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Comparative analysis of a cold standby system under different arrival conditions

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Abstract
In this paper two reliability models for 2-units redundant system are analyzed stochastically by taking into consideration the concept of arrival and conditional arrival time of the server. There is a single server who takes some time to arrive at the system with the condition that he has to attend the system immediately at its complete failure. The failed server under goes for treatment and resume the jobs with full efficiency after treatment. The Semi-Markov process and regenerative point techniques are used to drive the expansions for some reliability and economic measures such as repair rate, treatment rate and arrival time of the server are statistically independent with different probability density functions. The graphical behavior of MTSF and profit function has been observed for particular values to various parameters and costs. The profit of the present model has also been compared with that of the model Nandal and Malik [2016].

Keywords: redundant system, server failure, treatment, repair, arrival time, conditional arrival time and stochastic analysis

Introduction

In reliability theory, it is proved that performance and reliability of repairable systems can be improved of redundancy. But there are many systems in which a unit cannot be kept as spare due to its high cost. Therefore, reliability modeling of such systems has been done by many researchers including Murari and Goel (1984) and Cao and Wu (1989) [1, 2] developed reliability models for redundant system under different repair polices. But, most of these systems have been studied under a common assumption that repair facility neither fails nor deteriorates during jobs. But, this assumption seems to unrealistic when repair facility meets with an accident may because of the reasons like mishandling, electric shocks and carelessness on the part of the server. In such a situation, treatment to the server may be given in order to resume the jobs by the server. However, the concept of server failure has already been introduced in Dhankar (2010) [3] while analyzing a huge unit system. In such a situation server may be allowed to take some time (called arrival time) to reach at the system with the condition that he has to reach the system immediately at the failure of both units.

In view of the above, the purpose of the present study is to study a cold standby system of two identical units with server failure. There is a single server who is allowed to take some time to arrive at the system to carry out repair activities subject to the condition that server has to attend the system immediately at complete failure of the system. The server is subjected to failure while performing jobs and goes for treatment. And, the failed unit waits for repair till the server becomes in good condition after receiving treatment. The random variables associated with the failure rate, treatment and arrival rates are statistically independent. The distributions for failure time of the unit and the server are taken as negative exponential while that of repair rate, treatment rate and arrival time of the server are arbitrary with different probability density functions. The semi-Markov process and regenerative point technique are adopted to derive the expressions for some measures of system effectiveness in study state. The graphical behavior of some important reliability characteristic has been observed for arbitrary values of the various parameters and costs. The purpose of the present work is to develop the concept of comparison between two identical units by introducing simultaneously idea of arrival and conditional arrival time of the server.

Notations:

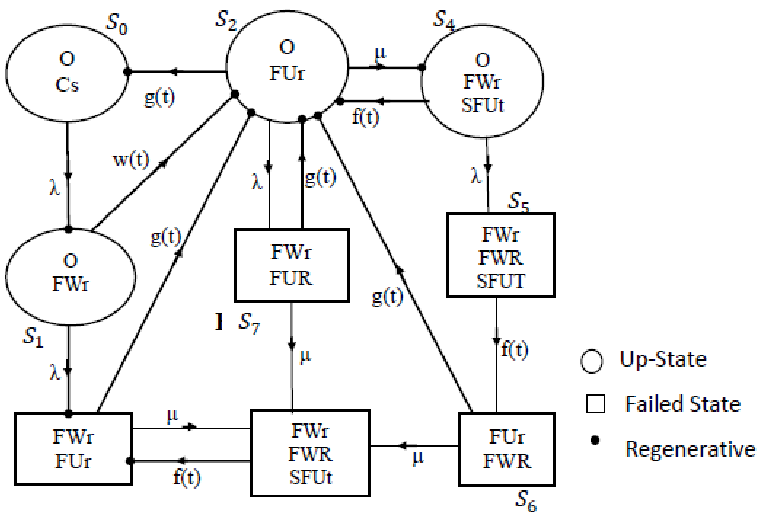
E : Set of regenerative states
$\bar{E}$: Set of non-regenerative states
λ : Constant failure rate
FUr /FWr : The unit is failed and under repair/waiting for repair
SFUt/SFUT : The server is failed and under treatment/continuously under treatment from previous state
FUR/FWR : The unit is failed and under repair / waiting for repair continuously from previous state
g(t)/G(t) : pdf/cdf of repair time of the unit
f(t)/F(t) : pdf/cdf of preventive maintenance time of the unit
w(t)/W(t) : pdf/cdf of arrival time of the server
q_{ij} (t)/ Q_{ij}(t) : pdf / cdf of passage time from regenerative state Si to a regenerative state Sj or to a failed state Sj without visiting any other regenerative state in (0, t]
$q_{ij.kr}$ (t)/$Q_{ij.kr}$(t) : pdf/cdf of direct transition time from regenerative state Si to a regenerative state Sj or to a failed state Sj visiting state Sk, Sr once in (0, t]
M_i(t) : Probability that the system up initially in state $S_i \in E$ is up at time t without visiting to any regenerative state
W_i(t) : Probability that the server is busy in the state S_i up to time 't' without making any transition to any other regenerative state or returning to the same state via one or more non-regenerative states.
μ_i : The mean sojourn time in state S_i which is given by

$$\mu_i = E(T) = \int_0^\infty P(T > t)\, dt = \sum_j m_{ij} ,$$

Where T denotes the time to system failure.
m_{ij} : Contribution to mean sojourn time (μ_i) in state S_i when system transits directly to state S_j so that $\mu_i = \sum_j m_{ij}$ and m_{ij} = $\int t dQ_{ij}(t) = -q_{ij}^{*\prime}(0)$

&/© : Symbol for Laplace-Stieltjes convolution/Laplace convolution
*/** : Symbol for Laplace Transformation /Laplace Stieltjes Transformation

The possible transition states of the system model are shown in fig.1
State Transition Diagram



1. Transition Probabilities and Mean Sojourn Times

Simple probabilistic considerations yield the following expressions for the non-zero elements as

$$p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t)dt \tag{1}$$

$p_{12} = w^*(\lambda)$, $p_{13} = 1 - w^*(\lambda)$, $p_{24} = \frac{\mu}{\lambda + \mu}(1 - g^*(\lambda + \mu))$, $p_{27} = \frac{\lambda}{(\lambda + \mu)}(1 - g^*(\lambda + \mu))$,

$p_{32} = g^*(\mu)$, $p_{38} = 1 - g^*(\mu)$, $p_{42} = f^*(\lambda)$, $p_{45} = 1 - f^*(\lambda)$, $p_{62} = g^*(\mu)$, $p_{68} = 1 - g^*(\mu)$,

$p_{72} = g^*(\mu)$, $p_{78} = 1 - g^*(\mu)$, $p_{01} = p_{56} = p_{83} = 1$

It can be easily verify that

$$p_{01}=p_{12}+p_{13}=p_{20}+p_{24}+p_{27}=p_{42}+p_{45}=1$$
$$p_{20}+p_{24}+p_{22.7}+p_{23.78}=p_{32}+p_{33.8}=p_{42}+p_{42.56}+p_{43.568}=1 \quad (2)$$

The mean sojourn times (μ_i) is in the state S_i are

$$\mu_0=m_{01},\ \mu_1=m_{12}+m_{13},\ \mu_2=m_{20}+m_{24}+m_{27},\ \mu_4=m_{42}+m_{45},$$
$$\mu_2'=m_{20}+m_{24}+m_{22.7}+m_{22.78},\ \mu_3'=m_{32}+m_{33.8},\ \mu_4'=m_{42}+m_{42.56}+m_{43.568} \quad (3)$$

2. Reliability and Mean Time to System Failure (MTSF)

Let $\phi_i(t)$ be the cdf of first passage time from regenerative state Si to a failed state.

Regarding the failed state as absorbing state, we have the following recursive relations for $\phi_i(t)$:

$$\phi_0(t)=Q_{01}(t)\&\phi_1(t)$$
$$\phi_1(t)=Q_{12}(t)\&\phi_2+Q_{13}(t)$$
$$\phi_2(t)=Q_{20}(t)\&\phi_0(t)+Q_{24}(t)\&\phi_4(t)+Q_{27}(t)$$
$$\phi_4(t)=Q_{42}(t)\&\phi_2(t)+Q_{45}(t) \quad (4)$$

Taking LST of above relation (4) and solving for $\Phi_0^{**}(s)$, we have

$$R^*(s)=\frac{1-\phi^{**}(s)}{s} \quad (5)$$

The reliability of the system model can be obtained by taking Inverse Laplace transform of (5).
The mean time to system failure (MTSF) is given by

$$MTSF=\lim_{s\to 0}\frac{1-\phi^{**}(s)}{s}=\frac{N}{D} \quad (6)$$

Where

$$N=\mu_0(1-p_{24}(1-p_{45}))+\mu_1(1-p_{24}p_{42})+\mu_2 p_{12}+\mu_4 p_{12}p_{24} \quad and \quad D=1-p_{24}p_{42}-p_{01}p_{12}p_{20} \quad (7)$$

3. Steady State Availability

Let A_i (t) be the probability that the system is in up-state at instant 't' given that the system entered regenerative state Si at t = 0.The recursive relations for $A_i(t)$ are given as:

$$A_0(t)=M_0(t)+q_{01}(t)©A_1(t)$$
$$A_1(t)=M_1(t)+q_{12}(t)©A_2(t)+q_{13}(t)©A_3(t)$$
$$A_2(t)=M_2(t)+q_{20}(t)©A_0(t)+q_{22.7}(t)©A_2+q_{23.78}(t)©A_3(t)+q_{24}(t)©A_4(t)$$
$$A_3(t)=q_{32}(t)©A_2(t)+q_{33.8}(t)©A_3(t)$$
$$A_4(t)=M_4(t)+(q_{42}(t)+q_{42.56}(t))©A_2(t)+q_{43.568}(t)©A_3(t) \quad (8)$$

Where

$$M_0(t)=e^{-(2\lambda+\alpha_0)t},\ M_1(t)=e^{-(\lambda+\alpha_0+\beta_0)t}\overline{G(t)},\ M_4(t)=e^{-(\lambda+\alpha_0)t}\overline{R(t)},\ M_6(t)=e^{-(\lambda+\alpha_0)t}\overline{F(t)} \quad (9)$$

Taking LT of above relations (8) and solving for $A_0^*(s)$. The steady state availability is given by

$$A_0(\infty)=\lim_{s\to 0} sA_0^*(s)=\frac{N_1}{D_1} \tag{10}$$

Where

$$N_1 = p_{32}\{p_{20}(\mu_0+\mu_1)+\mu_2+p_{24}\mu_4\}$$
$$D_1 = \text{p}_{32}\{p_{20}(\mu_0+\mu_1)+\mu_2^{'}+\mu_4^{'}p_{24}\}+\mu_3^{'}\{p_{23.78}+p_{13}p_{20}+p_{24}p_{43.568}\} \tag{11}$$

4. Busy Period Analysis for Server Due to Repair

Let $B_i^R(t)$ be the probability that the server is busy in repair the unit at an instant't' given that the system entered regenerative state Si at t=0.The recursive relations for $B_i^R(t)$ are as follows:

$$\begin{aligned}
&\text{B}_0^R(t)=\text{q}_{01}(t)©B_1^R(t)\\
&B_1^R(t)=\text{q}_{12}(t)©B_2^R(t)+q_{13}(t)©B_3^R(t)\\
&B_2^R(t)=W_2(t)+q_{20}(t)©B_0^R(t)+q_{24}(t)©B_4^R(t)+q_{22.7}(t)©B_2^R(t)+q_{23.78}(t)©B_3^R(t)\\
&B_3^R(t)=W_3(t)+q_{32}(t)©B_2^R(t)+\text{q}_{33.8}(t)©B_3^R(t)\\
&B_4^R(t)=q_{42}(t)©B_2^R(t)+\text{q}_{42.56}(t)©B_2^R(t)+\text{q}_{43.568}(t)©B_3^R(t)
\end{aligned} \tag{12}$$

Where

$$W_2(t)=e^{-(\lambda+\mu)t}\overline{G(t)}+(\lambda e^{-(\lambda+\mu)t}©1)\overline{G(t)} \quad \text{and} \quad W_3(t)=e^{-\mu t}\overline{G(t)} \tag{13}$$

Taking LT of above relations (12) and solving for $B_0^{R*}(s)$.The time for which server is busy due to repair is given by

$$B_0^R(\infty)=\lim_{s\to 0} sB_0^{R*}(\text{s})=\frac{N_2}{D_1} \tag{14}$$

Where

$$N_2 = W_2^*(0)p_{32}+W_3^*(0)(p_{23.78}+p_{13}p_{20}+p_{24}p_{43.568}) \quad \text{and } D_1 \text{ is already mentioned.} \tag{15}$$

$$N_3=\lambda\alpha[(\theta+\mu)(\alpha+\lambda)\{(\theta+\lambda)(\lambda+\beta)+\lambda\theta\}+\lambda\mu(\lambda+\beta)(\lambda+\mu+\alpha)]$$

5. Expected Number of Repairs

Let $R_i(t)$ be the expected number of repairs by the server in (0, t] given that the system entered the regenerative state Si at t = 0. The recursive relations for $R_i(t)$ are given as:

$$\begin{aligned}
&R_0(t)=Q_{01}(t)\&R_1(t)\\
&R_1(t)=Q_{12}(t)\&R_2(t)+Q_{13}(t)\&R_3(t)\\
&R_2(t)=Q_{20}(t)\&(1+R_0(t))+Q_{24}(t)\&R_4(t)+Q_{22.7}(t)\&(1+R_2(t))\\
&\qquad +Q_{23.78}(t)\&R_3(t)\\
&R_3(t)=Q_{32}(t)\&(1+R_2(t))+Q_{33.8}(t)\&R_3(t)\\
&R_4(t)=Q_{42}(t)\&R_2(t)+Q_{42.56}(t)\&\left(1+R_2(t)\right)+Q_{43.568}(t)\&R_3(t)
\end{aligned} \tag{16}$$

Taking LST of above relations (16) and solving for $R_0^{**}(s)$.The expected no. of repairs per unit time by the server are giving by

$$R_0(\infty)=\lim_{s\to 0} sR_0^{**}(s)=\frac{N_3}{D_1} \quad (17)$$

Where

$N_3 = 1 - P_{24}P_{42} + P_{13}P_{20}$ and D_1 is already mentioned. (18)

6. Expected Number of Treatments Given to the Server

Let $T_i(t)$ be the expected number of treatments given to the server in (0, t] given that the system entered the regenerative state S_i at t = 0. The recursive relations for $T_i(t)$ are given as:

$$\begin{aligned}
T_0(t) &= Q_{01}(t) \& T_1(t) \\
T_1(t) &= Q_{12}(t) \& T_2(t) + Q_{13}(t) \& T_3(t) \\
T_2(t) &= Q_{20}(t) \& T_0(t) + Q_{24}(t) \& T_4(t) + Q_{22.7}(t) \& (1+T_2(t)) \\
&\quad + Q_{23.78}(t) \& \left(1+T_3(t)\right) \\
T_3(t) &= Q_{32}(t) \& T_2(t) + Q_{33.8}(t) \& \left(1+T_3(t)\right) \\
T_4(t) &= Q_{42}(t) \& \left(1+T_2(t)\right) + Q_{42.56}(t) \& \left(1+T_2(t)\right) + Q_{43.568}(t) \& \left(1+T_3(t)\right)
\end{aligned} \quad (19)$$

Taking LST of above relations (19) and solving for $T_0^{**}(s)$ .The expected no. of repairs per unit time by the server are giving by

$$T_0(\infty)=\lim_{s\to 0} sT_0^{**}(s)=\frac{N_4}{D_1} \quad (20)$$

Where

$N_4 = P_{23.78}P_{32} + P_{33.8}(P_{23.78} + P_{13}P_{20} + P_{24}P_{43.568}) + P_{32}P_{24}$ and D_1 is already mentioned. (21)

7. Profit Analysis

The profit incurred to the system model in steady state can be obtained as

$$P = K_0A_0 - K_1B_0^R - K_2R_0 - K_3T_0 \quad (22)$$

Where

P = Profit of the system model

K_0 = Revenue per unit up-time of the system

K_1 = Cost per unit time for which server is busy due to repair

K_2 = Cost per unit time repair

K_3 = Cost per unit time for treatment of the server

8. Particular Case

Suppose $g(t)=\theta e^{-\theta t}$, $w(t)=\beta e^{-\beta t}$ and $f(t)=\alpha e^{-\alpha t}$

We can obtain the following results:

$$p_{12}=\frac{\beta}{\beta+\lambda},\ p_{13}=\frac{\lambda}{\beta+\lambda},\ p_{20}=\frac{\theta}{(\theta+\lambda+\mu)},\ p_{24}=\frac{\mu}{\theta+\lambda+\mu},\ p_{27}=\frac{\lambda}{\theta+\lambda+\mu},\ p_{32}=\frac{\theta}{(\theta+\mu)},\ p_{38}=\frac{\mu}{\theta+\mu},$$

$$p_{42}=\frac{\alpha}{\alpha+\lambda},\ p_{45}=\frac{\lambda}{\lambda+\alpha},\ p_{62}=\frac{\theta}{\theta+\mu},\ p_{68}=\frac{\mu}{\theta+\mu},\ p_{72}=\frac{\theta}{\theta+\mu},\ p_{78}=\frac{\mu}{\theta+\mu},$$

$$\mu_0=\mu_5=\mu_8=\frac{1}{\lambda},\ \mu_1=\frac{1}{\beta+\lambda},\ \mu_2=\frac{1}{\theta+\lambda+\mu},\ \mu_3=\mu_6=\mu_7=\frac{1}{\theta+\mu},\ \mu_4=\frac{1}{\lambda+\alpha},$$

$$\mu_2^{'} = \frac{\alpha(\theta+\mu)^2+\lambda\mu}{\alpha(\theta+\mu)^2(\theta+\lambda+\mu)},\ \mu_4^{'} = \frac{\alpha(\theta+\mu)+\lambda(\theta+\lambda+2\mu)}{\alpha(\theta+\mu)(\alpha+\lambda)},\ \mu_3^{'} = \frac{\alpha+\mu}{\alpha(\theta+\mu)},$$

Also

$$N = (\lambda+\beta+\alpha_0)(\theta+\lambda+\alpha_0+\beta_0)+2\lambda(\lambda+\beta+\alpha_0)+2\lambda\beta_0$$

$$D = (\lambda+\beta+\alpha_0)(\theta+\lambda+\alpha_0+\beta_0)(2\lambda+\alpha_0)-2\lambda\theta(\lambda+\beta+\alpha_0)-2\lambda\beta\beta_0$$

$$\begin{aligned} D_1 = {} & \beta(\alpha+\alpha_0)[\theta(\beta+\lambda)\{(\theta+\beta_0)(\alpha+\lambda)+\alpha\alpha_0\}+\beta\beta_0(\alpha+\lambda)(\theta+\alpha_0+\beta_0)] \\ & +\lambda\{2(\alpha+\lambda)+\alpha_0\}\{\alpha(\beta+\lambda)(\beta+\beta_0)(\theta+\lambda+\alpha_0+\beta_0)+\alpha_0\beta_0\beta(\theta+\lambda+\alpha_0+\beta_0+\beta)\} \\ & +\alpha_0\beta(\alpha+\lambda+\alpha_0)\{\theta(\beta+\lambda)(\theta+2\lambda+\alpha_0+\beta_0)+\beta\beta_0(\theta+\alpha_0+\beta_0)\} \end{aligned}$$

$$\begin{aligned} N_1 = {} & \alpha\beta[\theta(\beta+\lambda)\{(\theta+\beta_0)(\lambda+\alpha)+\alpha\alpha_0\}+\beta\beta_0(\alpha+\lambda)(\theta+\alpha_0+\beta_0) \\ & +\lambda\{2(\lambda+\alpha)+\alpha_0\}\{(\theta+\beta_0)(\lambda+\beta)+\beta_0(\theta+\alpha_0+\beta_0)\} \\ & +\alpha_0\{\theta(\beta+\lambda)(\theta+2\lambda+\alpha_0+\beta_0)+\beta\beta_0(\theta+\alpha_0+\beta_0)\}] \end{aligned}$$

$$N_2 = \theta^{-1}\alpha\beta\lambda(\theta+\beta_0)(\beta+\lambda)(\theta+\lambda+\alpha_0)(2(\lambda+\alpha)+\alpha_0))$$

$$N_3 = \lambda\beta_0\alpha(\theta+\alpha_0+\beta_0)(\beta+\lambda)\{2(\lambda+\alpha)+\alpha_0\}$$

$$\begin{aligned} N_4 = {} & \beta\alpha_0[\theta(\beta+\lambda)\{(\theta+\beta_0)(\alpha+\lambda)+\alpha\alpha_0\}+\beta\beta_0(\alpha+\lambda)(\theta+\alpha_0+\beta_0) \\ & +(\alpha+\lambda+\alpha_0)\{\theta(\beta+\lambda)(\theta+2\lambda+\alpha_0+\beta_0)+\beta\beta_0(\theta+\alpha_0+\beta_0)\} \\ & +\lambda\beta_0(\theta+\lambda+\alpha_0+\beta_0)(2(\lambda+\alpha)+\alpha_0)(\theta+\lambda+\alpha_0+\beta_0+\beta)] \end{aligned}$$

Graphs

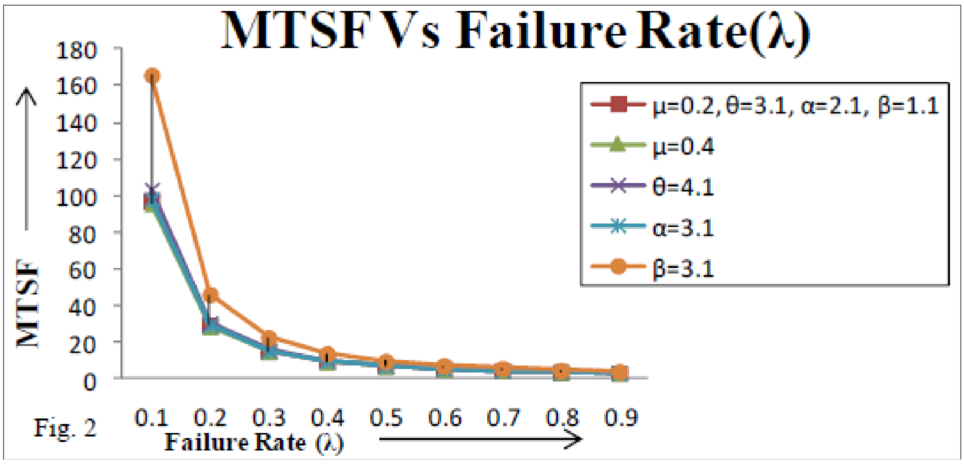


Fig. 2

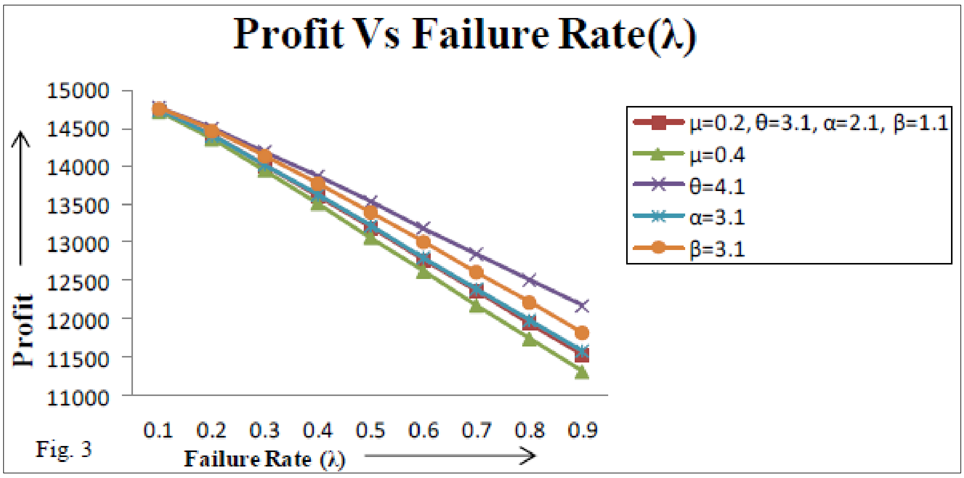


Fig. 3

Conclusion

a) For the particular case $g(t)=\theta e^{-\theta t}$, $w(t)=\beta e^{-\beta t}$ and $f(t)=\alpha e^{-\alpha t}$, the values of some important reliability measures including mean time to system failure (MTSF), availability and profit function are obtained for arbitrary values of various parameters and. The behavior of these measures with respect to failure rate is shown respectively in figures 2 and 3. It observed that MTSF and profit keep on decreasing with the increase of failure rates of the unit and server. However, they follow an upward trend with the increase of repair rate (θ), treatment rate (α) and arrival rate (β) of the server. Further, it can also be seen that MTSF declines rapidly with a slight positive change in failure rate. Hence, the study reveals that a cold stand by system of two identical units can be made more available and profitable to use either by calling the sever immediately to rectify the faults or by increasing the repair rate of the failed unit in case server takes some time to arrive the system.

Profit Difference Graph

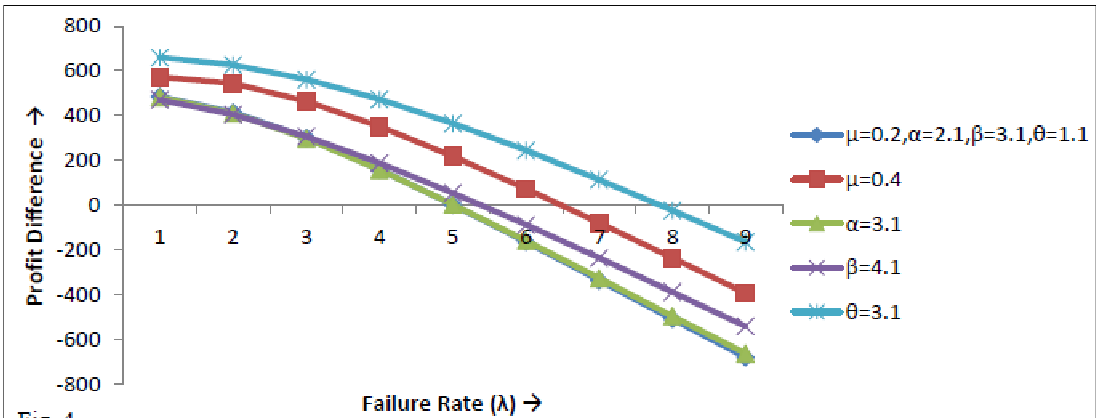


Fig. 4

b) Comparative Study of the Profit of the System Models

To make the study more concrete the behavior of some important reliability measures has been examined graphically for some specific values of the parameters. The mean time of system failure is same as that of the model in which server allowed to take some time to arrive (called arrival time) at the system to see the feasibility of repair. The trend of profit difference of the system models has been shown graphically in figure 4. The results show that profit difference goes on decreasing with the increase of failure rate of the unit, treatment rate of the server, repair rate of the unit while it increases with the increase of arrival rate of the server. It is revealed that the idea of calling the server immediately when system has no unit to work, is useful in making the system more profitable to use, in case failure of the unit is high.

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