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Improved estimators for population mean in presence of measurement error

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Abstract
This paper proposed some improved class of estimators of population mean in presence of measurement error using auxiliary variable based on arithmetic mean, geometric mean and harmonic mean of the usual unbiased estimator, Salabh estimator (1977) and estimators due to Bahal and Tuteja (1991) in case of simple random sampling without replacement (SRSWOR). The expressions of bias and MSEs have been obtained up to the first order of approximation. The performances of the proposed estimators are checked using real population data set. In addition, an empirical study is carried out in the support of theoretical results.

Keywords: Auxiliary information, auxiliary variable, measurement error, bias, mean square error

1. Introduction

In survey sampling, generally it is assumed that the observations have been recorded without any error. Such a supposition may not be tenable in actual practice and the data may contain observational or measurement errors. Measurement errors can occur due to many factors such as due to interviewer, due to respondent or due to the instrument etc. Measurement error is generally taken as a difference between estimated value and true value or deviation of measured values from their true values. Several authors have paid their attention towards the estimation of population parameters such as mean, median, variance a coefficient of variation etc. in the presence of measurement error.

Shalabh (1997) [11] considered the estimation of population mean using ratio method and analyzed its properties in the presence of measurement error. The problem of measurement error has also been studied by Dubey and Singh (2001) [9] etc.

The objective of this paper is to suggest an improved exponential ratio type estimator of population mean of the study variable using information of auxiliary variable in the presence of measurement error.

2. Notations

Let us consider a finite population $W = (W_1, W_2, ..., W_N)$ of N units out of which a sample of size n is drawn using simple random sampling without replacement (SRSWOR).Let Y and X be study and auxiliary variable respectively. Further assuming that x_i and y_i are observed values for the i^{th} sampling units with measurement error as opposed to their true values X_i and Y_i on two characteristics X and Y respectively for the i^{th} (i=1,2,…..,n) unit in the sample of size n. The measurement errors are defined as

$$u_i = y_i - Y_i \tag{2.1}$$

And

$$v_i = x_i - X_i \tag{2.2}$$

Where u_i and v_i are independent and stochastic in nature with mean zero and variance (σ_u^2, σ_v^2) respectively. Let the population means are defined by $\bar{X}$ and $\bar{Y}$. Further let $\rho = \rho_{xy}$ is the population correlation coefficient and $C_y = \frac{\sigma_y}{\bar{Y}}$ and $C_x = \frac{\sigma_x}{\bar{X}}$ are the population coefficient of variation and C_{xy} is the population coefficient of covariance in y and x.

Define,

$\bar{y} = \bar{Y}(1+e_0)$ and $\bar{x} = \bar{X}(1+e_1)$

Such that $E(e_0) = E(e_1) = 0$

and $E(e_0^2) = \frac{C_y^2}{n}\left(1+\frac{\delta u^2}{\delta y^2}\right)$, $E(e_1^2) = \frac{C_x^2}{n}\left(1+\frac{\delta v^2}{\delta x^2}\right)$, $E(e_0 e_1) = \rho_{xy}\frac{C_x C_y}{n}$

The usual unbiased estimator for population mean of the study variable Y is defined by

$$t_0 = \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \tag{2.3}$$

Where $\bar{y}$ is the sample mean of the study variable Y.

Using information on the population mean x_i of the auxiliary variable x Salabh (1977) proposed the following estimator in presence of measurement error

$$t_1 = \bar{y}(\bar{X}/\bar{x}) \tag{2.4}$$

Bahal and Tuteja (1991) [3] proposed the following exponential ratio type estimator in presence of measurement error as

$$t_2 = \bar{y}\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right) \tag{2.5}$$

We know that the variance of the usual unbiased estimator $\bar{y}$ under SRSWOR is approximately, for large n,

$$\mathrm{var}(t_0) = \frac{\bar{Y}^2}{n}C_y^2\left(1+\frac{\delta u^2}{\delta y^2}\right) \tag{2.6}$$

To the first degree of approximation the biases and mean squared errors (MSE's) of t_1 and t_2 are respectively given by

$$Bias(t_1) = \bar{Y}\left[\frac{1}{n}C_x^2\left(1+\frac{\delta v^2}{\delta x^2}\right) - \frac{1}{n}\rho_{xy}C_x C_y\right] \tag{2.7}$$

$$MSE(t_1) = \bar{Y}^2\left[\frac{C_y^2}{n}\left(1+\frac{\delta u^2}{\delta y^2}\right) + \frac{C_x^2}{n}\left(1+\frac{\delta v^2}{\delta x^2}\right) - 2\rho_{xy}\frac{C_x C_y}{n}\right] \tag{2.8}$$

$$Bias(t_2) = \bar{Y}\left[\frac{3}{8}\frac{C_x^2}{n}\left(1+\frac{\delta v^2}{\delta x^2}\right) - \frac{\rho_{xy}C_x C_y}{2n}\right] \tag{2.9}$$

$$MSE(t_2) = \bar{Y}^2\left[\frac{C_y^2}{n}\left(1+\frac{\delta u^2}{\delta y^2}\right) + \frac{C_x^2}{4n}\left(1+\frac{\delta v^2}{\delta x^2}\right) - \rho_{xy}\frac{C_x C_y}{n}\right] \tag{2.10}$$

Motivated by Adichwal *et al*. (2015), we have proposed some estimators of population mean using auxiliary information in form of variable of the study variable Y in the presence of measurement error based on arithmetic mean, geometric mean and harmonic mean of the estimators (t_0, t_1), (t_0, t_2), (t_1, t_2) and (t_0, t_1, t_2).

3. Suggested Estimator

In this section we have suggested some estimators of population mean based on usual unbiased estimator, Salabh (1977) estimator and Bahal and Tuteja estimator (1991) [3] in the presence of measurement error.

3.1 The estimators based on t_0 and t_1

Taking the arithmetic mean (AM), geometric mean (GM) and Harmonic mean (HM) of the estimators t_0 and t_1 we get the estimator of the population mean respectively as

$$t_3^{(AM)} = \frac{1}{2}(t_0 + t_1) = \frac{\bar{y}}{2}\left(1 + \frac{\bar{X}}{\bar{x}}\right) \tag{3.1}$$

$$t_3^{(GM)} = \left(t_0 t_1\right)^{1/2} = \bar{y}\left(\bar{X}/\bar{x}\right)^{1/2} \tag{3.2}$$

$$t_3^{(HM)} = \frac{2}{\left(\frac{1}{t_0} + \frac{1}{t_1}\right)} = \frac{2\bar{y}}{\left(1 + \frac{\bar{x}}{\bar{X}}\right)} \tag{3.3}$$

To the first order of approximation, the bias and the mean squared errors of $t_3^{(AM)}$, $t_3^{(GM)}$ and $t_3^{(HM)}$ are respectively given by

$$Bias(t_3^{(AM)}) = \frac{\bar{Y}}{2}\left[\frac{C_x^2}{n}\left(1 + \frac{\delta v^2}{\delta x^2}\right) - \rho_{xy} C_x C_y\right] \tag{3.4}$$

$$Bias(t_3^{(GM)}) = \bar{Y}\left[\frac{3C_x^2}{8n}\left(1 + \frac{\delta v^2}{\delta x^2}\right) - \frac{\rho_{xy} C_x C_y}{2n}\right] \tag{3.5}$$

$$Bias(t_3^{(HM)}) = \bar{Y}\left[\frac{C_x^2}{4n}\left(1 + \frac{\delta v^2}{\delta x^2}\right) - \frac{\rho_{xy} C_x C_y}{2n}\right] \tag{3.6}$$

$$MSE(t_3^{(AM)}) = MSE(t_3^{(GM)}) = MSE(t_3^{(HM)}) = \bar{Y}^2\left[\frac{C_y^2}{n}\left(1 + \frac{\delta u^2}{\delta y^2}\right) + \frac{C_x^2}{4n}\left(1 + \frac{\delta v^2}{\delta x^2}\right) - \rho_{xy}\frac{C_x C_y}{n}\right] \tag{3.7}$$

3.2 The estimator based on t_0 and t_2

The estimator of $\bar{Y}$ based on AM, GM and HM of the estimators t_0 and t_2 are respectively defined as

$$t_4^{(AM)} = \frac{1}{2}\left(t_0 + t_2\right) = \left(\frac{\bar{y}}{2}\right)\left(1 + \exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right)\right) \tag{3.8}$$

$$t_4^{(GM)} = \left(t_0 t_2\right)^{1/2} = \bar{y}\exp\left(\frac{1}{2}\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right)\right) \tag{3.9}$$

$$t_4^{(HM)} = \frac{2}{\left(\frac{1}{t_0} + \frac{1}{t_2}\right)} = \frac{2\bar{y}}{\left\{1 + \exp\left(\frac{\bar{x} - \bar{X}}{\bar{X} + \bar{x}}\right)\right\}} \tag{3.10}$$

To the first degree of approximation, the biases and the MSEs of $t_4^{(AM)}$, $t_4^{(GM)}$ and $t_4^{(HM)}$ are respectively given by

$$Bias\left(t_4^{(AM)}\right)=\bar{Y}\left[\frac{3C_x^2}{16n}\left(1+\frac{\delta v^2}{\delta x^2}\right)-\frac{\rho_{xy}C_xC_y}{4n}\right] \quad (3.11)$$

$$Bias\left(t_4^{(GM)}\right)=\bar{Y}\left[\frac{5C_x^2}{32n}\left(1+\frac{\delta v^2}{\delta x^2}\right)-\frac{\rho_{xy}C_xC_y}{4n}\right] \quad (3.12)$$

$$Bias\left(t_4^{(HM)}\right)=\bar{Y}\left[\frac{C_x^2}{8n}\left(1+\frac{\delta v^2}{\delta x^2}\right)-\frac{\rho_{xy}C_xC_y}{4n}\right] \quad (3.13)$$

$$MSE\left(t_4^{(AM)}\right)=MSE\left(t_4^{(GM)}\right)=MSE\left(t_4^{(HM)}\right)=\bar{Y}^2\left[\frac{C_y^2}{n}\left(1+\frac{\delta u^2}{\delta y^2}\right)+\frac{C_x^2}{16n}\left(1+\frac{\delta v^2}{\delta x^2}\right)-\rho_{xy}\frac{C_xC_y}{2n}\right] \quad (3.14)$$

3.3 The estimator based on t_1 and t_2

The estimator of S_y^2 based on AM, GM and HM of the estimators t_1 and t_2 are respectively defined as

$$t_5^{(AM)}=\frac{1}{2}\left(t_1+t_2\right)=\left(\frac{\bar{y}}{2}\right)\left(\frac{\bar{X}}{\bar{x}}+\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)\right) \quad (3.15)$$

$$t_5^{(GM)}=\left(t_1t_2\right)^{1/2}=\bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)^{1/2}\exp\left(\frac{1}{2}\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)\right) \quad (3.16)$$

$$t_5^{(HM)}=\frac{2}{\left(\frac{1}{t_1}+\frac{1}{t_2}\right)}=\frac{2\bar{y}}{\left\{\frac{\bar{x}}{\bar{X}}+\exp\left(\frac{\bar{x}-\bar{X}}{\bar{X}+\bar{x}}\right)\right\}} \quad (3.17)$$

To the first degree of approximation, the biases and the MSEs of $t_5^{(AM)}$, $t_5^{(GM)}$ and $t_5^{(HM)}$ are respectively given by

$$Bias\left(t_5^{(AM)}\right)=\bar{Y}\left[\frac{11C_x^2}{16n}\left(1+\frac{\delta v^2}{\delta x^2}\right)-\frac{3\rho_{xy}C_xC_y}{4n}\right] \quad (3.18)$$

$$Bias\left(t_5^{(GM)}\right)=\bar{Y}\left[\frac{21C_x^2}{32n}\left(1+\frac{\delta v^2}{\delta x^2}\right)-\frac{3}{4}\frac{\rho_{xy}C_xC_y}{n}\right] \quad (3.19)$$

$$Bias\left(t_5^{(HM)}\right)=\bar{Y}\left[\frac{9C_x^2}{16n}\left(1+\frac{\delta v^2}{\delta x^2}\right)-\frac{3\rho_{xy}C_xC_y}{4n}\right] \quad (3.20)$$

$$MSE\left(t_5^{(AM)}\right)=MSE\left(t_5^{(GM)}\right)=MSE\left(t_5^{(HM)}\right)=\bar{Y}^2\left[\frac{C_y^2}{n}\left(1+\frac{\delta u^2}{\delta y^2}\right)+\frac{9C_x^2}{16n}\left(1+\frac{\delta v^2}{\delta x^2}\right)-\rho_{xy}\frac{3C_xC_y}{2n}\right] \quad (3.21)$$

3.4 The estimator based on t_0, t_1 and t_2

The estimator of $\bar{Y}$ based on AM, GM and HM of the estimators t_0, t_1 and t_2 are respectively defined as

$$t_6^{(AM)}=\frac{1}{3}\left(t_0+t_1+t_2\right)=\left(\frac{\bar{y}}{3}\right)\left(1+\frac{\bar{X}}{\bar{x}}+\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)\right) \quad (3.22)$$

$$t_6^{(GM)}=\left(t_0t_1t_2\right)^{1/3}=\bar{y}\left[\left(\frac{\bar{X}}{\bar{x}}\right)\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)\right]^{1/3} \quad (3.23)$$

$$t_6^{(HM)} = \frac{3}{\left(\frac{1}{t_0}+\frac{1}{t_1}+\frac{1}{t_2}\right)} = \frac{3\bar{y}}{\left\{1+\frac{\bar{x}}{\bar{X}}+\exp\left(\frac{\bar{x}-\bar{X}}{\bar{X}+\bar{x}}\right)\right\}} \qquad (3.24)$$

To the first degree of approximation, the biases and the MSEs of $t_6^{(AM)}$, $t_6^{(GM)}$ and $t_6^{(HM)}$ are respectively given by

$$Bias\left(t_6^{(AM)}\right) = \bar{Y}\left[\frac{11C_x^2}{24n}\left(1+\frac{\delta v^2}{\delta x^2}\right) - \frac{\rho_{xy}C_xC_y}{6n}\right] \qquad (3.25)$$

$$Bias\left(t_6^{(GM)}\right) = \bar{Y}\left[\frac{11C_x^2}{72n}\left(1+\frac{\delta v^2}{\delta x^2}\right) - \frac{\rho_{xy}C_xC_y}{3n}\right] \qquad (3.26)$$

$$Bias\left(t_6^{(HM)}\right) = \bar{Y}\left[\frac{7C_x^2}{24n}\left(1+\frac{\delta v^2}{\delta x^2}\right) - \frac{\rho_{xy}C_xC_y}{2n}\right] \qquad (3.27)$$

$$MSE\left(t_6^{(AM)}\right) = MSE\left(t_6^{(GM)}\right) = MSE\left(t_6^{(HM)}\right) = \bar{Y}^2\left[\frac{C_y^2}{n}\left(1+\frac{\delta u^2}{\delta y^2}\right) + \frac{C_x^2}{4n}\left(1+\frac{\delta v^2}{\delta x^2}\right) - \rho_{xy}\frac{C_xC_y}{n}\right] \qquad (3.28)$$

4. Efficiency Comparison

From (2.6), (2.8), (2.10), (3.7), (3.14), (3.21) and (3.28), we have

$$MSE(t_1) - Var(t_0) < 0$$

or

$$C_x(1+\frac{\delta_v^2}{\delta_x^2}) < 2\rho_{xy}C_y \qquad (4.1)$$

$$MSE(t_2) - Var(t_0) < 0$$

or

$$C_x(1+\frac{\delta_v^2}{\delta_x^2}) < 4\rho_{xy}C_y \qquad (4.2)$$

$$MSE(t_3^{(h)}) - Var(t_0) < 0$$

or

$$C_x(1+\frac{\delta_v^2}{\delta_x^2}) < 4\rho_{xy}C_y \qquad (4.3)$$

$$MSE(t_4^{(h)}) - Var(t_0) < 0$$

or

$$C_x(1+\frac{\delta_v^2}{\delta_x^2}) < 8\rho_{xy}C_y \qquad (4.4)$$

$$MSE(t_5^{(h)}) - Var(t_0) < 0$$

or

$$C_x(1+\frac{\delta_v^2}{\delta_x^2}) < 8/3\rho_{xy}C_y \qquad (4.5)$$

$$MSE(t_6^{(h)}) - Var(t_0) < 0$$
$$or$$
$$C_x(1+\frac{\delta_v^2}{\delta_x^2}) < 4\rho_{xy}C_y \tag{4.6}$$

Where (h=AM, GM, HM)

From (4.1)-(4.6) we get that the estimators t_1, t_2 and $t_i^{(h)}$, (i=4, 5, 6; h=AM, GM, HM) are better than usual unbiased estimator $t_0 = \bar{y}$ respectively if

$$T < 2 \tag{4.7}$$
$$T < 4 \tag{4.8}$$
$$T < 4 \tag{4.9}$$
$$T < 8 \tag{4.10}$$
$$T > 8/3 \tag{4.11}$$
$$T < 4 \tag{4.12}$$

Where T= $C_x(1+\delta_v^2/\delta_x^2)/C_y\rho_{xy}$

From (2.8), (2.10), (3.7), (3.14), (3.21) and (3.28), we have

$$MSE(t_2) - MSE(t_1) < 0$$
$$or$$
$$4\rho_{xy}C_y < 3C_x(1+\frac{\delta_v^2}{\delta_x^2}) \tag{4.13}$$

$$MSE(t_3^{(h)}) - MSE(t_1) < 0$$
$$or$$
$$4\rho_{xy}C_y < 3C_x(1+\frac{\delta_v^2}{\delta_x^2}) \tag{4.14}$$

$$MSE(t_4^{(h)}) - MSE(t_1) < 0$$
$$or$$
$$4\rho_{xy}C_y < 5C_x(1+\frac{\delta_v^2}{\delta_x^2}) \tag{4.15}$$

$$MSE(t_5^{(h)}) - MSE(t_1) < 0$$
$$or$$
$$8\rho_{xy}C_y < 7C_x(1+\frac{\delta_v^2}{\delta_x^2}) \tag{4.16}$$

$$MSE(t_6^{(h)}) - MSE(t_1) < 0$$
$$or$$
$$4\rho_{xy}C_y < 3C_x(1+\frac{\delta_v^2}{\delta_x^2}) \tag{4.17}$$

Where (h=AM, GM, HM)

From (4.13)-(4.17), it is observed that the estimators t_2 and $t_i^{(h)}$, (i=3, 4, 5, 6; h=AM, GM, HM) are better than the estimator t_1 respectively if

$$T > 4/3 \quad (4.18)$$
$$T > 4/3 \quad (4.19)$$
$$T > 4/5 \quad (4.20)$$
$$T > 8/7 \quad (4.21)$$
$$T > 4/3 \quad (4.22)$$

From (2.10), (3.7), (3.14), (3.21) and (3.28), we have

$$MSE(t_3^{(h)}) - MSE(t_2) = 0 \quad (4.23)$$

$$MSE(t_4^{(h)}) - MSE(t_2) < 0$$
$$or$$
$$8\rho_{xy}C_y < 3C_x(1+\frac{\delta_v^2}{\delta_x^2}) \quad (4.24)$$

$$MSE(t_5^{(h)}) - MSE(t_2) < 0$$
$$or$$
$$8\rho_{xy}C_y > 5C_x(1+\frac{\delta_v^2}{\delta_x^2}) \quad (4.25)$$

$$MSE(t_6^{(h)}) - MSE(t_2) = 0 \quad (4.26)$$

Where (h=AM, GM, HM).

From (4.23) and (4.26) it is observed that the estimators $t_3^{(h)}$ and $t_6^{(h)}$, (h=AM, GM, HM) are equally efficient. Further it is also observed from (3.24) and (3.25) that the estimators $t_4^{(h)}$ and $t_5^{(h)}$, (h=AM, GM, HM) are better than the estimators t_2 respectively if

$$T > 8/3 \quad (4.27)$$
$$T < 8/5 \quad (4.28)$$

From (3.7), (3.14), (3.21) and (3.28), we have

$$MSE(t_4^{(h)}) - MSE(t_3^{(h)}) < 0$$
$$or$$
$$8\rho_{xy}C_y < 3C_x(1+\frac{\delta_v^2}{\delta_x^2}) \quad (4.29)$$

$$MSE(t_5^{(h)}) - MSE(t_3^{(h)}) < 0$$
$$or$$
$$8\rho_{xy}C_y > 5C_x(1+\frac{\delta_v^2}{\delta_x^2}) \quad (4.30)$$

$$MSE(t_6^{(h)}) - MSE(t_3^{(h)}) = 0 \quad (4.31)$$

Where (h=AM, GM, HM).

here $t_6^{(h)}$ is equally efficient. From (4.29) and (4.30) it is observed that the estimators $t_4^{(h)}$, $t_5^{(h)}$ are better than the estimators $t_3^{(h)}$ respectively if

$$T > 8/3 \quad (4.32)$$

$$T < 8/5 \quad (4.33)$$

From (3.14), (3.21) and (3.28), we have

$$MSE(t_5^{(h)}) - MSE(t_4^{(h)}) < 0$$

$$or$$

$$C_x(1+\frac{\delta_v^2}{\delta_x^2}) < 2\rho_{xy}C_y \quad (4.34)$$

$$MSE(t_6^{(h)}) - MSE(t_4^{(h)}) < 0$$

$$or$$

$$3C_x(1+\frac{\delta_v^2}{\delta_x^2}) < 8\rho_{xy}C_y \quad (4.35)$$

Where (h=AM, GM, HM).

From (4.34) and (4.35) it is observed that the estimators $t_5^{(h)}$ and $t_6^{(h)}$ are better than the estimators $t_4^{(h)}$, (h=AM, GM, HM) respectively if

$T < 2$ (4.36)
$T < 8/3$ (4.37)

Further from (3.21) and (3.28), we have

$$MSE(t_6^{(h)}) - MSE(t_5^{(h)}) < 0$$

$$or$$

$$8\rho_{xy}C_y < 5C_x(1+\frac{\delta_v^2}{\delta_x^2}) \quad (4.38)$$

Thus the estimator $t_6^{(h)}$ are better than $t_5^{(h)}$, (h=AM, GM, HM) if $T > 8/5$

Where h= AM, HM, GM and T = $C_x(1+\delta_v^2/\delta_x^2)/C_y\rho_{xy}$

5. Empirical Study

In this section we compare the performance of the estimators using real population data set. The description of population data set are as follows.

Population [Source : Gujrati and Sangeetha (2007), p.539].

Y_i = True consumption expenditure,

X_i = True income,

y_i = Measured consumption expenditure,

x_i = Measured income.

The values of the parameters are

n =10, $\overline{X}=170$, $\overline{Y}=127$, $\sigma_x^2=3300$, $\sigma_y^2=1278$, $\sigma_u^2=32.4001$, $\sigma_v^2=32.3998$, $C_y=0.2815$ $C_x=0.3379$, $\rho_{xy}=0.9641$.

Table 5.1: MSE's of estimators with and without measurement errors

Estimators	MSE with measurement error	MSE without measurement error	Contribution of measurement error in MSE
t_0	131.0400	127.8000	3.2400
$t_1^{(h)}$	21.2297	16.1815	5.0482
$t_2^{(h)}$	29.6398	25.9477	3.6921
$t_3^{(h)}$	29.6398	25.9477	3.6921
$t_4^{(h)}$	68.7161	65.3631	3.3530
$t_5^{(h)}$	13.8110	9.5539	4.2571
$t_6^{(h)}$	29.6398	25.9477	3.6921

(h=AM, GM, HM)

Using real data set, we have computed the percent relative efficiencies (PREs) of the various estimators of $\overline{Y}$ with respect to usual unbiased estimator $\overline{y}$ by using the formula:

$$PRE(t,\overline{y})=\frac{Var(\overline{y})}{MSE(t)}\times 100, \quad (4.1)$$

Where t= t_1, t_2, $t_i^{(h)}$, (i= 3, 4, 5, 6; h=AM, GM, HM)

Table 5.2: PRE's of estimators with and without measurement errors

Estimators	With measurement error	Without measurement error
t_0	100.0000	100.0000
t_1	617.2483	789.7921
t_2	442.1083	492.5285
$t_3^{(h)}$	442.1083	492.5285
$t_4^{(h)}$	190.6976	195.5231
$t_5^{(h)}$	948.8097	1337.6798
$t_6^{(h)}$	442.1083	492.5285

6. Conclusion

This paper suggests some estimator of population mean in presence of measurement error using information of auxiliary variable based on arithmetic mean, geometric mean and harmonic mean of the usual unbiased estimator, Salabh estimator (1977) and estimators due to Bahal and Tuteja (1991) [3]. From table 5.1 it is clear that the MSE of the estimator $t_5^{(h)}$ is less as compared to usual mean estimator t_0 and other existing estimators t_1, t_2, $t_3^{(h)}$, $t_4^{(h)}$ and $t_6^{(h)}$. In terms of PREs, from table 5.2 it is observed that the PRE of the estimator $t_5^{(h)}$ is higher compared to usual mean estimator and other existing estimators for both the cases with and without measurement error. Hence it is recommended to use in practice.

7. References

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