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R Geetha
Assistant Professor,
Department of Mathematics,
KG College of Arts and Science,
Coimbatore, Tamil Nadu, India

Correspondence
R Geetha
Assistant Professor,
Department of Mathematics,
KG College of Arts and Science,
Coimbatore, Tamil Nadu, India

A study on soft GSR-closed sets in soft topological spaces

R Geetha

Abstract
In this paper the concept of soft -gsr-closed and open sets in soft topological s paces are defined. The properties on soft closed set, soft regular open, soft-gsr-closed sets and soft-gsr open sets are discussed.

Keywords: R Soft set, soft topology, soft-gsr-closed sets, soft-gsr-open sets

1. Introduction
A new mathematical tool of the concept of soft set theory was initiated by Molodtsov. Muhammad Shabir and Munazza Naz introduced the soft topological spaces and the notions of soft open set, soft closed set, soft closure, soft interior points. Levine introduced generalized closed and open sets in topological spaces. The properties of soft semi open sets and soft semi closed sets are introduced by Bin Chen. Soft regular open set and soft regular closed sets are introduced by Saziye Yuksel.

2. Preliminaries
Definition: 2.1
Let U be the initial universe, P(U) be the power set of U and E be the set of all parameters
Let A be a non -empty subset of E. A pair (F, A) is called a soft set over U, where F is a mapping given by
F: A→ P(U). In other words, a soft set over U is a parameterized family of subsets of the universe U. F (Ɛ) may be considered as the set Ɛ- approximate elements Of the soft set (F, A) for Ɛ ϵ A. For two soft sets (F,A) and (G,B) over the common universe U, (F,A) is a soft subset of (G,B) if
(i) A $\subseteq$ B and (ii) for all e ϵ A, F (e) and G(e) are identical approximations. We write (F, A) $\tilde{\subset}$ (G, B). (F, A) is said to be a soft superset of (G, B), if (G, B) is a soft subset of (F,A).
Two soft sets (F, A) and (G, B) over a common universe U are said to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

Definition: 2.2
The union of two soft sets of (F, A) and (G, B) over the common universe U is soft set (H, C), where C = A U B and for all e ϵ C
(i) H(e) = F(e) if e ϵ A-B, (ii) H(e) = G(e) if e ϵ B-A and (iii) H(e) = F(e) U G(e) if e ϵ A ∩ B, (F, A) U (G, B) = (H,C).

Definition: 2.3
The Intersection (H, C) of two soft sets (F, A) and (G,B) over a common universe U denoted (F,A) ∩ (G,B) is defined as C = A∩B and H (e) = F (e) ∩ G (e) for all e ϵ C.

Definition: 2.4
For a soft set (F, A) over the universe U, the relative complement of (F, A) is denoted by $(F,A)^c$ and is defined by $(F, A)^c = (F^c, A)$, where F^c: A → P(U) is a mapping defined by F^c (e) = U - F(e) for all e ϵ A.

Definition: 2.5
Let τ be the collection of soft sets over X, then τ is called a soft topology on X if τ satisfies
(i) ϕ, X belong to τ.
(ii) The union of any number of soft sets in τ belongs to τ.
(iii)The intersection of any two soft sets in τ belongs to τ.
The triplet (X, τ, E) is called a soft topological s pace over X and it is denoted by X (For simplicity).

Definition: 2.6
Let (X, τ, E) be soft s pace over X. A soft set (F, E) over X is said to be soft closed in X, if its relative complement $(F, E)^c$ belongs to τ. The relative complement is a mapping F^c: E→ P(X) defined by F^c (e) = X - F(e) for a ll e ϵ A.

Definition: 2.7
Let X be an initial universe set, E be the set of parameters and τ = {ϕ, X}. Then τ is called the soft indiscrete topology on X and (X, τ, E) is said to be a soft indiscrete s pace over X. If τ is the collection of all soft sets defined over X, then τ is called the soft discrete topology on X and (X, τ, E) is said to be a soft discrete s pace over X.

Definition: 2.8
Let (X, τ, E) be a soft topological s pace over X and the soft interior of (F,E) denoted by Int (F,E) is the union of all soft open subsets of (F,E). Clearly, (F,E) is the largest soft open set over X which is contained in (F,E).The soft closure of (F,E) denoted by Cl(F,E) is the inters ection of all closed sets containing (F, E). Clearly (F, E) is smalles t soft closed set containing (F, E).

Int (F, E) = U{(O, E): (O, E) is soft open and (O,E) $\tilde{\subset}$ (F, E)}.

Cl (F, E) = ∩ {(O, E): (O, E) is soft closed and (F, E) $\tilde{\subset}$ (O, E)}.

Definition: 2.9
Let U be the common universe set and E be the set of all parameters. Let (F, A) and (G, B) be soft sets over a common universe set U and A, B is a subset of E. Then (F, A) is a subset of (G, B), denoted by (F, A) $\tilde{\subset}$ (G, B). (F, A) equals (G, B), denoted by (F, A) = (G, B) if (F,A) $\tilde{\subset}$ (G,B) and (G,B) $\tilde{\subset}$ (F, A).

Definition: 2.10
A soft subset (A, E) of X is called
(i) a soft generalized closed (Soft g-closed) in a soft topological s pace (X, τ, E) if Cl(A, E) $\tilde{\subset}$ (U,E) whenever (A, E) $\tilde{\subset}$ (U,E) and (U,E) is soft open in X.
(ii) a soft s emi open if (A, E) $\tilde{\subset}$ Int(Cl(A, E))
(iii) a soft regular open if (A, E)= Int(Cl(A, E)).
(iv) a soft α-open if (A, E) $\tilde{\subset}$ Int(Cl(Int(A, E)))
(v) a soft b-open if (A, E) $\tilde{\subset}$ Cl(Int(A, E)) U Int(Cl(A, E))
(vi) a soft pre-open set if (A, E) $\tilde{\subset}$ Int (Cl(A, E)).
(vii) a soft clopen is (A, E) is both soft open and soft closed.

The complement of the soft semi open, soft regular open, soft α-open, soft b-open, soft pre-open sets are their respective soft semi closed, soft regular closed, soft α-closed, soft b-closed and soft pre -closed sets.

Definition: 2.11
The soft semi closure of (A, E) is the intersection of all soft semi closed sets containing (A, E). (i.e) The smalles t soft semi closed set containing (A,) and is denoted by sscl (A, E).
The soft semi interior of (A,E) is the union of all soft semi open set contained in (A, E) and is denoted by ss int(A, E).
Similarly, we define soft regular-closure, soft α-closure, soft pre-closure, soft semi closure and soft b -closure of the soft set (A, E) of a topological s pace X and are denoted by srcl (A, E), sαcl (A, E), spcl(A, E), sscl (A, E) and Sbcl (A, E) respectively.

Definition: 2.12
A sub set (A, E) of a soft topological s pace X is called
(i) a soft rg-closed set if Cl(A, E) $\tilde{\subset}$ (U,E) whenever (A, E) (U,E) and (U,E) is soft regular open.
(ii) a soft αg-closed set if α Cl(A, E) $\tilde{\subset}$ (U,E) whenever (A, E) (U,E) and (U,E) is soft open.
(iii) a soft gr-closed if srcl(A, E) $\tilde{\subset}$ (U,E) whenever (A, E) (U,E) and (U,E) is soft - open.

3. Soft-GSR-closed sets
Definition: 3.1 A soft subset (A, E) of a soft topological s pace *X* is called soft-gsr-closed set in X if *sscl(A, E)* $\tilde{\subset}$ *(U,E)* whenever *(A,E)* $\tilde{\subset}$ *(U,E)* and *(U,E)* is soft regular open in *X*.

The ore m: 3.1
Every soft closed set is soft-gsr-closed, converse is not true.

Example: 3.1 Let X = {a, b, c, d}, E = {e_1, e_2}
$(F,E)_1$ = {(e_1{c}), (e_2{a})}
$(F,E)_2$ = {(e_1{d}), (e_2{b})}
$(F,E)_3$ = {(e_1{c,d}), (e_2{a,b})}
$(F,E)_4$ = {(e_1{a,d}), (e_2{b,d})}
$(F,E)_5$ = {(e_1{b,c,d}), (e_2{a,b,c})}
$(F,E)_6$ = {(e_1{a,c,d}), (e_2{a,b,d})}
τ= {ϕ, X, (F_1, *E*), (F_2, *E*)……………...(F_6, *E)*} defines a soft topology on X.
Where (*A*, *E*) = {{ϕ},{a}} is soft-gsr-closed set, but not soft-closed.

The ore m: 3.2
Every soft g-closed set is soft-gsr-closed. The convers e need not be true.

Example: 3.2
Let X = {a, b,c,d}, E = {e_1, e_2}
$(F,E)_1$ = {(e_1{c}), (e_2{a})}
$(F,E)_2$ = {(e_1{d}), (e_2{b})}
$(F,E)_3$ = {(e_1{c,d}), (e_2{a,b})}
$(F,E)_4$ = {(e_1{a,d}), (e_2{b,d})}
$(F,E)_5$ = {(e_1{b,c,d}), (e_2{a,b,c})}
$(F,E)_6$ = {(e_1{a,c,d}), (e_2{a,b,d})}
τ= {ϕ, X, (F_1, *E*), (F_2, *E*)……………...(F_6, *E)*} defines a soft topology on X.
Where (*A*, *E*) = {{ϕ}, {b,d}} is soft-gsr-closed set, but not soft- closed.

Theorem: 3.3
Every soft α - closed set is soft-gsr-closed but not convers ely.

Example: 3.3
Let X = {a, b,c,d}, E = {e_1, e_2}
$(F,E)_1$ = {(e_1{c}), (e_2{a})}
$(F,E)_2$ = {(e_1{d}), (e_2{b})}
$(F,E)_3$ = {(e_1{c,d}), (e_2{a,b})}
$(F,E)_4$ = {(e_1{a,d}), (e_2{b,d})}
$(F,E)_5$ = {(e_1{b,c,d}), (e_2{a,b,c})}
$(F,E)_6$ = {(e_1{a,c,d}), (e_2{a,b,d})}
τ= {ɸ, X, (F_1, *E*), (F_2, *E*)……………...(F_6, *E)*} defines a soft topology on X.
Where (*A*, *E*) = {{ɸ}, {b}} is soft-gsr-closed set, but not soft-α closed.

Theorem: 3.4
soft s emi-closed set is soft-gsr -closed but not convers ely.

Example: 3.4
Let X = {a, b,c,d}, E = {e_1, e_2}
$(F,E)_1$ = {(e_1{c}), (e_2{a})}
$(F,E)_2$ = {(e_1{d}), (e_2{b})}
$(F,E)_3$ = {(e_1{c,d}), (e_2{a,b})}
$(F,E)_4$ = {(e_1{a,d}), (e_2{b,d})}
$(F,E)_5$ = {(e_1{b,c,d}), (e_2{a,b,c})}
$(F,E)_6$ = {(e_1{a,c,d}), (e_2{a,b,d})}
τ= {ɸ, X, (F_1, *E*), (F_2, *E*)……………...(F_6, *E)*} defines a soft topology on X.
Where (*A*, *E*) = {{ɸ}, {a,b,c}} is soft-gsr-closed set, but not soft semi closed.

The ore m: 3.5
Every soft- g - closed set is soft-gsr -closed. Convers e need not be true as seen by the following example.

Example: 3.5
Let X = {a, b, c, d}, E = {e_1, e_2}
$(F, E)_1$ = {(e_1{c}), (e_2{a})}
$(F, E)_2$ = {(e_1{d}), (e_2{b})}
$(F, E)_3$ = {(e_1{c,d}), (e_2{a,b})}
$(F, E)_4$ = {(e_1{a,d}), (e_2{b,d})}
$(F, E)_5$ = {(e_1{b,c,d}), (e_2{a,b,c})}
$(F, E)_6$ = {(e_1{a,c,d}), (e_2{a,b,d})}
τ= {ɸ, X, (F_1, *E*), (F_2, *E*)……………...(F_6, *E)*} defines a soft topology on X.
Where (*A*, *E*) = {{a,d},{ɸ}} is soft-gsr-closed set, but not soft- αg closed.

The ore m: 3.6
Every soft rg - closed set is soft- gsr - closed. Convers e need not be true as seen by the following example.

Example: 3.6
Let X = {a, b,c,d}, E = {e_1, e_2}
$(F,E)_1$ = {(e_1{c}), (e_2{a})}
$(F,E)_2$ = {(e_1{d}), (e_2{b})}
$(F,E)_3$ = {(e_1{c,d}), (e_2{a,b})}
$(F,E)_4$ = {(e_1{a,d}), (e_2{b,d})}
$(F,E)_5$ = {(e_1{b,c,d}), (e_2{a,b,c})}
$(F,E)_6$ = {(e_1{a,c,d}), (e_2{a,b,d})}
τ= {ɸ, X, (F_1, *E*), (F_2, *E*)……………...(F_6, *E)*} defines a soft topology on X.
Where (*A*, *E*) = {{a}, {b}} is soft-gsr-closed set, but not soft γg -closed.

The ore m: 3.7
Every soft πgr- closed set is soft-gsr- closed. Convers e need not be true as seen by the following example.

Example: 3.7
Let X = {a, b,c,d}, E = {e_1, e_2}
$(F,E)_1$ = {(e_1{c}), (e_2{a})}
$(F,E)_2$ = {(e_1{d}), (e_2{b})}
$(F,E)_3$ = {(e_1{c,d}), (e_2{a,b})}
$(F,E)_4$ = {(e_1{a,d}), (e_2{b,d})}
$(F,E)_5$ = {(e_1{b,c,d}), (e_2{a,b,c})}
$(F,E)_6$ = {(e_1{a,c,d}), (e_2{a,b,d})}
τ= {ɸ, X, (F_1, *E*), (F_2, *E*)……………...(F_6, *E)*} defines a soft topology on X.
Where (*A*, *E*) = {{a, d},{ɸ}} is soft-gsr-closed but not soft πgr-closed.

The ore m: 3.8
If (*A,E)* is soft regular open and soft-gsr -closed then (A, E) is soft s emi- closed.

Proof:
Assume that (A, E) is soft regular open and soft-gsr -closed.
sscl(*A*, *E*) $\widetilde{\subset}$ (A, E) (By definition).
But a lways (*A*, *E*) $\widetilde{\subset}$ *scl* (A, E).
Thus (*A*, *E*) = *s scl*(*A*,*E)*
Therefore (*A*, *E*) is soft semi –closed.

The ore m: 3.9
If (*A*, *E*) is soft-gsr-closed in X and (A, E) $\widetilde{\subset}$ (*B*,*E)* $\widetilde{\subset}$ *sscl*(*A*,*E)*. Then (*B*, *E*) is also soft-gsr- closed.

Proof:
Assume that (A, E) is soft-gsr – closed in X and (*A*,*E*) $\widetilde{\subset}$ (*B*,*E*) $\widetilde{\subset}$ *sscl*(*A*, *E*).
Let (*B*, *E*) $\widetilde{\subset}$ (*U*,*E)* and (*U*,*E*) is soft regul r open in X.
Since (A, E) $\widetilde{\subset}$ (*B*,*E)* and (*B*,*E*) $\widetilde{\subset}$ (*U*,*E*), we have (*A*,*E)* $\widetilde{\subset}$ (*U*,*E*).
Hence *sscl*(A, E) $\widetilde{\subset}$ (*U*,*E*) (s ince (*A*,*E)* is soft-gsr- closed).
Since (*B*,*E)* $\widetilde{\subset}$ *s scl*(*A*,*E)*, we have *sscl*(*B*,*E*) $\widetilde{\subset}$ *sscl*(*A*,*E)* (*U*,*E*).
Therefore, (*B*, *E*) is soft-gsr – closed.

Theorem: 3.10
Let (*A*,*E)* be soft-gsr-closed set in X. Then ss cl(*A*,*E)* - (*A*,*E)* contain only null soft regular closed set.

Proof:
Suppos e that (A, E) be soft-gsr-closed in X.
Let (*H*,*E)* be soft reguar closed of ss cl(A, E) $\widetilde{\subset}$ (*A*,*E)*. Then (*H*,*E*) $\widetilde{\subset}$ ss cl(A, E) *A*,*E*) and s o (A, E) $\widetilde{\subset}$ (*H*,*E)*.
Since (*A*,*E)* is soft-gsr-closed and X - (*H*,*E*) is soft regular open. ss cl(*A*,*E)* $\widetilde{\subset}$ X - (*H*,*E)*.
cons equently (*H*,*E)* $\widetilde{\subset}$ X - s s cl(A, E).
We have (*H*,*E*) $\widetilde{\subset}$ s s cl(A, E).
Hence we obtain (*H*,*E)* $\widetilde{\subset}$ s cl(A, E) ∩ (X - s s cl(A, E)) = ɸ.
This implies (*H*,*E)* = ɸ.
Therefore, s s cl(*A*,*E)* - (A, E) contains only null soft regular closed set.

Remark: 3.1
The converse of the above theorem does not hold as shown in the following example.

Example: 3.8
Let X = {a, b, c, d}, E = {e $_1$, e$_2$}.
Let us take the soft topology on X and the soft set (A, E)={{b},{c}}. Sscl (*A,E)* - (*A, E*) contains only null soft regular closed set. But (A, E) is not a soft-gsr-closed set in (X,).

The ore m: 3.28 Let (X, E,) be a soft topological s pace over X and (A, E) be soft-gsr – closed in X. (*A,E)* is soft semi-closed if and only if s s cl(*A,E)* - (*A, E*) is soft regular closed.

Proof:
Let (*A, E*) be soft-gsr - closed.
If (*A, E*) is soft s emi- closed then ss cl(A, E) = (A, E). s s cl(A, E) - (*A,E)* = ϕ
which is soft regular closed.
Convers ely, Suppos e that ss cl(*A,E)* - (A, E) is soft regular closed.
Since (*A,E)* be soft-gsr-closed, then ss cl(A, E) - (*A,E)* = ϕ.
That is ss cl(*A,E)* = (*A, E*).
Hence (*A,E)* is soft semi- closed.

4. Soft GSR – Open Sets

Definition: 4. 1 Let (X, E, τ) be a soft topological s pace over X. A soft set (*A, E*) is called s of tgsr -open set in X if the relative complement (A, E) ^{c} is s of tgsr – closed.

Theorem: 4.1
A soft set (A, E) is soft gsr-open set in a soft topological s pace X if and only if
(*H,E*) $\breve{\subseteq}$ s s int(A, E) whenever (*H,E*) is soft regular closed in X and (*H,E)* $\breve{\subseteq}$ (*A,E).*

Proof:
Suppose that (*H,E)* is soft regular closed and (*H,E)* (*A,E)* imp lies (*H,E)* $\breve{\subseteq}$ s s int(A, E).
Let (A, E) ^{c} $\breve{\subseteq}$ (*U,E)*, where (*U,E*) is soft regular open.
Then (U,E) ^{c} $\breve{\subseteq}$ (*A, E*) where is soft regular closed.
By hypothes is (U,E) ^{c} $\breve{\subseteq}$ s s int(A, E).
That is (ssint(A, E))^c $\breve{\subseteq}$ (*U,E).*
Equivalently (s s cl(A, E))^c $\breve{\subseteq}$ (*U,E).*
Thus (A, E) ^{c} is soft-gsr-closed. Hence we obtain (*A, E*) is soft- gsr-open.
Convers ely, Suppos e that (A, E) is soft gsr-open, (*H,E*) $\breve{\subseteq}$ (*A,E)* and (*H,E*) is soft regular closed.
Then (*H,E)*^{c} is soft regular open.
Then (A, E) ^{c} $\breve{\subseteq}$ (*H,E)*^{c}. Since (A, E) ^{c} is soft-gsr-closed.
Hence ((sscl(A, E)^c $\breve{\subseteq}$ (H,E)^c
Therefore (H,E) $\breve{\subseteq}$ ((sscl(A, E)^c)^c = ssint(A, E).

The ore m: 4.2 Let (X, τ) be a soft topological s pace and (A, E) be a soft set over X. If a soft set (A, E) is soft-gsr-closed in x then ss cl(*A,E)* - (*A, E*) is soft-gsr- open.

Re mark: 4.1 Revers e implication does not hold as shown in the following example.

Example: 4.1
Let X = {a, b, c, d}, E = {e$_1$, e$_2$}. Consider the the soft topology τ on X and the soft set (A, E) ={{b},{c}}. s s cl(A, E)- (*A,E)* soft gsr-open set. But (A, E) is not a soft gsr-closed set in(X, τ).

Theorem: 4.3
If (*A,E)* is soft-gsr -open in X and sint (A, E) $\breve{\subseteq}$ (A, E) $\breve{\subseteq}$ (B,*E*) then (*B,E*) is soft-gsr-open set.

Proof:
Suppose (A, E) is soft-gsr -open in X and s int(A, E) $\breve{\subseteq}$ (A, E) $\breve{\subseteq}$ (B,*E)*
Let (*H,E)* $\breve{\subseteq}$ and (*H,E*) is soft regular closed in X.
Since (*B,E*) $\breve{\subseteq}$ (A, E) and (*H,E*) $\breve{\subseteq}$ (*B,E*) s o we have(*H,E*) $\breve{\subseteq}$ (A, E).
Hence (*H,E*) $\breve{\subseteq}$ s int(A, E). and s int(A, E) $\breve{\subseteq}$ (*B,E*). Therefore (*B,E*) is soft-gsr-open.

Theorem: 4.4
The intersection of two soft- gsr- open sets is again a soft-gsr-open set.

5. Conclusion

The concept of soft set is very rich in both theoretical and experimental. Many parameters have been investigated by different authors. Further the authors proposed to introduce new parameters in soft closed and soft open sets.

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