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Inference in generalized linear model with inequality

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Abstract
In this paper we are develop new criteria to estimate parameters of liner regression model under certain linear equality and inequality restrictions on regression coefficients. A new procedure has been suggested for testing inequality hypotheses on regression coefficients of linear model. in this context, the covariance matrix for heteroscedastic disturbances has been specifies and estimated by choosing structure for the error variance. The likelihood ratio test has been discussed for testing linear inequality restrictions on parameters of the generalized linear regression model.

Keywords: Generalized linear, inequality

1. Introduction

Since, the last four decades there has been a considerable growth in the research about multiple regression analysis with linear restrictions on parameters. Recently, several econometricians have developed restricted estimation procedures for linear models subject to (i) multiple equality constraints, (ii) multiple inequality constraints and (iii) multiple equality and inequality constraints.

In recent years there has been heavy emphasis on the estimation of linear models that are subject to inequality restrictions on the regression coefficients. Most of the econometric testing problems are potentially one-sided because economic theory is usually rather good at providing information about the signs of parameters. Many econometric problems are formulated as the generalized stochastic linear model.

In the literature, the various problems of multiple regression analysis with linear restrictions were studied by many American as well as British statisticians and econometricians. In India, a few econometricians have made contributions in this direction.

The main contributors regarding the present study are: Wallace (1972) [15], Dent (1980) [5], Clarke, Giles and Wallace (1987) [4], Ohtani (1987) [12], Farebrother (1988) [6], Firoozi (1993) [8], Bera (1997) [2], Michelis (1999) [11] and others.

2. Tests for Inequality Restrictions In General Linear Model

Consider a general linear model

$$Y = X\beta + \in, \quad \in \sim N\left[0, \Omega(r)\right] \qquad \text{...(2.1)}$$

Where Y, X, β and ∈ are n×1, n×k, k×1 and n×1 matrices; X has rank k;

$\Omega(r)$ is an (n×n) positive definite matrix which is a function of the g elements in the (g×1) matrix γ. Under the problem of jointly testing, we have,

1. Equality Tests: $H_0 : R\beta = r \sim H_1 : R\beta \neq r$ ...(2.2)

2. Inequality Tests: $H_0 : R\beta \geq r \sim H_1 : R\beta \ngeq r$ ...(2.3)

Ω is a given (q×1) matrix and R is a given q×k matrix of rank q.

a) **Nonlinear Equality Tests**

Let L(θ) = L* (y, 0) be the joint density of an (n×1) matrix of random variables $Y = \left[y_1,\ y_2,\ ...,\ y_n\right]^{|}$ which is characterised by a (P×1) matrix of parameters θ and let h(θ) be a (q×1) matrix function of the P parameters.
Consider the problem of testing

h (θ) = 0 ~ h(θ) $\neq$ 0 ...(2.4)

or h (θ) = 0 ~ h(θ) $\geq$ 0 ...(2.5)

or h (θ) ≥ 0 ~ h(θ) $\not\geq$ 0 ...(2.6)

Let $\breve{\theta}$, $\tilde{\theta}$ and be the values of θ which maximize L(θ)R subject to h(θ) = 0, h(θ) $\geq$ 0 and h(θ) $\neq$ 0, respectively.

Also let $D(\theta) = \frac{\partial \log L(\theta)}{\partial \theta}$ ...(2.7)

$$F(\theta) = \frac{-\partial^2 \log L(\theta)}{\partial \theta \partial \theta^{|}} \quad ...(2.8)$$

$$H(\theta) = \frac{\partial h(\theta)}{\partial \theta} \quad ...(2.9)$$

and $K(\theta) = H(\theta)\left[F(\theta)\right]^{-1} H(\theta)^{|}$ ..(2.10)

Now usual Likelihood Ratio (LR), Wald (W) and Lagrange Multiplier (LM) test statistics are given by

1. $LR = 2\log\left[L(\hat{\theta}) \big/ L(\breve{\theta})\right]$ ...(2.11)

2. $W = h(\hat{\theta})^{|}\left[k(\hat{\theta})\right]^{-1} h(\hat{\theta})$ ...(2.12)

and iii. $LM = D(\breve{\theta})^{|}\left[F(\breve{\theta})\right]^{-1} D(\breve{\theta})$ ...(2.13)

where it is assumed that F(θ) is positive definite and H(θ) has full row rank in same open neighbourhood of the true value of θ.

b) Linear Equality Tests

Consider the problem of testing q linear restrictions on the parameters θ = [β| σ²]| of the standard linear regression model

$$Y = X\beta + \in, \quad \in\sim N\left[0, \sigma^2 I_n\right]$$

As H_0: Rβ = r

Now, by substituting h(θ) = Rβ – r and

$$\log L(\theta) = \frac{-n}{2}\log\left(2\sigma\sigma^2\right) - \frac{(Y - X\beta)^{|}(Y - X\beta)}{2\sigma^2} \quad ...(2.14)$$

We can write the three test statistics as

1. $LR = n\log\left(\frac{\breve{\sigma}^2}{\hat{\sigma}^2}\right)$ ...(2.15)

2. $W = \left(R\hat{\beta} - r\right)' \left[R\left(X'X\right)^{-1} R'\right]^{-1} \left(R\hat{\beta} - r\right) / \hat{\sigma}^2$ ...(2.16)

and iii. $LM = \left(Y - X\breve{\beta}\right)' X \left(X'X\right)^{-1} X' \left(Y - X\breve{\beta}\right) / \breve{\sigma}^2$ ...(2.17)

or $LM = \left(\hat{\beta} - \breve{\beta}\right)' \left(X'X\right)^{-1} \left(\hat{\beta} - \breve{\beta}\right) / \breve{\sigma}^2$ ...(2.18)

By defining $\hat{\sigma}^2 = \left(Y - X\hat{\beta}\right)' \left(Y - X\hat{\beta}\right) / n$ ...(2.19)

$\tilde{\sigma}^2 = \left(Y - X\tilde{\beta}\right)' \left(Y - X\tilde{\beta}\right) / n$ ...(2.20)

$\breve{\sigma}^2 = \left(Y - X\breve{\beta}\right)' \left(Y - X\breve{\beta}\right) / n$ ...(2.21)

Amemia derived the three statistics as

1. $LR = n \log\left(\breve{\sigma}^2 / \hat{\sigma}^2\right)$ ...(2.22)

2. $W = n\left(\breve{\sigma}^2 - \hat{\sigma}^2 / \hat{\sigma}^2\right)$ ...(2.23)

and iii. $LM = n\left(\breve{\sigma}^2 - \hat{\sigma}^2 / \breve{\sigma}^2\right)$ ...(2.24)

3. Tests For Testing Inequality Restrictions Of The Form H_0: h(θ) = 0 ~ H_1: h(θ) ≥ 0

Consider the problem of testing
H_0: h(θ) = 0 ~ H_1: h(θ) ≥ 0. Farebrother (1988) [6] has suggested the three general statistics as

1. $LR = 2\log\left[L\left(\tilde{\theta}\right) / L\left(\breve{\theta}\right)\right]$ ...(3.1)

2. $W = h\left(\tilde{\theta}\right)' \left[K\left(\tilde{\theta}\right)\right]^{-1} h\left(\tilde{\theta}\right)$ ...(3.2)

and iii. $LM = D\left(\breve{\theta}\right)' \left[F\left(\breve{\theta}\right)\right]^{-1} D\left(\breve{\theta}\right)$ ...(3.3)

where the unconstrained estimator $\hat{\theta}$ is replaced with the inequality constrained estimator $\tilde{\theta}$ to reflect the change in the H_1. Gourieroux, Holly and Monfort (1982) [10] have proposed the Kuhn – Tucker test statistic by replacing $D\left(\breve{\theta}\right)$ with $\left[D\left(\breve{\theta}\right) - D\left(\tilde{\theta}\right)\right]$ and it is given by

$KT = \left[D\left(\breve{\theta}\right) - D\left(\tilde{\theta}\right)\right]' \left[F\left(\breve{\theta}\right)\right]^{-1} \left[D\left(\breve{\theta}\right) - D\left(\tilde{\theta}.\right)\right]$ ...(3.4)

This statistic is a function of both $\breve{\theta}$ and $\tilde{\theta}$.
By considering h(θ) = Rβ – r for the standard linear model, the three statistics for testing H_0: Rβ – r ~ H_1: Rβ ≥ r are given by

1. $LR = n \log\left(\breve{\sigma}^2 / \hat{\sigma}^2\right)$ ...(3.5)

2. $W = \left(\sim / \hat{\sigma}^2 \right)$ ...(3.6)

and iii. $KT = \left(\sim / \breve{\sigma}^2 \right)$ ...(3.7)

where, $\sim = n\left(\breve{\sigma}^2 - \tilde{\sigma}^2 \right)$ ...(3.8)

or $\sim = \left(\tilde{\beta} - \breve{\beta} \right)^{|} \left(X^{|} X \right) \left(\tilde{\beta} - \breve{\beta} \right)$ ...(3.9)

or $^{-} = \left(R\tilde{\beta} - r \right)^{|} \left[R\left(X^{|} X \right)^{-1} R^{|} \right]^{-1} \left(R\tilde{\beta} - r \right)$ ...(3.10)

Suppose that the inequality constrained least squares estimator $\tilde{\beta}$ satisfied q_j inequality constraints

$J^{|} R\tilde{\beta} \geq J^{|} r$ as equalities, where J is an ($n \times q_j$) sub matrix of I_n, then $\tilde{\beta}$ is given by

$$\tilde{\beta} = \breve{\beta} - \left(X^{|} X \right)^{-1} R^{|} J \left[J^{|} R \left(X^{|} X \right)^{-1} R^{|} J \right]^{-1} \left(J^{|} R\breve{\beta} - J^{|} r \right) \quad ...(3.11)$$

and $R\tilde{\beta} - r = Q\breve{B}_j \left(R\breve{\beta} - r \right)$ ...(3.12)

where $Q = R\left(X^{|} X \right)^{-1} R^{|}$

and $\breve{B}_j = Q^{-1} - J\left(J^{|} Q J \right)^{-1} R^{|}$ ...(3.13)

Thus we have,

$$\sim = \left(R\hat{\beta} - r \right)^{|} \breve{B}_j / \left(R\hat{\beta} - r \right) \quad ...(3.14)$$

The Kuhn-Tucker statistics is given by

$$KT\left(\sim / \breve{\sigma}^2 \right) \quad ...(3.15)$$

3. Generalization Of The Wald Test For Inequality Restrictions Of The Form H_0: h(θ) = 0 ~ H_1: h(θ) ≥ 0

We have, $\hat{\theta} \overset{Asy}{\sim} N\left[\theta, \left[F(\theta) \right]^{-1} \right]$ ...(4.1)

And $h\left(\hat{\theta} \right) \overset{Asy}{\sim} N\left[0, K(\theta) \right]$ ...(4.2)

Where $K(\theta) = H(\theta) \left[F(\theta) \right]^{-1} H(\theta)^{|}$

Also, one may obtain consistent estimator for K(θ) as $K\left(\hat{\theta} \right)$ and Wald statistic is given by

$$W = h\left(\hat{\theta} \right)^{|} \left[K\left(\hat{\theta} \right) \right]^{-1} h\left(\hat{\theta} \right) \quad ...(4.3)$$

Here, under the H_0: h(θ) = 0 we follows an asymptotically χ^2 distribution with ‘q’ degrees of freedom.
Consider an unconstrained maximum likelihood estimator $\tilde{\theta}$ and its distribution as

$$\tilde{\theta} \overset{Asy}{\sim} N\left[\theta, v(\theta)\right] \quad ...(4.4)$$

Where n v(θ) is a consistent estimator of ψ(θ).
Here, ψ(θ) is such that

$$\sqrt{n}\left(\tilde{\theta}-\theta\right) \overset{Asy}{\sim} N\left[0, \psi(\theta)\right] \quad ...(4.5)$$

Thus, $h\left(\tilde{\theta}\right) \overset{Asy}{\sim} N\left[0, M(\theta)\right]$ ...(4.6)

Where $M(\theta) = H(\theta)V(\theta)H(\theta)^{|}$ ...(4.7)

Hence, the generalized Wald statistic is given by

$$GW = h\left(\tilde{\theta}\right)^{|}\left[M\left(\tilde{\theta}\right)\right]^{-1} h\left(\tilde{\theta}\right) \overset{Asy}{\sim} \chi^2(q) \text{ Under } H_0: \quad ...(4.8)$$

5. Tests For Testing Inequality Restrictions Of The Form H_0: h(θ) ≥ 0 ~ H_1: h(θ) $\not\geq$ 0

Consider the problem of testing H_0: h(θ) ≥ 0 ~ H_1: h(θ) $\not\geq$ 0.
The three tests in this case are given by

1. $LR = 2\log\left[L\left(\hat{\theta}\right) / L\left(\breve{\theta}\right)\right]$ ...(5.1)

2. $W = h\left(\hat{\theta}\right)^{|}\left[K\left(\hat{\theta}\right)\right]^{-1} h\left(\hat{\theta}\right)$ ...(5.2)

and iii. $KT = D\left(\tilde{\theta}\right)^{|}\left[F\left(\tilde{\theta}\right)\right]^{-1} D\left(\tilde{\theta}\right)$ ...(5.3)

Farebrother (1988) [6] has suggested a modified Wald test statistic as

$$WA = \left[h\left(\hat{\theta}\right) - h\left(\tilde{\theta}\right)\right]^{|}\left[K\left(\hat{\theta}\right)\right]^{-1}\left[h\left(\hat{\theta}\right) - h\left(\tilde{\theta}\right)\right] \quad ...(5.4)$$

By considering h(θ) = Rβ – r, the three test statistics for testing H_0: Rβ ≥ r ~ H_1: Rβ $\not\geq$ r are given by

1. $LR = n\log\left(\tilde{\sigma}^2 / \hat{\sigma}^2\right)$ ...(5.5)

2. $W = \left(R\hat{\beta} - r\right)^{|}\left[R\left(X^{|}X\right)^{-1} R^{|}\right]^{-1}\left(R\hat{\beta} - r\right) / \hat{\sigma}^2$ ...(5.6)

and iii. $KT = \left(\tilde{\beta} - \hat{\beta}\right)^{|}\left(X^{|}X\right)\left(\tilde{\beta} - \hat{\beta}\right) / \tilde{\sigma}^2$ ...(5.7)

The modified Wald test statistic is given by

$$WA = \left(R\tilde{\beta} - R\hat{\beta}\right)^{|}\left[R\left(X^{|}X\right)^{-1} R^{|}\right]^{-1}\left(R\tilde{\beta} - R\hat{\beta}\right) / \hat{\sigma}^2 \quad ...(5.8)$$

Also, Farebrother (1988) [6] suggested a “Distance type Wald test statistic” as

$$WD = \left(R\tilde{\beta} - r\right)^{|}\left[R\left(X^{|}X\right)^{-1}R^{|}\right]^{-1}\left(R\tilde{\beta} - r\right)/\hat{\sigma}^2 \qquad ...(5.9)$$

$$= \left(R\tilde{\beta} - r\right)^{|}\left[R\left(X^{|}X\right)^{-1}R^{|}\right]^{-1}\left(R\tilde{\beta} - r\right)/\breve{\sigma}^2 \qquad ...(5.10)$$

Consider the general linear model

$$Y = X\beta + \in, \qquad \in \sim N\left[0, \Omega(r)\right] \qquad ...(5.11)$$

Gourieroux, Holly and Monfort (1982) [10] have established a common asymptotic distribution for the three statistics for testing H_0: $R\beta = r$ against H_1: $R\beta \geq r$, is given by

$$P\left[\xi_A < C\right] = \sum_{i=0}^{q} W_i P\left[\chi^2(i) < C\right] \qquad ...(5.12)$$

Under the null hypothesis, where $W_1, W_2, ..., W_n$ are known weights. Farebrother (1986) [7] has identified the finite sample null distribution of these statistics where $\Omega(r) = d^2 I_n$.

6. Relationship Among Likelihood Ratio (Lr), Wald (W) and Kuhn-Tucker Tests For Testing Inequality Restrictions

Consider the classical linear regression model

$$Y_{n\times 1} = X_{n\times k}\beta_{k\times 1} + \in_{n\times 1} \qquad ...(6.1)$$

Where $\in \sim N\left[0, \sigma^2\Omega(\eta)\right]$

Here, Y is n×1, X is an n×k nonstochastic matrix of rank k, k<n, $\Omega(\eta)$ is a positive definite matrix for the η values of interest and the k×1 vector β and the scalar σ^2 are unknown nuisance parameters. Without loss of generality, we generally assume that $\Omega(0) = I_n$.

Consider the problem of testing,

H_0: $R\beta = r$

Against H_1: $R\beta > r$

Where R is a known q×k, with rank q<k, and r is q×1 and known.

Gourieroux, Holly and Monfort (1982) [10] have proposed the three tests namely, Likelihood Ratio, Wald and Kuhn – Tucker tests, when $\sigma^2\ \Omega(\eta)$ is known, are

1. $$LR = 2\left[L\left(\tilde{\beta}, \sigma^2, \eta\right) - L\left(\hat{\beta}_0, \sigma^2, \eta\right)\right] \qquad ...(6.2)$$

Or

$$LR = -\left(Y - X\tilde{\beta}\right)^{|}\left[\sigma^2\Omega(\eta)\right]^{-1}\left(Y - X\tilde{\beta}\right) + \left(Y - X\hat{\beta}_0\right)^{|}\left[\sigma^2\Omega(\eta)\right]^{-1}\left(Y - X\hat{\beta}_0\right) \qquad ...(6.3)$$

2. $$W = \left(R\tilde{\beta} - \Omega\right)^{|}\left\{R\left[X^{|}\left(\sigma^2\Omega(\eta)\right)^{-1}X\right]^{-1}R^{|}\right\}^{-1}\left(R\tilde{\beta} - \Omega\right) \qquad ...(6.4)$$

3. $$KT = \left(\hat{\lambda}_0 - \tilde{\lambda}\right)^{|}\left\{R\left[X^{|}\left(\sigma^2\Omega(\eta)\right)^{-1}X\right]\right\}R^{|}\left(\hat{\lambda}_0 - \tilde{\lambda}\right)/4 \qquad ...(6.5)$$

Such that LR = W = KT, where $\hat{\beta}_0$ is the equality constrained (under H_0) estimator of β.

These tests have exact null distributions as probability mixtures of χ^2 – distributions and the degenerate distribution at zero. Wu and King (1994) [17] have shown that, if $\sigma^2\Omega(\eta)$ is unknown, σ^2 and η in LR, W and KT above are replaced in the conventional way by the Maximum Likelihood estimators $\hat{\sigma}_0^2$ and $\hat{\eta}_0$ under H_0 and $\tilde{\sigma}^2$ and $\tilde{\eta}$ under H_1 respectively.

The familiar finite sample inequality $W \geq LR \geq KT$ still holds and the above null distribution holds asymptotically.

Farebrother (1986) [7] considered the case where $\Omega(\eta) = I_n$, so that error covariance matrix is unknown upto the scalar parameter σ^2. He has shown that

1. $$LR = n\log\left(1+\frac{W}{n}\right) \qquad ...(6.6)$$

and ii. $$LR = -n\log\left(1-\frac{KT}{n}\right) \qquad ...(6.7)$$

so that W≥LR≥KT.

For the linear regression model $Y = X\beta + \in$, Wolak (1987, 1989) [16] considered the problem of testing H_0: Rβ > r against H_1: Rβ ≠ r. This problem is of interest when one cannot be sure about ruling out the possibility of Rβ > r not being true. Wolak (1989) [16] was able to find the null distributions of the KT, Wald and LR test statistics for respectively testing Rβ = r ~ Rβ > r; Rβ ≠ r as probability mixtures of products of two independent χ^2 – distributions when $\sigma^2\Omega$ (η) is known. If $\sigma^2\Omega(\eta)$ is unknown upto finite parameters, these null distributions hold asymptotically. Also,

LR = W =KT if $\sigma^2\Omega(\eta)$ is known

LR ≤ W ≤ KT if $\sigma^2\Omega(\eta)$ is unknown;

If finite samples and the three tests are equivalent asymptotically.

7. Inference In Generalized Linear Model With Inequality Restrictions On Parameters

Consider the standard generalized linear regression model

$$Y_{n\times1} = X_{n\times k}\beta_{k\times1} + \in_{n\times1} \qquad ...(7.1)$$

with $E(\in)=0$ and $E(\in\in^{|})=\psi$ ...(7.2)

where ψ is a (n×n) positive definite covariance matrix.

A prior information usually provides a set of linear inequality restrictions on the parametric vector β, which is given by

$$R\beta \geq r \qquad ...(7.3)$$

Where R is a (q×k) matrix of known constants;

r is a (q×1) vector of known constants.

When an unrestricted estimate of β does not satisfy the restrictions, a test may be performed to test the compatibility of the restrictions with the sample data.

Now, we wish to test the joint inequality Rβ ≥ r.

One may state the null and alternative hypotheses as

H_0: $R\beta \geq r$ ...(7.4)

H_1: $\beta \in R^k$ ...(7.5)

Where R^k is the k – dimensional real vector space $R_{q\times k}$ is assumed to have full row rank.

The unrestricted maximum likelihood estimator (MLE) of β can be obtained as a solution to the minimization problem:

$$\min_{\beta}\left\|Y - X\beta\right\|_{\psi}^{2} \qquad ...(7.6)$$

Where $\left\|Y - X\beta\right\|_{\psi}^{2} = \left(Y - X\beta\right)^{|}\psi^{-1}\left(Y - X\beta\right)$ ...(7.7)

The unrestricted MLE of β is given by

$$\tilde{\beta} = \left[X^{|}\psi^{-1}X\right]^{-1}X^{|}\psi^{-1}Y \qquad ...(7.8)$$

Premultiplying equation (7.1) by $R\left[X^{|}\psi^{-1}X\right]^{-1}X^{|}\psi^{-1}$ and subtracting r gives

$$\left(R\tilde{\beta} - r\right) = \left(R\beta - r\right) + \in^{*} \qquad ...(7.9)$$

Where $\in^* = R\left[X^{|}\psi^{-1}X\right]^{-1} X^{|}\psi^{-1} \in$ ...(7.10)

We have, $E\left(\in^*\right) = 0$ ...(7.11)

and $\operatorname{var}\left(\in^*\right) = R\left[X^{|}\psi^{-1}X\right]^{-1} R^{|}$ ...(7.12)

Defining $\delta = R\beta - r$, $\tilde{\delta} = R\tilde{\beta} - r$

and $\Omega = R\left[X^{|}\psi^{-1}X\right]^{-1} R^{|}$,

The equation (7.9) can be written as

$$\tilde{\delta} = \delta + \in^* \qquad ...(7.13)$$

with $\in^* \sim N\left[0, \Omega\right]$

In terms of δ, (7.4) can be written as

H_0: $\delta \geq 0$
H_1: $\delta \in R^k$ ...(7.14)

Thus, the required testing problem can be expressed as:

under $\delta^* = \delta + \in^*$, $\in^* \sim N\left[0, \Omega\right]$ ...(7.15)

test the hypotheses, H_0: $\delta \geq 0$ against H_1: $\delta \in R^k$.
Generally, the positive definite covariance matrix ψ in Ω is unknown. Here, ψ may represent either autoregressive structure or heteroscedastic structure for the disturbances.
Assume that ψ refers to the covariance matrix for the heteroscedastic disturbances defined as follows:

$$\psi = \operatorname{diag}\left(\sigma_1^2, \sigma_2^2, \ldots \sigma_n^2\right) \qquad ...(7.16)$$

Also, it is assumed that

$$\sigma_i^2 = Z_i^{|}\gamma \qquad ...(7.17)$$

Where Z_i is (p×1) vector of known observations on certain explanatory variables γ is (p×1) vector of corresponding unknown parameters. An estimator for γ can be obtained by regressing the squares of the residuals on Z_i's.
Generally OLS or BLUS or Recursiveor Studentized or predicted residuals may be considered for this regression.
By considering the squares of Internally studentized residuals (e^{**2}), one can write the regression equation to obtain an estimator for γ as

$$e_i^{**2} = Z_i^{|}\gamma + \eta_i, \; i=1,2,...,n \qquad ...(7.18)$$

or $e^{**} = Z\gamma + \eta$ ...(7.19)

where e^{**} is (n×1) vector of squares of Internally studentized residuals;
Z is (n×p) matrix of known constants;
γ is (p×1) vector of unknown parameters related to heteroscedastic variances;
and η is (n×1) vector of error observations.
The OLS estimator of γ is given by

$$\hat{\gamma} = \left(Z^{|}Z\right)^{-1} Z^{|}e^{**} \qquad ...(7.20)$$

Now, an estimator for σ_i^2 is given by

$$\sigma_i^{*2} = Z_i^{|}\hat{\gamma} \quad ...(7.21)$$

Thus, an estimator for ψ is given by

$$\psi^* = \text{diag}\left(\sigma_1^{2*}, \sigma_2^{2*}, \ldots, \sigma_n^{2*}\right) \quad ...(7.22)$$

An estimated GLS (EGLS) estimator for β is given by

$$\tilde{\beta}^* = \left[X^{|}\psi^{*-1}X\right]^{-1} X^{|}\psi^{*-1}Y \quad ...(7.23)$$

Also, the dispersion matrix of EGLS estimator is given by

$$\text{var}\left(\tilde{\beta}^*\right) = \left[X^{|}\psi^{*-1}X\right]^{-1} \quad ...(7.24)$$

The proposed Likelihood Ratio test statistic for testing H_0 in (7.14) is given by

$$\Lambda = \left(\tilde{\delta}^* - \tilde{\delta}_R\right)^{|}\Omega^{*-1}\left(\tilde{\delta}^* - \tilde{\delta}_R\right) \quad ...(7.25)$$

Where $\tilde{\delta}^* = R\tilde{\beta}^* - \Omega$ and $\tilde{\delta}_R = R\tilde{\beta}_R^* - r$

and $\Omega^* = R\left[X^{|}\psi^{*-1}X\right]^{-1} R^{|}$

Here, $\tilde{\beta}^*$ is unrestricted EGLS estimator of β and $\tilde{\beta}_R^*$ is inequality restricted EGLS estimator of β.
Here, under $\delta \geq 0$, the Likelihood Ratio test statistic Λ follows χ^2 distribution with q degrees of freedom.
In other words, the Λ can be expressed as

$$\Lambda = \left\{\left(Y - X\tilde{\beta}_R^*\right)^{|}\psi^{*-1}\left(Y - X\tilde{\beta}_R^*\right)\right\} - \left\{\left(Y - X\tilde{\beta}^*\right)^{|}\psi^{*-1}\left(Y - X\tilde{\beta}^*\right)\right\} \quad ...(7.26)$$

Remark:1 The Wald test statistic which is equivalent to the proposed LR test statistic given in (7.26) can be written as,

$$W^* = \left(R\tilde{\beta}_R^* - R\tilde{\beta}^*\right)\left[R\left[X^{|}\psi^{*-1}X\right]^{-1} R^{|}\right]^{-1}\left(R\tilde{\beta}_R^* - R\tilde{\beta}^*\right) \quad ...(7.27)$$

8. Conclusions

A new procedure has been suggested for testing joint inequality hypotheses on regression coefficients of linear model. In this context, the covariance matrix for heteroscedastic disturbances has been specified and estimated by choosing a structure for the error variance. The Likelihood Ratio test has been discussed for testing linear inequality restrictions on parameters of the generalized linear regression model.

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