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Application of value at risk on Moroccan exchange rates

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Abstract
Currency risk is, nowadays, identified as an essential component of the international firm's environment, but this awareness is relatively recent. However, modeling and forecasts of exchange rate developments remain a weak point in macroeconomic analysis. Despite their theoretical coherence, models fail to do better than random process. Market expectations have no predictive power. On the other hand, several methods deserve to be tested to measure the said risk.
Indeed, this article presents an application of the Value at Risk method on Moroccan currency risk. The methods used to calculate the VaR are:
- The parametric method or variance-covariance approach;
- The parametric method adjusted by the Cornish-Fisher method;
- The historical VaR;
- The Monte Carlo VaR;
- The BoostrapVaR.

Keywords: exchange rate, currency risk, VaR, variance-covariance approach, Cornish-Fisher method, Monté Carlo method, Boostrap method, financial mathematics, Statistical methods

1. Introduction

Morocco is the 5th largest economy in Africa. Its currency is not freely convertible. However, the Moroccan course is established on the basis of a basket of currencies Euro and Dollard which are the main country partner's currencies. The basket consists of 80% Euro and 20% Dollar. As a result, all foreign-based companies are exposed to a currency risk related to these two currencies. For this reason, this article presents a modeling of the exchange rate of the two currencies in relation to the Moroccan currency according to the Value at Risk method.

2. Theoretical overview

Foreign exchange risk is one of the market risks, it is linked to "the uncertainty of the exchange rate of one currency compared to another in the short or medium term. Not knowing what will be the evolution of an exchange rate at three or six months causes difficulties. "[1]. It is therefore the risk related to the variation of exchange rate between the date of the commitment and the date of settlement.

Types and sources of foreign exchange risk: Foreign exchange risk may arise from commercial or financial transactions carried out by the company internationally. It can also depend on the international development of the company and the investments it has made abroad.

Sources of foreign exchange risk can thus be categorized as following:
- Risks arising from commercial operations;
- Risks resulting from financial activities;
- Economic risks sometimes called competitiveness risks or induced risks;
- Risks related to cross-border investments.

2.1. Some reminders of probability and statistics

Before presenting the Value at Risk method, it is important to recall some probabilistic definitions.

- Density and distribution function

We consider a real stochastic variable or continuous real random variable (r.r.v), noted X, whose probability law, for an occurrence x, is defined by the density function $f_X(x)$, assumed to be continuous, such that $f_X(x) \geq 0$:

$$\forall\ (a,b) \in R^2\ P(a \leq X \leq b) = \int_a^b f_X(x)dx$$

With:

$$\int_{-\infty}^{+\infty} f_X(x)dx = 1$$

We denote by $F_X(.)$ the distribution function or cumulative distribution function associated withX, such as:

$$F_X(a) \equiv P(X \leq a) \equiv (P(X \leq a) = \int_{-\infty}^{a} f_X(x)dx$$

- Some Moments:

For a real random variable (r.r.v) X, the mean and the variance are two particular moments, but more generally it is possible, under certain assumptions, to define the population of moments and the population of centered moments in the following way:

$$E(X^k) = \int_{-\infty}^{+\infty} x^k f_X(x)dx$$

- The moment of order 1: the mean

The mean, denoted E (X), corresponds to the moment of order one (k = 1):

$$E(X) = \int_{-\infty}^{+\infty} x f_X(x)dx = \mu$$

The moment of order 2: The variance

The variance, denoted by V (X), is defined by the central moment of order two (k = 2):

$$V(X) = \mu_2 = E[(X-\mu)^2] = \int_{-\infty}^{+\infty} (x-\mu)^2 f_X(x)dx$$

The central moment of order 3: Skewness

For the purpose of the VAR, it is important to define the 3rd order centered moment.

$$Skewness = \mu_3 = E[(X-\mu)^3]$$

Skewness is a distribution asymmetry measure from which we can define a new measure, namely the Skewness coefficient:

$$Skewness\ Coefficient = \frac{\mu_3}{\sigma^3}$$

- The central moment of order 4: Kurtosis

The 4th order centered moment is defined as follows:

$$Kurtosis = \mu_4 = E[(X-\mu)^4]$$

The new measure to deduce from Kurtosis is the degree of Kurtosis excess:

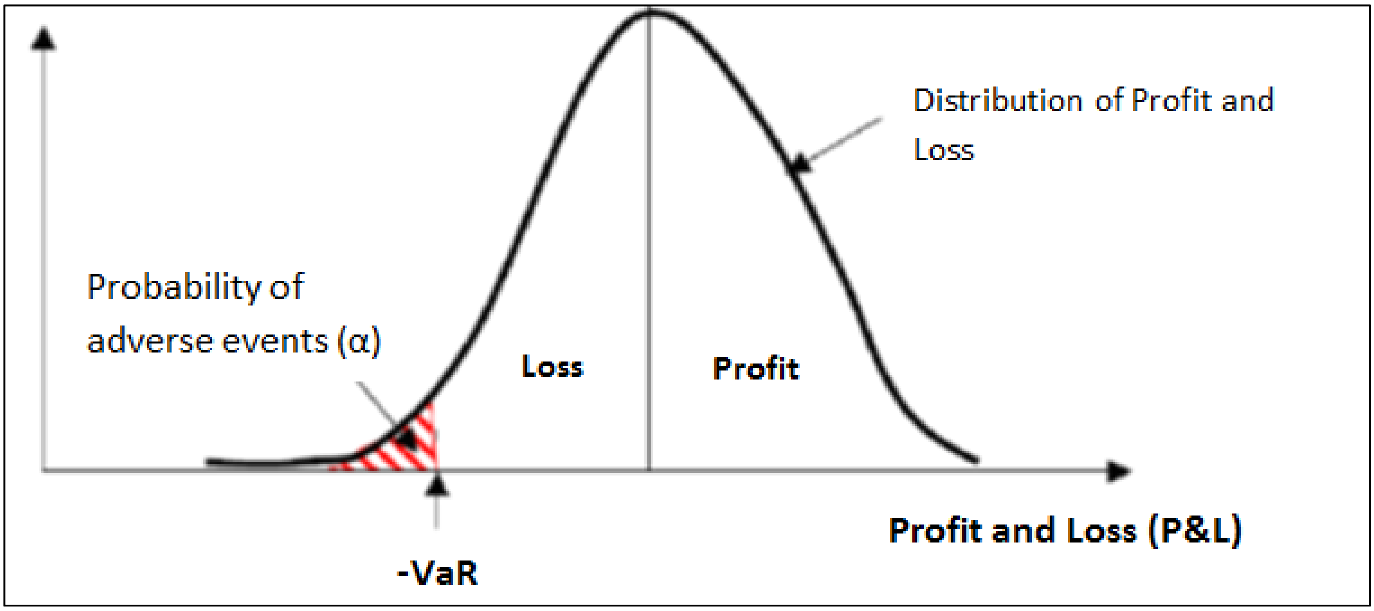
$$Degree\ of\ excess\ Kurtosis = \frac{\mu_4}{\sigma^4} - 3$$

2.2. Evaluation of currency risk by the VaR 'Value at Risk' method

By definition, VaR is the maximum loss that a portfolio manager may experience during a certain period of time with a given probability. Statistically, the Value at Risk can be defined as a quantile of the distribution of the theoretical P & L (Profit and Loss) of a portfolio, resulting from the possible movements of market risk factors, over a fixed time horizon. The P & L of a portfolio value P over a horizon of h days is given by:

$$P\&L(t\ ;\ h) = \Delta_h P_t = P_{t+h} - P_t$$

By definition, the VaR over h days with a confidence level (1-α) is the value R such as the probability of losing R at the most, beyond h days, is equal to (1-α). VARα; h = R such as the Probability
P (ΔhPt<R) = 1-α.
This is illustrated by the graph:



Graph of the density function P&L

2.3. The parametric method or variance-covariance approach

The parametric approach tries to define a formula describing the distribution of gains / losses. To calculate a day's VaR we use the formula$\boldsymbol{VaR_{\alpha\%} = \mu - q * \sigma}$, with q a quantile taken from the normal law table (1.96 for α = 5% and 2, 33 for α = 1%), whereas for 10 days the formula becomes $\boldsymbol{VaR_{\alpha\%} = \sqrt{10}(\mu - q * \sigma)}$.

This method is based on several assumptions, mainly:

- The variations of risk factors follow a normal law.
- The relation between the variations of value's portfolio and the variations of market variables is linear.

2.4. The Cornish-Fisher VAR

Under the assumption of normality of returns, asymmetry and kurtosis are zero. In reality, returns generally have negative asymmetries and high kurtosis. Using a normal quantile's limited development, we can take asymmetry and kurtosis into consideration; this is the Cornish-Fisher approximation:

$$\boldsymbol{z^{Cornish-Fisher} \approx z + \frac{1}{6}(z^2 - 1)S + \frac{1}{24}(z^3 - 3z)K - \frac{1}{36}(2z^3 - 5z)S^2}$$

z is the normal quantile at α: N (z) = α, S is asymmetry and K is kurtosis. In the case of zero asymmetry, the preceding expression is reduced to:

$$z^{Cornish-Fisher} \approx z + \frac{1}{24}(z^3 - 3z)K$$

The Cornish Fisher VaR with a confidence level of 1-α (for example 99%) and a horizon T (for example 10 days) is then:

$$VaR(\alpha, T) \approx \mu T + z^{Cornish-Fisher} * \sigma * \sqrt{T}$$

2.5. The historical method

The historical simulation approach does not make assumptions about the distribution of portfolio returns. However, it is based on the hypothesis of stationarity of returns, in other words the distribution of the portfolio value variations for the horizon on which we calculate the VaR is well estimated by observations of these variations from historical data.

2.6. The Monte Carlo simulation

The method using Monte Carlo simulation is similar to the historical method as it is a total valuation method based on different scenarios. Like the historical method, the Monte Carlo method does not assume that market variables follow a normal distribution.

In the historical method, the scenarios have already occurred in the past. With the Monte Carlo method, however, the scenarios are generated at random. They follow a form similar to those that took place in the past but they are not limited by history.

Each risk factor has a dynamic that can be modeled with a term of uncertainty. By taking into account the dynamics of each risk factor as well as their correlation, it is possible to generate probable scenarios of collective evolution of these risk factors.

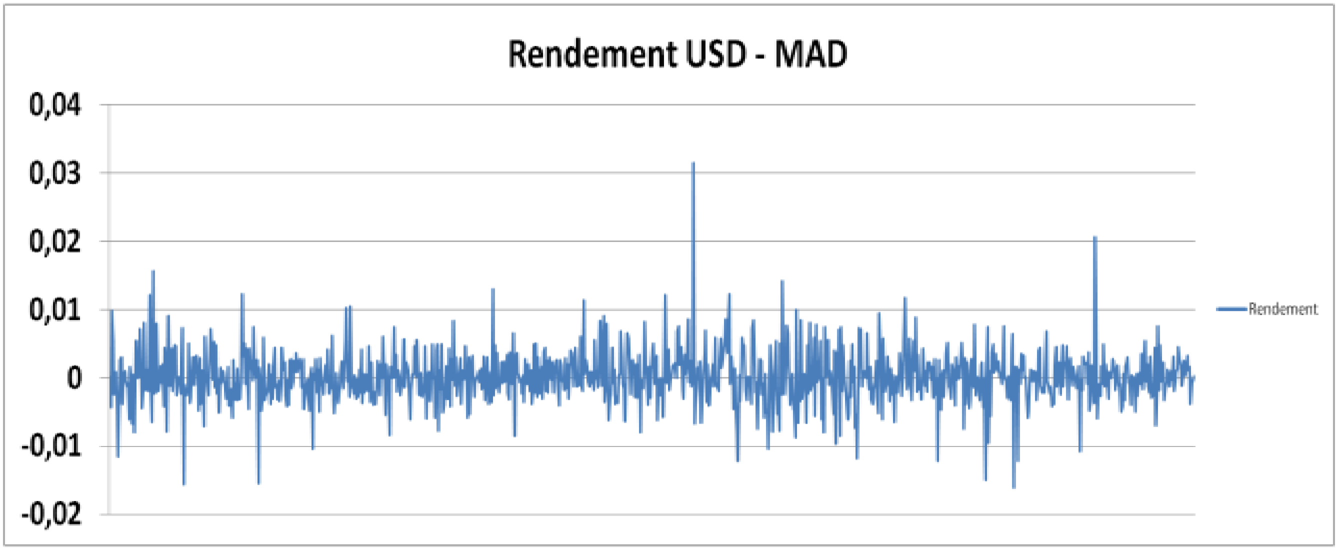
2. 7. VAR using the Boostrap technique

The Boostrap technique is a statistical inference method dating from the late 1970s. This method is based on simulations (we are talking about resampling). Its principle is to generate on the basis of an initial series several other randomly reordered series, with or without replacement, so as to decrypt the distribution of the initial series.

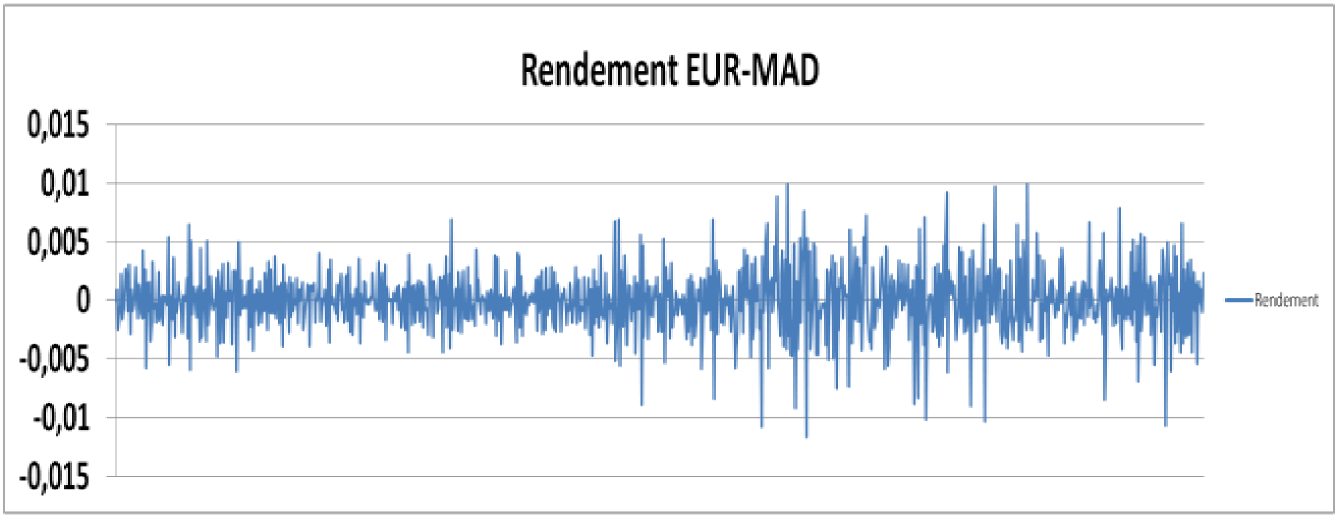
3. Database and Applications

The exchange rate covered by this article is the exchange rate of the Moroccan currency "MAD" compared to the US Dollar and the Euro. Our series contains the exchange rates of the two currencies for the daily period from 01/01/2013 to 31/10/2016. To quantify the volatility of exchange rates, we have used the daily return of the two currencies as an indicator of exchange rate risk:

$$R_{USD(t)} = \frac{USD_t - USD_{t-1}}{USD_{t-1}} \text{ et } R_{EUR(t)} = \frac{EUR_t - EUR_{t-1}}{EUR_{t-1}}.$$



Return USD-MAD from 01/01/2013 to 31/10/2016



Return EUR-MAD from 01/01/2013 to 31/10/2016

The trajectory of the two portfolios is very volatile around an approximately zero value.

	Return USD-MAD	Return EUR-MAD
Mean	0.000164	-0.00003
Standard deviation	0.00399	0.00272

Descriptive Statistics of Returns

3.1. Normality test

The fundamental hypothesis to be tested is the normality of our series.

-Statistical test:

Normality test of Return USD-MAD

Normality Tests				
Test	**Statistical**		**p-Value**	
Shapiro-Wilk	W	0.950897	Pr < W	<0.0001
Kolmogorov-Smirnov	D	0.058247	Pr > D	<0.0100
Cramer-von Mises	W-Sq	1.299431	Pr > W-Sq	<0.0050
Anderson-Darling	A-Sq	7.693207	Pr > A-Sq	<0.0050

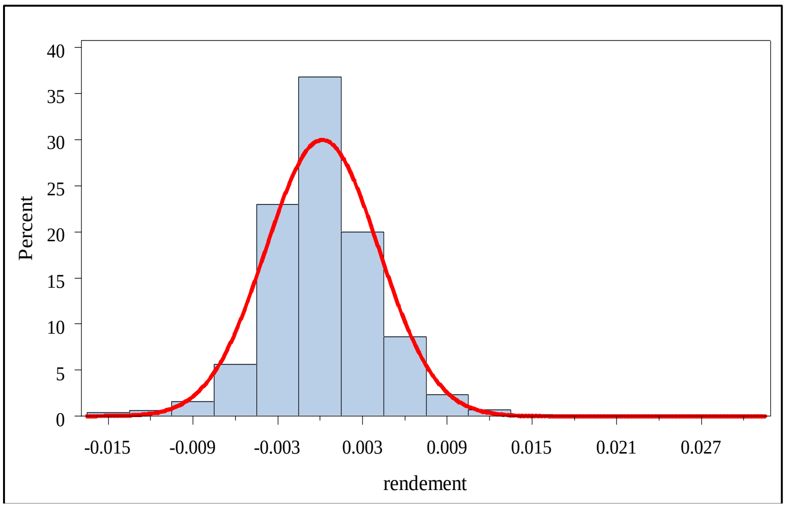
Normality test of Return EUR-MAD

NormalityTests				
Test	**Statistical**		**p-Value**	
Shapiro-Wilk	W	0.978346	Pr < W	<0.0001
Kolmogorov-Smirnov	D	0.046331	Pr > D	<0.0100
Cramer-von Mises	W-Sq	0.737173	Pr > W-Sq	<0.0050
Anderson-Darling	A-Sq	4.521396	Pr > A-Sq	<0.0050

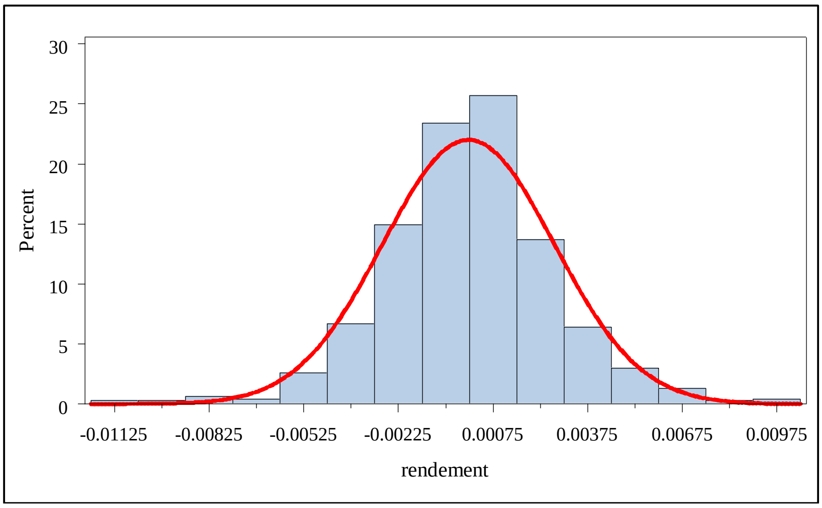
The normality tests on our data show the rejection of normality, which is the case almost for all financial series. However, the incompatibility with the normal law is quasi-systematically decided on large size, even if the differences of distributions are weak. As a result, empirical approaches, especially graphical ones, remain important.

-Graphic illustration:

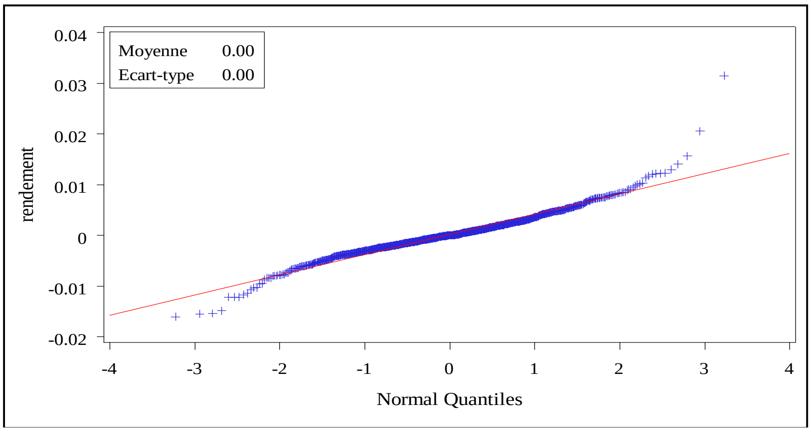
On the two graphs below, we note that the distribution of our series is close to the normal distribution.



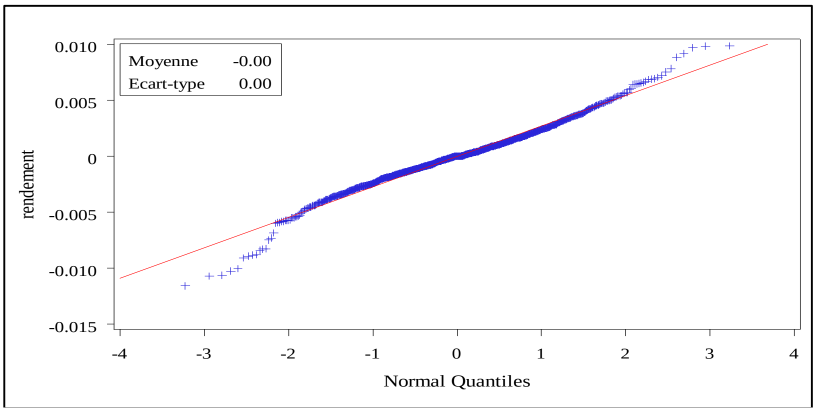
Return USD-MAD



Return EUR-MAD



Q-Q plot Return USD-MAD



Q-Q plot Return EUR-MAD

Thus, the Q-Q Plots of the two distributions show that the two series are close to the normal distribution.

3.2. Parametric VaR and VaR according to the Cornish-Fisher approximation

a) Parametric VaR

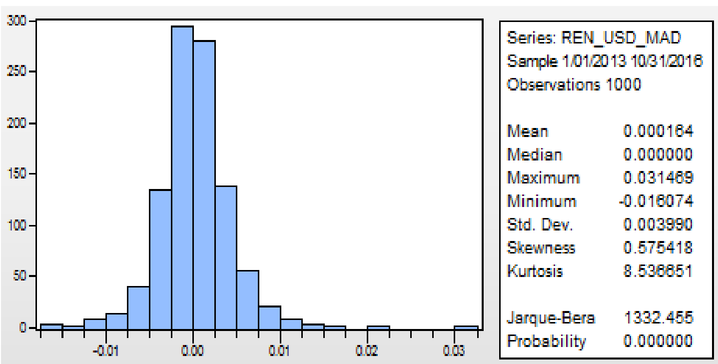
The parametric VaR calculation is based on the normality hypothesis. But our two series do not adjust to a normal distribution. Nevertheless we will calculate the parametric VaR and then we will correct it with the Cornish-Fisher approximation.

	USD/MAD	EUR/MAD
Mean	1.64E-04	-3.00E-05
Forecastvolatility	0,00399	0.00272
$VaR_{1d,95\%}$	-0.007656	-0.005361
$VaR_{1d,99\%}$	-0.009133	-0.006368

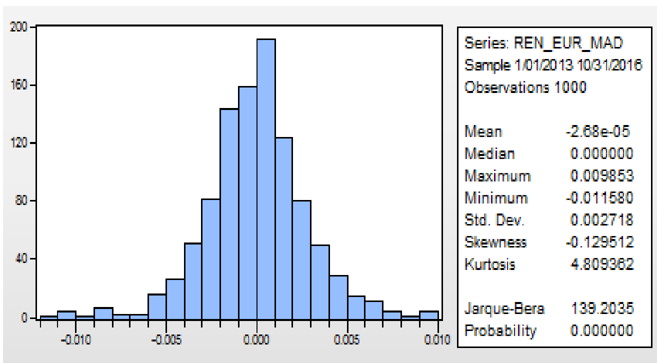
Daily VaR USD/MAD & EUR/MAD correct it with Cornish-Fisher method

a) The Cornish-Fisher approximation VaR

If we consider that our distributions are not very far from a normal distribution (graphs above), in this case we can calculate the Cornish-Fisher approximation VaR to correct the asymmetry and the thickness of our series tails.



USD/MAD



EUR/MAD

Normality test of Returns Dollar and EURO.

As shown in the fig above (Eviews output), the asymmetry (skewness) and flattening (Kurtosis) coefficients of the portfolio'sreturns distribution are, respectively, higher than 0 and 3. Also, The Jarque-Berastatistic is above its critical value which is around 5, 991. This shows that the distribution of the portfolio's daily returns is leptokurtic, which means that it has thick tails in comparison with the normal distribution.

Daily VaR USD/MAD & EUR/MAD with Cornish-Fisher method

	USD/MAD	**EUR/MAD**
Skewness	0.575418	-0.129512
Kurtosis	8.536651	4.809382
Cornish (α=1%)	4.642203	3.362121
Cornish (α=5%)	2.770877	2.226764
Mean	1.64E-04	-3.00E-05
Forecastvolatility	0,00399	0.00272
$VaR_{1d,95\%}$	-0.01121980	-0.00602680
$VaR_{1d,99\%}$	-0.0186864	-0.0091150

We note that the adjustment of the distribution of returns parameters (the asymmetry coefficient and the flattening coefficient) allows to increase the maximum loss value at the level of significance 95% and 99% in relation to the VaR. parametric calculated forward. This implies that on a portfolio of 1 000 000 MAD the VaR becomes:

Comparison between Parametric VaR and VaR with Cornish-Fisher method

	USD/MAD		**EUR/MAD**	
	ParametricVaR	**VaR with Cornish-Fisher method**	**ParametricVaR**	**VaR with Cornish-Fisher method**
$VaR_{1d,95\%}$	7 656	11 220	5 361	6 027
$VaR_{1d,99\%}$	9 133	18 686	6 368	9 115

3.3. Historical VaR

The historical VaR calculation is based on a simple descending sorting of the return values, then we choose the value corresponding to the 99% and 95% quantiles, which means for a risk of 1% and 5%. On the basis of a portfolio of value of 1 000 000 MAD on the two currency portfolios, the historical VaR is as follows:

The Historic VaR of returns USD-MAD and EUR-MAD

Historical VaR		USD-MAD	EUR-MAD
% with 95%	1 day	0.005769	0.004130
	10 days	0.018243	0.013060
% with 99%	1 day	0.010346	0.008300
	10 days	0.032717	0.026247
VaR with 95%	1 day	5 769	4 130
	10 days	18 243	13 060
VaR with 99%	1 day	10 346	8 300
	10 days	32 716	26 246

On a 1Million dirham wallet we note that:

- On the USD portfolio: the maximum loss over a one-day horizon is 5,769Dh for a risk threshold of 5%, 10,346Dh for a risk of 1% and 18,243Dh in 10 days for a risk of 5% and 32 716Dh for a risk of 1%.
- On the EUR portfolio: the maximum loss over a one-day horizon is 4 130Dh for a risk threshold of 5%, 8 300Dh for a risk of 1% and 13 060Dh in 10 days for a risk of 5% and 26 246Dh for a risk of 1%.

3.4. VaR according to Monte Carlo simulation

The VaR's calculation according to a Monte Carlo simulation is based on the following steps:
Generation of 1000 returns scenarios through the Alea function on Excel., "Generation of random numbers of uniform laws [0,1]".

- Generation of independent inverse normal random variables based on the statistical characteristics of historical series (USD and EUR).
- The decreasing sorting of the different values obtained.
- The search for "VaR" that correspond to the risk thresholds selected (5% and 1%).

On the basis of a portfolio of value of 1 000 000 MAD on the two currency portfolios, the VaR according to Monte Carlo simulation is as follows:

The Monte Carlo VaR of returns USD-MAD and EUR-MAD

Monte Carlo Value at Risk		USD-MAD	EUR-MAD
% with 95%	1 jour	0.006696	0.004526
	10 jours	0.021175	0.014312
% with 99%	1 jour	0.008581	0.006328
	10 jours	0.027135	0.02001
VaR with 95%	1 jour	6 696	4 526
	10 jours	21 175	14 312
VaR with 99%	1 jour	8 581	6 328
	10 jours	27 135	20 010

On a 1Million dirham portfolio we note that:

- On the USD portfolio: the maximum loss over a one-day horizon is 6,696Dh for a risk threshold of 5%, 8,581Dh for a risk of 1% and 21,175DH in 10 days for a risk of 5% and 27 135Dh for a risk of 1%.
- On the EUR portfolio: the maximum loss over a one-day horizon is 4,526Dh for a risk threshold of 5%, 6,328Dh for a risk of 1% and 14,312DH in 10 days for a risk of 5% and 2010Dh for a risk of 1%.

3.5. VaR according to the Boostrap technique

The VaR'sCalculation according to the Boostrap technique is done on the basis of the following steps:

- Generation of 1000 returns scenarios through the Alea function on Excel., "Generation of random numbers of uniform laws [0,1]" each scenario contains 1000 random values (size of our portfolios).
- Ranking in ascending order random numbers of each scenario, which gives us a reordered series of exchange rates (USD and EUR).
- Calculation of a return relative to our redeveloped series according to the following formula:

$$R_{USD} = \frac{USD_{end} - USD_{beginning}}{USD_{beginnin}} \& \ R_{EUR} = \frac{EUR_{end} - EUR_{beginning}}{EUR_{beginning}}.$$

- Sort decreases 1000 returns calculated on the basis of the 1000 redeveloped series.
- The search for "VaR" which correspond to the risk thresholds selected (5% and 1%).

On the basis of a value portfolio of 1 000 000 MAD on the two currency portfolios, the VaR according to the Boostrap simulation is as follows:

The Boostrap VaR of returns USD-MAD and EUR-MAD

Boostrap Value at Risk		USD-MAD	EUR-MAD
% with 95%	1 jour	0.157056	0.036616
	10 jours	0.496654	0.115899
% with 99%	1 jour	0.174542	0.046874
	10 jours	0.551950	0.148228
VaR with 95%	1 jour	157 056	36 316
	10 jours	496 654	114 841
VaR with 99%	1 jour	174 542	46 874
	10 jours	551 950	148 228

On a 1Million dirham wallet we find that:

- On the USD portfolio: the maximum loss over a one-day horizon is 157,056Dh for a risk threshold of 5%, 174,542Dh for a risk of 1% and 496,654Dh in 10 days for a risk of 5% and 551,950DH for a risk of 1%.

- On the EUR portfolio: the maximum loss over a one-day horizon is 36 316Dh for a risk threshold of 5%, 46 874Dh for a risk of 1% and 114 841Dh in 10 days for a risk of 5% and 148 228Dh for a risk of 1%.

4. Synthesis

The Moroccan exchange rate risk estimation, according to the different techniques of the Value at Risk method, has shown that the degree of exchange risk aversion varies according to the technique applied. Indeed, the Boostrap technique is the most averse method with a very important VaR, then the correction of the parametric method by the Cornish-Fisher technique also gives also a quite significant probable loss. However, the historical, Monté Carlo and Parametric techniques give probable losses of close size. On the other hand, Value At Risk is a simple and applicable method for measuring currency risk.

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