) [25] Analysed a finite buffer renewal input single working vacation and vacation interruption queue with state dependent vacations. They also presented an efficient computation algorithm and computed the stationary queue length for the above model along with different performance measures. Sree Parimala and Palaniammal (2014) [24] studied bulk service queueing model with server's single and delayed vacation. For this model, the steady state solutions and the system characteristics are derived and analysed. Ibe (2015) [13] has considered a single server vacation queueing system with server time out and derived expressions for the mean waiting time and studied N-policy scheme.

Bharathidass (2013) [8] has analysed a Markovian bulk service queue under accessibility rules in which a single server Markovian queue is considered. The arriving units served in batches by using Accessibility and Non-Accessibility with varying service rates. The expressions for the steady state probabilities when the server is busy as well as idle are derived. The mean and variance for the number of units in the queue are obtained. The expected waiting time of units is also attained. Also the numerical results for number of units in the queue are computed for various value of arrival rate and exhibited the corresponding graphs when the remaining parameters are fixed. Balasubramanian *et al* (2015) [8] have analysed a finite size single server Markovian queue with restoration, catastrophe and general bulk service. The expression for the expected number of times the system reaches to the capacity, expected number of customers in the system and respective factorial moments have been derived under different conditions. Balasubramanian *et al* (2016) [7] have studied a Markovian queue with blocking and catastrophes in which a queueing system with two servers is considered and the arrival times of units and service times corresponding to both servers follow Markovian nature. The expressions for the expected queue length in the system and loss probability of units have been derived and also the numerical results are provided. Arivukkarasu *et al* (2016) [1] have studied customer impatience in bulk arrival queue with optional service in which considered a single server queue and customers arrive in batches of different sizes. The arrival times of batches follow Poisson process and the service times of essential and optional services are distributed according to exponential and general distributions respectively. The principles of customer's impatience, server's vacation and bulk size rule have been employed. The expressions for the probability generating functions, steady state probabilities and expected system size have been derived when the server is on vacation, busy with essential service and busy with optional service.

Vacation queueing systems with service interruption have also been investigated in the recent years by researchers. No system is found perfect in the real world since all the devices fail more or less frequently. It is an inevitable phenomenon which can be realized in computer, communication and manufacturing systems. Several day to day queueing problems on industry and society have been studied by using queueing concepts under the assumptions of server vacation and unreliable server. Many researchers have analyzed single server queueing system with service interruption. Other denominations of an unreliable server queueing model have been analysed by a list of queueing experts like Wang (2004) [26], Choudhury (2009) [9], Wang and Haung (2009) [27] and Jain and Jain (2010) [14]. Maraghi *et al* (2009) [20] have studied a batch arrival queueing system with random breakdowns and Bernoulli schedule server vacations having general vacation time distribution and obtained the steady state results for queue size, mean queue size and mean waiting time of the customer in the queue along with some special cases discussed and some known results have been derived. Khalaf *et al* (2011) [23] have discussed an $M^{[X]}/G/1$ queueing system with Bernoulli schedule server vacations and random system breakdowns with general delay and general repair times. The steady state solutions have been found by using supplementary variables technique. Ayyappan and Shyamala (2013) [3] have considered a queueing model with batch arrival with second optional service, Bernoulli ssssssssssk-optional vacation and balking. The time dependent solution and the corresponding steady state solutions have been derived along with performance measures, the mean queue size and the average waiting time explicitly. Balamani (2014) [5] has studied a two stage batch arrival queue with compulsory server vacation and second optional repair and has derived the steady state solutions also computed the mean queue length and the mean waiting time. Ebenesar Anna Bagyam *et al* (2015) [10] have considered bulk arrival multi-stage retrial queue with Bernoulli vacation, the steady state distributions of the server state and the queue size have been obtained along with important measures. Maragathasundari and Karthikeyan (2015) [19] have studied about a Non Markovian queueing model with extended server vacation, random breakdowns and general repair. Also derived the steady state results in explicit and closed form in terms of the probability generating functions for the number of customers in the queue, average number of customers and mean waiting time in the queue.

In real life situations, the server may fail and require repair. The interruption of service to the customers makes negative impact on the performance of any system. In this kind of situation, there is a basic need to develop a reliable queueing system for maintaining the better performance of service. The multi optional repair concept is more realistic to model the unreliable server queue. When the server is not restored with the first essential repair, then the provision of subsequent repairs may be helpful for restoring the server; this phenomenon is known as 'multi optional repair'.

In this paper, the unreliable $M/E_k/1$ queue using the multi optional repair has been studied. The concepts of working vacation along with service interruption due to the server breakdown incorporated make our queueing model more robust and versatile. The queueing analysis carried out by using an analytical method and multi-optional repair based on generating function. The rest of the paper is organized as follows. Section two describes the model by stating the necessary assumptions and notations which are needed to develop the model. The steady state equations governing the model are constructed. In Section three, the steady state equations governing the model are constructed. In section four, the different partial probability generating functions are defined. Section five is dedicated to estimating the probability generating functions. In section six, state probabilities are obtained for different positions of the server. Also in section seven, average number of units in the system is derived based on the positions of the server.

2. Model Description

Consider an M/E_k/1 queueing system with single removable and non-perfect server. Based on the above stated concepts, the assumptions and notations employed in the text are presented here.

The state of the system is denoted by the triplet (n, i, m); n (= 0, 1, 2, …) denotes the number of the customers present in the system, i = 0 denotes that the unit is not in service, i(= 1, 2, …, k) denotes that the unit is undergoing in the i^{th} service phase and m(= V, B, D, R_j) describes the position of the server as:

$$m = \begin{cases} \text{V, Working vacation state} \\ \text{B, Turn} - \text{on and busy state} \\ \text{D, Breakdown state} \\ \text{R}_\text{j}\text{, Under j}^{\text{th}}(1 \le \text{j} \le \text{r})\text{phase repair state} \end{cases}$$

The customers arrive according to Poisson process with state dependent arrival rate Λ_j given by

$$\Lambda_j = \begin{cases} \lambda_0\text{, if the server is on working vacation} \\ \lambda_1\text{, if the server is turned on and in operation state} \\ \lambda_2\text{, if the server is turned on and in break down state} \\ \lambda_\text{j}, 3 \le \text{j} \le \text{r} + 2 \text{ if the server is turned on and under} \\ \quad (\text{j} - 2)^{\text{th}} \text{ phase optional repair} \end{cases}$$

The units are served according to k-phase Erlang distribution. The Erlang distribution has k identical and independent phases each with mean $1/_{k\mu_0}$ and $1/_{k\mu_1}$ while the server is on working vacation and busy states, respectively. The server provides the service according to first come first serve (FCFS) discipline.

The vacation time of the server follows an exponential distribution with mean 1/η. During the busy state, the server is subjected to breakdown according to Poisson distribution with breakdown rate α. As soon as the server is broken-down, it is immediately sent for repairing at the repair facility wherein the repairman provides the first essential repair (FER) with rate β. After the completion of FER, the server may opt for secondary first optional repair with probability σ or may join the system with complementary probability $\bar{\sigma} = (1 - \sigma)$ to render the service to the units. In general, the server may opt any of j (2≤ j ≤ r) optional repairs with probability σ after completing the (j-1)th optional repair, or may join the system with complementary probability $\bar{\sigma} = (1 - \sigma)$.

After the completion of the repair, the server provides service with the same efficiency as before according to FCFS discipline.

The steady state probabilities of the system states are defined as follows:

$P_{0,0,V}$: Probability that there are no units in the system and the server is on vacation.

$P_{n,i,V}$: Probability that there are n (≥ 0) units in the system when the server is rendering i^{th} (1≤ i ≤ k) phase of service while on working vacation.

$P_{n,i,B}$: Probability that there are n (≥ 0) units in the system when the server is rendering i^{th} (1≤ i ≤ k) phase of service and the server is in busy state.

$P_{n,i,D}$: Probability that there are n (≥ 0) units in the system when the server is rendering i^{th} (1≤ i ≤ k) phase of service and the server is in break down state.

$P_{n,i,Rj}$: Probability that there are n (≥ 0) units in the system when the server is rendering i^{th} (1≤ i ≤ k) phase of service and the server is under j^{th} (1≤ j ≤ r) phase of repair.

3. Steady State Equations

Based on the above descriptions, the steady state equations for the present queueing model are designed here.

$$\lambda_0 P_{0,0,v} = k\mu_0 \sum_{\xi=a}^{b} P_{\xi,1,V} + k\mu_1 \sum_{\xi=a}^{b} P_{\xi,1,B} \quad (1)$$

$$(\lambda_0 + \eta + k\mu_0)P_{1,i,V} = k\mu_0 P_{1,i+1,V} \ ; i = 1,2, \dots, k-1 \quad (2)$$

$$(\lambda_0 + \eta + k\mu_0)P_{1,k,V} = k\mu_0 P_{b+1,1,V} + \lambda_0 P_{0,1,V} \quad (3)$$

$$(\lambda_0 + \eta + k\mu_0)P_{n,i,V} = k\mu_0 P_{n,i+1,V} + \lambda_0 P_{n-1,i,V} \ ; n \ge 2, 1 \le i \le k-1 \quad (4)$$

$$(\lambda_0 + \eta + k\mu_0)P_{n,k,V} = k\mu_0 P_{n+b,1,V} + \lambda_0 P_{n-1,k,V} \ ; n \ge 2 \quad (5)$$

$$(\lambda_1 + \alpha + k\mu_1)P_{1,i,B} = k\mu_1 P_{1,i+1,B} + \sigma\beta P_{1,i,D} + (1-\sigma)\beta \sum_{j=1}^{l-1} P_{1,i,R_j} + \beta P_{1,i,R_r} \ ; 1 \le i \le k-1, 1 \le j \le r \quad (6)$$

$$(\lambda_1 + \alpha + k\mu_1)P_{1,k,B} = k\mu_1 P_{b+1,1,B} + \sigma\beta P_{1,k,D} + (1-\sigma)\beta \sum_{j=1}^{l-1} P_{1,k,R_j} + \beta P_{1,k,R_r} + \eta \sum_{i=1}^{k} P_{1,i,V} \quad (7)$$

$$(\lambda_1 + \alpha + k\mu_1)P_{n,i,B} = k\mu_1 P_{n,i+1,B} + \sigma\beta P_{n,i,D} + (1-\sigma)\beta \sum_{j=1}^{l-1} P_{n,i,,R_j} + \beta P_{n,i,R_r} + \lambda_1 P_{n-1,i,B} \ ; n \ge 2, 1 \le i \le k-1 \quad (8)$$

$$(\lambda_1 + \alpha + k\mu_1)P_{n,k,B} = k\mu_1 P_{b+n,1,B} + \sigma\beta P_{n,k,D} + (1-\sigma)\beta \sum_{j=1}^{l-1} P_{n,k,,R_j} + \beta P_{n,k,R_r} + \eta \sum_{i=1}^{k} P_{n,i,V} + \lambda_1 P_{n-1,k,B} \ ; n \ge 2 \quad (9)$$

$$(\lambda_2 + \beta)P_{1,i,\eta} = \alpha P_{1,i,B} \ ; 1 \le i \le \text{k} \quad (10)$$

$$(\lambda_2 + \beta)P_{1,i,\eta} = \alpha P_{n,i,\eta} = \alpha P_{n,i,B} + \lambda_2 P_{n-1,i,\eta} \ ; n \ge 2, 1 \le i \le \text{k} \quad (11)$$

$$(\lambda_3 + \beta)P_{1,i,R_1} = \sigma\beta P_{1,i,\eta} \ ; 1 \le i \le \text{k} \quad (12)$$

$$(\lambda_3 + \beta)P_{n,i,R_1} = \sigma\beta P_{n,i,\eta} + \lambda_3 P_{n-1,i,R_1} \ ; n \ge 2, 1 \le i \le \text{k} \quad (13)$$

$$(\lambda_{j+2} + \beta)P_{1,i,R_j} = \sigma\beta P_{1,i,R_{j-1}} \ ; 2 \le j \le r-1, 1 \le i \le \text{k} \quad (14)$$

$$(\lambda_{j+2} + \beta)P_{n,i,R_j} = \sigma\beta P_{n,i,R_{j-1}} + \lambda_{j+2} P_{n-1,i,R_j} \ ; n \ge 2, 2 \le j \le r-1, 1 \le i \le \text{k} \quad (15)$$

$$(\lambda_{r+2} + \beta)P_{1,i,R_r} = \sigma\beta P_{1,i,R_{r-1}} \ ; 1 \le i \le \text{k} \quad (16)$$

$$(\lambda_{r+2} + \beta)P_{n,i,R_r} = \sigma\beta P_{n,i,R_{r-1}} + \lambda_{r+2} P_{n-1,i,R_r} \ ; n \ge 2, 1 \le i \le \text{k} \quad (17)$$

4. Probability Generating Functions

In this section, the different partial probability generating functions are defined when the server is on working vacation, busy, break down under first essential repair and j[th] $(2 \leq j \leq r)$ phase of optional repair respectively.

$$P_V(z) = \sum_{i=1}^{k} X_i(z) = \sum_{n=1}^{\infty} \sum_{i=1}^{k} z^n P_{n,i,V} \tag{18}$$

$$P_B(z) = \sum_{i=1}^{k} Y_i(z) = \sum_{n=1}^{\infty} \sum_{i=1}^{k} z^n P_{n,i,B} \tag{19}$$

$$P_D(z) = \sum_{i=1}^{k} T_i(z) = \sum_{n=1}^{\infty} \sum_{i=1}^{k} z^n P_{n,i,D} \tag{20}$$

$$P_{R_1}(z) = \sum_{i=1}^{k} N_{i1}(z) = \sum_{n=1}^{\infty} \sum_{i=1}^{k} z^n P_{n,i,R_1} \tag{21}$$

$$P_{R_j}(z) = \sum_{i=1}^{k} N_{ij}(z) = \sum_{n=1}^{\infty} \sum_{i=1}^{k} z^n P_{n,i,R_j},\ (2 \leq j \leq r) \tag{22}$$

Here$X_i(z)$, $Y_i(z)$, $T_i(z)$, $N_{i1}(z)$ and $N_{ij}(z)$ are the corresponding probability generating functions in the i[th] service phase.

5. Estimation of Probability Generating Functions

This section is devoted to obtain various probability generating functions. Initially, by using steady equations and stated probability generating functions, the probability generating functions at i[th] service phase and the over all probability generating functions are derived when the server is on different levels.

The equations (2) and (4) are multiplied by proper powers of **z** and taking the summation for n = 1, 2, 3, . . . ∞ and get

$$X_{i+1}(z) = u(z)X_i(z)\ ;\ 1 \leq i \leq k-1 \tag{23}$$

where

$$u(z) = b_0(1-z) + 1 + c_0$$

$$b_0 = \frac{\lambda_0}{k\mu_0}\ ;$$

$$c_0 = \frac{\eta}{k\mu_0}\ ;$$

Again, considering the equations (1), (3) and (5) and multiplied them by proper powers of z and their sum gives

$$X_1(z) = z^b u(z)X_k(z) + b_0\left(z^{\xi} - z^{b+1}\right)P_{0,1,V} + \sum_{n=1}^{a-1} P_{n,1,V}\ z^n - \frac{\mu_1}{\mu_0}\sum_{\xi=a}^{b} P_{\xi,1,B}\ z^{\xi} \tag{24}$$

Follow the above technique in equations (6) and (8) and get

$$Y_{i+1}(z) = (\alpha_1 + 1 - b_1 z)Y_i(z) - d_0 T_i(z) - d_0 \sum_{j=1}^{l-1} N_{1j}(z) - \frac{d_0}{\bar{\sigma}} N_{1r}(z)\ ;\ 1 \leq i \leq k-1 \tag{25}$$

where

$$\alpha_1 = \frac{\lambda_1 + \alpha}{k\mu_1}\ ;$$

$$b_j = \frac{\lambda_j}{k\mu_1}\ ;$$

$$d_0 = \frac{(1-\sigma)\beta}{k\mu_1}$$

By adding the equations (7) and (9) after multiplying them by z^{b+1} and z^{b+n}, (n = 2, 3, 4,…) respectively, and get

$$Y_1(z) = z^b(\alpha_1 + 1 - b_1 z)Y_k(z) + \sum_{n=1}^{b} P_{n,1,B} z^n - z^b d_0 T_k(z) - z^b d_0 \sum_{j=1}^{r-1} N_{k,j}(z)$$
$$-z^b \frac{d_0}{\bar{\sigma}} N_{k,r}(z) - c_1 z^b \sum_{i=1}^{k} X_i(z) \tag{26}$$

where

$$c_1 = \frac{\eta}{k\mu_1}$$

The equations (10) and (11) are multiplied by z and z^n respectively and summing over n = 1, 2,…. Then,

$$\{\bar{\sigma} b_2(1-z) + d_0\}T_i(z) = (\alpha_1 - b_1)\bar{\sigma} Y_i(z),\ 1 \leq i \leq k \tag{27}$$

The equations (12), (14), (15) and (16) are multiplied by z and both (13) and (17) are multiplied by z^n (n = 2, 3, …). The sum of the above equations is obtained as

$$N_{ij}(z) = \frac{(\sigma d_0)^j}{\prod_{m=1}^{j}\{(1-z)b_{m+2}\} + d_0} T_i(z) \tag{28}$$

In this stage, using the method of iteration in the equation (23) and get

$$X_i(z) = u^{i-1}(z)X_1(z)\ ;\ i = 1,2, \ldots k,\ 1 \leq i \leq k \tag{29}$$

Substitute (29) in the expression (24) which yields

$$X_1(z) = \frac{b_0(z^{\xi} - z^{b+1})P_{0,0,V} + \sum_{n=1}^{a-1} P_{n,1,V}\ z^n - \frac{\mu_1}{\mu_0}\sum_{\xi=a}^{b} P_{\xi,1,B}\ z^{\xi}}{[1 - z^b u^k(z)]} \tag{30}$$

Apply the expression for $X_1(z)$ given in (30) in the equation (29) and get

$$X_i(z) = u^{i-1}(z)\left[\frac{b_0(z^{\xi}-z^{b+1})P_{0,0,V}+\sum_{n=1}^{a-1}P_{n,1,V}\,z^{n}-\frac{\mu_1}{\mu_0}\sum_{\xi=a}^{b}P_{\xi,1,B}\,z^{\xi}}{[1-z^{b}u^{k}(z)]}\right];\ i = 1,2,\dots k \tag{31}$$

Now, using the equation (31) in the stated relation (18) which yields the partial probability generating function for the server is on working vacation.

$$P_V(z) = \left[\frac{b_0(z^{\xi}-z^{b+1})P_{0,1,V}+\sum_{n=1}^{a-1}P_{n,1,V}\,z^{n}-\frac{\mu_1}{\mu_0}\sum_{\xi=a}^{b}P_{\xi,1,B}\,z^{\xi}}{[1-z^{b}u^{k}(z)]}\right]\left[\frac{1-u^{k}(z)}{1-u(z)}\right] \tag{32}$$

Again, consider the expression (25) in which the expressions (27) and (28) are used. The resultant expression for $Y_{i+1}(z)$ is given by

$$Y_{i+1}(z) = V(z)Y_i(z)\ ;\ i = 1,2,\dots k \tag{33}$$

where

$$V(z) = \left[(1+\alpha_1-b_1z) - \frac{d_0\bar{\sigma}(\alpha_1-b_1)}{\bar{\sigma}b_2(1-z)+d_0} - \frac{d_0\bar{\sigma}(\alpha_1-b_1)}{\bar{\sigma}b_2(1-z)+d_0}\sum_{j=1}^{l-1}\frac{(\sigma d_0)^{j}}{\prod_{m=1}^{j}\{(1-z)b_{m+2}\}+d_0} - \frac{d_0(\alpha_1-b_1)}{\bar{\sigma}b_2(1-z)+d_0}\frac{(\sigma d_0)^{l}}{\prod_{m=1}^{l}\{(1-z)b_{m+2}\}+d_0}\right]$$

As before, using the method of iteration in (33) and get

$$Y_i(z) = V^{i-1}(z)Y_1(z)\ ;\ i = 1,2,\dots,k \tag{34}$$

Substitute the expression (34) in (26) which yields

$$Y_1(z) = \frac{\sum_{n=1}^{b}P_{n,1,B}\,z^{n}+c_1\,z^{b}\sum_{i=1}^{k}X_i(z)}{[1-z^{b+1}V^{k}(z)]},\ \text{where } c_1 = \frac{\eta}{k\mu_1} \tag{35}$$

Using the expression for $Y_1(z)$ given in (35) in the relation (34) and get,

$$Y_i(z) = V^{i-1}(z)\left[\frac{\sum_{n=1}^{b}P_{n,1,B}\,z^{n}+c_1\,z^{b}\sum_{i=1}^{k}X_i(z)}{[1-z^{b+1}V^{k}(z)]}\right];\ i = 1,2,\dots,k \tag{36}$$

Summing the equation (36) by plugging $i = 1,2,\dots,k$ and the equation (19) gives the required probability generating function for the server is busy.

$$P_B(z) = \left[\frac{\sum_{n=1}^{b}P_{n,1,B}\,z^{n}+c_1\,z^{b}P_V(z)}{[1-z^{b+1}V^{k}(z)]}\right]\left[\frac{1-V^{k}(z)}{1-V(z)}\right] \tag{37}$$

Similarly, summing the equation (27) for $i = 1,2,\dots,k$ and get the probability generating function for the server is breakdown.

$$P_D(z) = \frac{\bar{\sigma}(\alpha_1-b_1)}{[\bar{\sigma}b_2(1-z)+d_0]}P_B(z) \tag{38}$$

Consider the relation (28) for j = 1 and using the stated relation (21) which provides the probability generating function for the server is under first essential repair.

$$P_{R_1}(z) = \frac{\sigma d_0}{[(1-z)b_3+d_0]}\frac{\bar{\sigma}(\alpha-b_1)}{[\bar{\sigma}b_2(1-z)+d_0]}P_B(z) \tag{39}$$

By extending the values for $j = 2,3,\dots,r$ in (28) through (22) and get the probability generating functions for the server is under optional repair.

$$P_{R_j}(z) = \frac{(\sigma d_0)^{j}}{\left[\prod_{m=1}^{j}\{(1-z)b_{m+2}\}+d_0\right]}\frac{\bar{\sigma}(\alpha_1-b_1)}{[\bar{\sigma}b_2(1-z)+d_0]}P_B(z)\ ;\ 2\le j\le r \tag{40}$$

6. State Probabilities

The state probabilities, by considering the server is on working vacation, busy, breakdown, first essential repair and j[th] (j = 2,3, …,r) optional repair, are obtained by assuming the relation $\lim_{z\to 1}P_k(z) = P_k$ in the equations (32), (37) to (40).

$$P_V = \frac{1}{c_0}\left\{\frac{\mu_1}{\mu_0}\sum_{\xi=a}^{b}P_{\xi,1,B} - \sum_{n=1}^{a-1}P_{n,1,V}\right\} \tag{41}$$

$$P_B = \frac{\sum_{n=1}^{b}P_{n,1,B}+c_1P_V}{(\alpha_1-b_1)\left[-1+\bar{\sigma}\left\{1+\sum_{j=1}^{r-1}(\sigma d_0)^{j}f(m,j)\right\}+(\sigma d_0)^{l}f(m,l)\right]} \tag{42}$$

$$P_D = \frac{\bar{\sigma}}{d_0}(\alpha_1-b_1)P_B \tag{43}$$

$$P_{R_1} = \frac{\sigma\bar{\sigma}}{d_0}(\alpha_1-b_1)P_B \tag{44}$$

and

$$P_{R_j} = \sigma^{j}{d_0}^{j-1}\bar{\sigma}(\alpha_1-b_1)f(m,j)P_B \tag{45}$$

7. Queueing Performance Measures

The most important performance measure is the average number of units.

Let L_V, L_B, L_D, L_{R_1} and L_{R_j} be the expected number of units in the system when the server is on vacation, busy, breakdown, first essential repair and j^{th} $(j = 2,3,\ldots,r)$ optional repair respectively. Symbolically, $L_k = dP_k'(z)\ at\ z = 1$ is applied in the equations (32), (37) to (40) which provide as follows,

$$L_V = \frac{c_0\left[\alpha k b_0(1+c_0)^{k-1}+\beta\{1-(1+c_0)^k\}\right]+\alpha\left[b_0+(1+c_0)^{k-1}\{(bc_0-b_0)(1+c_0)-kb_0c_0\}\right]}{c_0^2\{(1+c_0)^k-1\}} \tag{46}$$

where

$$\alpha = \sum_{n=1}^{a-1} P_{n,1,V} - \frac{\mu_1}{\mu_0}\sum_{\xi=a}^{b} P_{\xi,1,B}$$

and

$$\beta = b_0(\xi - b - 1)P_{0,1,V} + \sum_{n=1}^{a-1} nP_{n,1,V} - \frac{\mu_1}{\mu_0}\sum_{\xi=a}^{b} \xi P_{\xi,1,B}$$

$$L_B = \frac{(1-V(1))(1-V^k(1)\left\{\sum_{n=1}^{b} nP_{n,1,B}+c_1(L_V+bP_V)\right\} + \left\{\sum_{n=1}^{b} P_{n,1,b}+c_1P_V\right\}\left[V^1(1)(1-V^k(1)+(1-V(1))V^{k-1}(1)\{kV^1(1)+(b+1)V(1)\}\right]}{\left(1-V^k(1)\right)\left(1-V(1)\right)^2} \tag{47}$$

$$L_D = \frac{\bar{\sigma}}{d_0^2}(\alpha_1 - b_1)[d_0L_B + \bar{\sigma}b_2P_B] \tag{48}$$

$$L_{R_1} = \sigma\bar{\sigma}(\alpha_1 - b_1)d_0^{-2}[d_0L_B + (\bar{\sigma}b_2 + b_3)P_B] \tag{49}$$

$$L_{R_j} = \sigma^j d_0^{j-2}\bar{\sigma}(\alpha_1 - b_1)f(m,j)[d_0L_B + \{\bar{\sigma}b_2 + d_0b_{m+2}f(m,j)\}P_B] \tag{50}$$

8. Conclusion

A single server Markovian arrival and Erlangian bulk service queue with state dependable rates have been studied. The additional concepts are included in the model for achieving valuable results. The state probabilities and expected number of units in the system are explicitly derived. This present work is more applicable for various industrial applications. The derived results are useful to study the reliability of the server. There is a chance to extend this work by introducing multiple servers.

9. References

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