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Extended vertex product cordial (evpc) labeling of graphs

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Abstract
In this paper we discuss a new type of labeling called as extended vertex product cordial Labeling. (evpcl). We allow the vertices of a graph to take values from 0 to p-1,p being the number of vertices of a graph, and restrict the edges to take values 0 or 1 only. We show that path Pn, Star $K_{1,n}$., Crown of Cn, Path union of C_3 i.e. Pn(C_3), Antena (C_3,m) are evpc graphs.

Keywords: labeling, extended vertex, product, cordial, product cordial

Introduction
A labeling which assigns number 0 or 1 to vertices (or edges) by some function is binary or cordial labeling. Let G be a (p,q) graph .The word cordial was used first time by Cahit in 1996.[3] In the present case we define a bijective function f : V(G) →{0,1,2..,p-1}. This introduces f^+:E(G)→{0,1}such that (uv)єE(G) then f^+(uv) = f(u)f(v)(mod 2). Further the condition is satisfied that $|e_f(0)-e_f(1)|\leq 1$. This condition is called as pairity condition. Thus the edge labels are restricted to take values 0 or 1 only. The label is cordial labeling. In discription that follows further we use the letter f for describing edge labels when confusion with f as a vertex label function as above is not possible. A graph that satisfies the conditions for evpc is called as evpc graph and the corresponding function f as above is evpc function. Further f is evpc function means that the graph G is evpc graph. We use $e_f(0,1)$= (a,b) to indicate number of edges with label 0 are a in number and that with label 1 are b in number under the evpc labeling function f.

1. Definitions
1.1 Path Pn It is sequences of vertices and edges given by v_1, e_1, v_2, e_2,...e_{n-1},v_n It has n vertices and n-1 edges.

1.2 Antena graph Consider a G=(p, q) graph. At each of it's vertex attach a path of length m.then we get a antena graph antena (G, m).If we attach K antennas of different length at each vertex of G then it is k-antena(G).

1.3 Tail graph Tail(G, m) has a path of length m attached at a suitable vertex on it with end point of path of degree at least.

1.4 Crown of G is obtained by attaching a pendent edge at each vertex of G. It was defined for cycle Cn and hence the name.

1.5 Path union of G. Pn(G) is obtained by attaching a copy of G at every vertex of Pn. The same fixed point on G is used to obtain Pn(G). Here we discuss Pn(C3).

2.1 Theorem A path Pn is evpcl iff n is an even number.
Proof: Let the graph G = Pn with ordinary labeling be given as (v_1, e_1, v_2, e_2,..,e_{n-1},v_n).Note that |V(Pn)|= n and |E(Pn)|= n-1.Let n be even number given by 2x. Define a function f:V(P_{2x})→{0,1,2..,2x-1} as follows:

f(vi)= 2(i-1) for i = 1,2,..,x
$f(v_{x+j})$=2(j-1)+1 for j = 1,2..,x.
Thus $e_f(0)$= x and $e_f(1)$ = x-1.And the graph is evpcl. For even n the graph is evpc graph.

When n is an odd number and we include number 0 for vertex labeling, there are $\frac{n+1}{2}$ even numbers that must be assigned to as many vertices. We have to label consecutive vertices as even number to produce smallest number of edges with even number as a label. This produces $e_f(0)= \frac{n+1}{2}$ out of maximum n-1 edges available. Thus there can be at most $\frac{n-3}{2}$edges with label 1. Thus $|e_f(0)-e_f(1)|$ =2 and the graph is not evpcl. #

2.2 Theorem $K_{1,n}$ is evpcl iff n is a odd number.

Proof: Let n be an odd number. In ordinary labeling of $K_{1,n}$ label the central vertex as u and the pendent vertices be $u_1,u_2,..,u_n$. Define a function f:V($K_{1,n}$)→{0,1,2...,n} as follows, f(u) = 1, $f(u_1)$ = 0, $f(u_i)$= i for i= 2,3,..n. We have and $e_f(0)=\frac{n+1}{2}$ and $e_f(1) = \frac{n-1}{2}$ Now if n is an even number we have $\frac{n}{2}$ +1 even numbers (as 0 is to be included) and $\frac{n}{2} - 1$ odd numbers. Further the vertex u can't be labeled as an even number as that will produce all edge labels as an even number. Therefore we must label vertex u as an odd number. Consequently the number of edges with odd label numbers is smaller by 2 than $e_f(0)$.

2.3 Theorem Crown of Cn is evpc graph.

Proof: The ordinary labeling of C_n be given as (v_1, e_1, v_2, e_2,..,e_{n-1},v_n,e_n,v_1) The pendent vertex at v_i be u_i and the corresponding edge be e_i'=(v_iu_i).G=Crown (Cn). Then |V(G)| = 2n= |E(G)|. Define a function f:V(G)→{0,1,2...,2n-1} as follows:f(ui)= 2(i-1) for i = 1,..n. f(vi)=2(i-1)+1, i = 1,2,..n. This gives $e_f(0,1)$=(n,n).

2.4 Theorem: for n is an even number Path union of C_3 i.e.Pn (C_3) is evpc graph.

Proof: Let the ordinary labeling of path be (v_1, e_1, v_2, e_2,.., e_{n-1},v_n). Between vi and vi+1 two new vertices ui and wi are taken.the new edges are (v_iu_i),(u_iw_i),(w_iv_{i+1}).

Define a function f:V(G)→{0,1,2...,3n-1}. Let n = 2x. f(vi) = 2(i-1) for i = 1,2,..,x. and $f(v_{x+i})$ = 2(i-1)+1for i = 1,2,..,x. f(ui) = $f(v_x)$+2i for i =1,2,..x. and f(wi) = $f(u_x)$+2i for i = 1,2,..x. $f(u_{x+i})$ = $f(v_n)$+2i for i =1,2,..x. and $f(w_{x+i})$ = $f(u_n)$+2i for i = 1,2,..x.

We have ef(0,1) = $(\frac{q+1}{2}, \frac{q-1}{2})$ where q = 4n-1.The graph is evpc.In the fig 4.1 below the labeling is explained for n =6 and G = C_3#

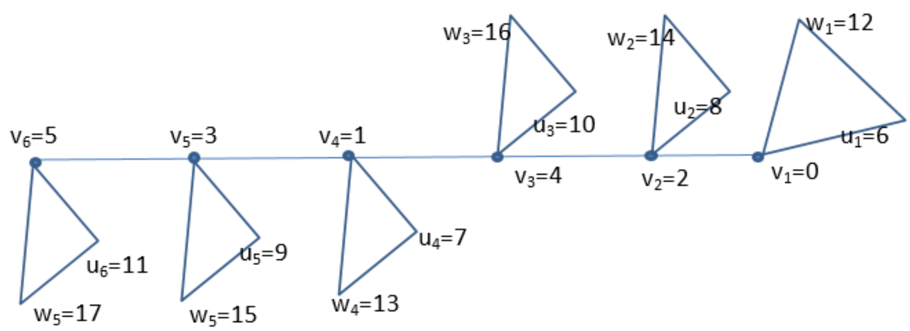

Fig 4.1: $P_6(C_3)$: A labeled copy.

Theorem 2.5 Antena (C_3,m) is evpc.

Proof. Let the three vertices on C_3 be v_1,v_2 and v_3.The path P_{m+1} at vertex vi be ($v_{i,1}$=v_i, $v_{i,2}$,$v_{i,3}$, ...,$v_{i,m}$, $v_{i,m+1}$) for i =1,2,3.Note that |V(G)| = 3m+3.Define f as follows:
f(vi)=2(i-1)+1 for i =1,2,3.
$f(v_{1,j})$=2(j-2),j = 2,3,..,m+1.

Case m is even number say 2x.

Then $f(v_{2,j})$=$f(v_{1,m+1})$+2(t+1) for j = x+t,t=0,1,2..x,x+1
$f(v_{2,x+j})$ = 5+ 2(j-1) for j= 2,3,..,x-1
$f(v_{3,j})$=$f(v_{2,x-1})$+2(j-1), j=2, 3, 4,..m+1. The number distribution is $e_f(0,1)$ = (t+1,t), t = $\frac{3m+2}{2}$

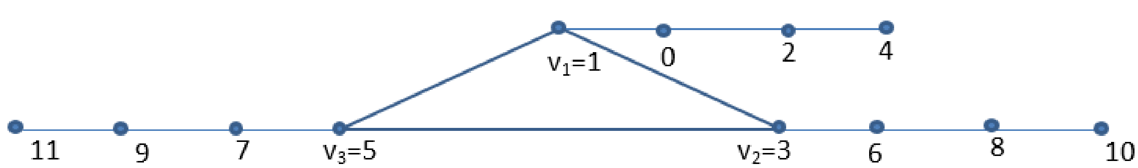

Fig 4.2: Antena (C_3, 3) A labeled evpc graph

Case m is odd number say 2x+1

$f(v_{2,j}) = f(v_{1,m+1})+2(j-x)$ for $j=x+1,x+2,...,2x+1$
$f(v_{2,j})=5+2(j-1), j=2,3,..x.$
$f(v_{3,j})=f(v_{2,x})+2(j-1)$, $j=2,3,4,..m+1$. The number distribution is $e_f(0,1) = (t,t), t = \frac{3m+3}{2}$

Theorem 4.6 Tail(C_3,m) is evpc graph.(m=1,2,..)
Proof: Tail(C_3,m)has a path of length m attached at a vertex and the end vertex of the path is of degree 3 or more. We consider the end degree vertex of path of degree 3 .The ordinary labeling of the graph be u,v, w as cycle vertices and tail attached at point u be ($u=u_1,u_2,u_3,..,u_{m+1}$). The two pendent vertices be x and y respectively. Define a function as $f:V(G)\rightarrow\{0,1,2..,m+4\}$ given as $f(u)=1,f(v)=3,f(w)=5,f(x)=0,f(y)=2,f(u_{m+1})=4$ for m is even number say 2x then, $f(u_j)= 5+2(j-1)$ for $j = 2,3,..,x$
$f(u_{x+j})=2x+4-2(j-1)$, $j = 1,2,..,x$. Number distribution is $e_f(0,1) = (3+x,2+x)$.
For m is a odd number say 2x+1(x=0,1,2,..) we have, f(uj) = 5+2(j-1) for j = 2,3,..,x+1 and $f(u_{x+j})=2x+4-(j-2)2$, j =2,3,..x,x+1.We have number distribution $e_f(0,1) = (2+x,2+x)$#

5. Conclusions
In this paper we discuss a new type of graph labeling. We show that certain graph families are evpc families. A well-known graph Cn is not found to be evpc.

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