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Peristaltic flow of a carreau fluid model through porous medium in an asymmetric channel

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Abstract
There is a vivid display of Mathematical modeling and analytical solution in the system of flowing an incompressible Carreau fluid through an asymmetric channel in the porosity. Diverse amplitudes and phase are found in the peristaltic wave train on the channel walls. A long wavelength approximation plays an important role in the solution of the flow problems. The clear forms of the axial pressure gradient, pressure drop and frictional force can be obtained over a wavelength by using a perturbation technic for a small Weissenberg number. In this process the main focus is laid on the effect of various parameters on the pumping characteristics. Besides this it is found that the size of the trapped bolus would react inversely in conformity to the increament in the magnetic numbers.

Keywords: carreau fluid, porous medium

Introduction

The system of pumping is the most common form of fluid transportation. The fluid transportation take place mainly due to the differences in pressure. This kind of fluid transportation takes place from low pressure region to high pressure region. This can be achieved in a region where there is a cross sectional area along the passage. Peristaltic transportation of fluids in biological bodies is a natural phenomenon.

Latham (1966) [3] is one of the earliest investigators of the study of peristalsis mechanism in relation to mechanical pumping. The earlier researchers confined their study to. The symmetric channels. Their attention was not tuned towards the study of peristaltic flow through non-symmetric channels. In the recent times, the research work by physiologists proved that the intra-uterine fluid flow through myometrical channel can be possible in both ways namely through the symmetric and non-symmetric channels.

The progressive waves which more independently on the outer and inner walls of the channel are considered to be out-of-phase. They bring about intra uterine fluid motion in a sagittal cross section of the uterus. The research theories and hypothesis put for forth by Ramachandra Rao (1995) [7] and Usha (1995) [7], Jaffin and Shapiro (1971), Brasseur *et al*. (1987) [1], Srivastava (1984) [10], Provost and Schwarz (1994) [5], Shukla and Gupta (1982) [9], and Vajravelu (2005) [11] reflect on the flow of viscous fluids through passages neglecting the permeability of the flexible walls under peristaltic motion. A study carried out by Mishra *et al*. (2004) says that the progressive wave set up along the channel has a long wavelength in a two-dimensional non-symmetric tube.

Now, It is proposed to consider the incompressible Carreau fluid model for the peristaltic transport of a non-Newtonian fluid in an asymmetric channel through porous medium.

Carreau fluid model

The constitute equation for the Carreau fluid model is

$$\tau = [\eta_\infty + (\eta_0 - \eta_\infty)(1 + (\Gamma \dot{\gamma})^2)^{\frac{n-1}{2}}]\dot{\gamma}$$

where τ is the extra stress tensor, η_0 is the zero shear rate viscosity, η_∞ is the infinite shear rate viscosity, Γ is the time constant, n is the dimensional less power law index and

$$\dot{\gamma} = \sqrt{\frac{1}{2}\sum_i\sum_j \dot{\gamma}_{ij}\dot{\gamma}_{ij}} = \sqrt{\frac{1}{2}\pi}$$

where π is the second invariant of strain-rate tensor. For $\eta_\infty = 0$, then we get

$$\tau = -\eta_0(1+(\Gamma\dot{\gamma})^2)^{\frac{n-1}{2}}\dot{\gamma}$$

Mathematical formulation of the problem

Let us consider the peristaltic transport of an incompressible Carreau fluid flow generated by the sinusoidal waves moving with constant speed c along a two dimensional channel of width $d_1 + d_2$.

The wall geometries are given by

$$h_1(\bar{X},\bar{t}) = \bar{d}_1 + \bar{a}_1 \sin\left[\frac{2\pi}{\lambda}(\bar{X} - c\bar{t})\right] \quad \text{(upper wall)} \tag{1}$$

$$h_2(\bar{X},\bar{t}) = -\bar{d}_2 - \bar{b}_1 \sin\left[\frac{2\pi}{\lambda}(\bar{X} - c\bar{t} + \phi)\right] \quad \text{(lower wall)} \tag{2}$$

in which $\bar{a}_1$ and $\bar{b}_1$ are amplitudes of the waves, λ is the wave length and c is the wave speed, $\phi(0 \le \phi \le \pi)$ is the phase difference, $\bar{X}$ and $\bar{Y}$ are the rectangular coordinates with $\bar{X}$ measured along the axis of the channel and $\bar{Y}$ perpendicular to $\bar{X}$.

Let $(\bar{U},\bar{V})$ be the velocity components in the fixed wave frame of reference $(\bar{X},\bar{Y})$.

It should be noted that $\phi = 0$ corresponds to symmetric channel with waves out of the phase and $\phi = \pi$ the waves are in phase and further $\bar{a}_1, \bar{b}_1, \bar{d}_1, \bar{d}_2$ and ϕ satisfies the condition, $\bar{a}_1^2 + \bar{b}_1^2 + 2\bar{a}_1\bar{b}_1\cos\phi \le (\bar{d}_1 + \bar{d}_2)^2$.

In the laboratory frame $(\bar{X},\bar{Y})$ the flow is unsteady. However, if observed in a coordinate system moving at the wave speed c wave frame $(\bar{x},\bar{y})$ it can be treated as steady. The coordinates and velocities in the two frames are

$$\bar{x} \equiv \bar{X} - c\bar{t},\ \bar{y} = \bar{Y},\ \bar{u} = \bar{U} - c,\ \bar{v} = \bar{V} \tag{3}$$

where $\bar{u}$ and $\bar{v}$ are the velocity components in the wave frame.

The equations of motion governing the flow are given by

$$\frac{\partial \bar{U}}{\partial \bar{X}} + \frac{\partial \bar{V}}{\partial \bar{Y}} = 0 \tag{4}$$

$$\rho\left(\frac{\partial \bar{U}}{\partial \bar{t}} + \bar{U}\frac{\partial \bar{U}}{\partial \bar{X}} + \bar{V}\frac{\partial \bar{V}}{\partial \bar{Y}}\right) = -\frac{\partial \bar{P}}{\partial \bar{X}} - \frac{\partial \bar{\tau}_{\bar{X}\bar{X}}}{\partial \bar{X}} - \frac{\partial \bar{\tau}_{\bar{X}\bar{Y}}}{\partial \bar{Y}} \tag{5}$$

$$\rho\left(\frac{\partial \bar{V}}{\partial \bar{t}} + \bar{U}\frac{\partial \bar{V}}{\partial \bar{X}} + \bar{V}\frac{\partial \bar{V}}{\partial \bar{Y}}\right) = -\frac{\partial \bar{P}}{\partial \bar{Y}} - \frac{\partial \bar{\tau}_{\bar{X}\bar{Y}}}{\partial \bar{X}} - \frac{\partial \bar{\tau}_{\bar{Y}\bar{Y}}}{\partial \bar{Y}} \tag{6}$$

Consider the following non-dimensional quantities.

$$x = \frac{\bar{x}}{\lambda},\ y = \frac{\bar{y}}{d_1},\ u = \frac{\bar{u}}{c},\ v = \frac{\bar{v}}{c},\ t = \frac{c}{\lambda}\bar{t},$$

$$h_1 = \frac{\bar{h}_1}{\bar{d}_1}, h_2 = \frac{\bar{h}_2}{\bar{d}_1}, \tau_{XX} = \frac{\lambda}{\eta_0 c}\bar{\tau}_{\overline{XX}},\ \tau_{XY} = \frac{\bar{d}_1}{\mu_0 c}\bar{\tau}_{\overline{XY}},\ \tau_{YY} = \frac{\bar{d}_1}{\eta_0 c}\bar{\tau}_{\overline{YY}},$$

$$\delta = \frac{\bar{d}_1}{\lambda},\ \mathrm{Re} = \frac{\rho c \bar{d}_1}{\eta_0}, We = \frac{\Gamma c}{\bar{d}_1},\ p = \frac{\bar{d}_1^{\,2}}{c\lambda\eta_0}\bar{p},\ \gamma = \frac{\bar{\gamma}\bar{d}_1}{c} \tag{7}$$

In view of the Equation (7), the Equations (5) and (6), after dropping bars, reduce to

$$\delta\,\mathrm{Re}\left(u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} - \delta^2\frac{\partial \tau_{xx}}{\partial x} - \frac{\partial \tau_{xy}}{\partial y} - \left(\frac{1}{Da} + M^2\right)(u+1) \tag{8}$$

$$-\delta^3\,\mathrm{Re}\left[\left(u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y}\right)\right] = -\frac{\partial p}{\partial y} - \delta^2\frac{\partial \tau_{xy}}{\partial x} - \frac{\partial \tau_{yy}}{\partial y} \tag{9}$$

Where

$$\tau_{xx} = -2\left[1 + (\frac{n-1}{2})We^2\dot{\gamma}^2\right]\frac{\partial u}{\partial x}$$

$$\tau_{xy} = -\left[1 + (\frac{n-1}{2})We^2\dot{\gamma}^2\right]\left(\frac{\partial u}{\partial y} + \delta^2\frac{\partial v}{\partial x}\right)$$

$$\tau_{yy} = -2\left[1 + (\frac{n-1}{2})We^2\dot{\gamma}^2\right]\frac{\partial v}{\partial y}$$

$$\dot{\gamma} = \left[2\delta^2(\frac{\partial u}{\partial x})^2 + \left(\frac{\partial u}{\partial y} + \delta^2\frac{\partial v}{\partial x}\right) + 2\delta^2(\frac{\partial v}{\partial y})^2\right]^{\frac{1}{2}}$$

here Da, M^2, δ, Re and We represent the Darcy number, Magnetic parameter, the wave number, Reynolds number and Weissenberg numbers respectively.

Under the assumptions of long wave length and low Reynolds number, neglecting the terms of order δ and higher in equations (8) and (9) then the form is

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial y}\left[\left(1 + \frac{n-1}{2}we^2\left(\frac{\partial u}{\partial y}\right)^2\right)\frac{\partial u}{\partial y}\right] - \left(\frac{1}{Da} + M^2\right)(u+1) \tag{10}$$

$$\frac{\partial p}{\partial y} = 0 \tag{11}$$

Corresponding non-dimensional boundary conditions are

$$u = -1 \quad \text{at} \quad y = h_1(x) \tag{12}$$

$$u = -1 \quad \text{at} \quad y = h_2(x) \tag{13}$$

The volume flow rate q

$$q=\int_{h_1}^{h_2} u dy$$

The instantaneous flow $Q(x,t)$ in a fixed frame

$$Q=\int_{h_1}^{h_2} u dy=\int_{h_1}^{h_2}(u+1)dy$$

$$Q=q+(h_1-h_2)$$

The time average flux $\bar{Q}$ over one period of the peristaltic wave is

$$\bar{Q}=\frac{1}{T}\int_0^T Q dT$$

Perturbation Solution
We used the Perturbation method to find the solution.
Consider the following expansions

$$u=u_0+We^2 u_1+o\left(We^3\right) \tag{14}$$

$$p=p_0+We^2 p_1+o\left(We^3\right) \tag{15}$$

$$q=q_0+We^2 q_1+o\left(We^3\right) \tag{16}$$

By using Eqns. (14)-(16) in Eqns.(10)-(13),we resolve the problem into zeroth order and first order as discussed below
Zeroth order (We^0)

$$\frac{\partial^2 u_0}{\partial y^2}-\left(\frac{1}{Da}+M^2\right)\left(u_0+1\right)=\frac{\partial p_0}{\partial x} \tag{17}$$

$$u_0=-1 \text{ at } y=h_1(x) \tag{18}$$

$$u_0=-1 \text{ at } y=h_2(x) \tag{19}$$

First order (We^1)

$$\frac{\partial^2 u_1}{\partial y^2}+3\left(\frac{n-1}{2}\right)\left(\frac{\partial^2 u_0}{\partial y^2}\right)\left(\frac{\partial u_0}{\partial y}\right)^2-\left(\frac{1}{Da}+M^2\right)u_1=\frac{\partial p_1}{\partial x} \tag{20}$$

$$u_1=-1 \quad \text{at} \quad y=h_1(x) \tag{21}$$

$$u_1=-1 \quad \text{at} \quad y=h_2(x) \tag{22}$$

Zeroth order Solution
On solving Eqn. (17), along with boundary conditions Eqns. (18) and (19), we get

$$u_0=c_1\cosh\left(\sqrt{A}y\right)+c_2\sinh\left(\sqrt{A}y\right)-\left(\frac{1}{A}\frac{\partial p_0}{\partial x}+1\right) \tag{23}$$

where $A=\frac{1}{Da}+M^{2}$

$$c_1=\left(\frac{1}{A}\frac{\partial p_0}{\partial x}\right)\left(\frac{\sinh\left(\sqrt{A}h_2\right)-\sinh\left(\sqrt{A}h_1\right)}{\sinh\left(\sqrt{A}\left(h_2-h_1\right)\right)}\right) \tag{24}$$

$$c_2=-\left(\frac{1}{A}\frac{\partial p_0}{\partial x}\right)\left(\frac{\cosh\left(\sqrt{A}h_2\right)-\cosh\left(\sqrt{A}h_1\right)}{\sinh\left(\sqrt{A}\left(h_2-h_1\right)\right)}\right) \tag{25}$$

The volume flow rate of zeroth order is given by,

$$q_0=\int_{h_2}^{h_1}\left(u_0+1\right)dy=\frac{\partial p_0}{\partial x}\frac{1}{A\sqrt{A}}\left[\frac{2-2\cosh\left(\sqrt{A}\left(h_1-h_2\right)\right)+\sqrt{A}\left(h_1-h_2\right)\sinh\left(\sqrt{A}\left(h_1-h_2\right)\right)}{\sinh\left(\sqrt{A}\left(h_2-h_1\right)\right)}\right] \tag{26}$$

From Eqn. (26), we get

$$\frac{\partial p_0}{\partial x}=\frac{A\sqrt{A}\sinh\left(\sqrt{A}\left(h_2-h_1\right)\right)q_0}{2-2\cosh\left(\sqrt{A}\left(h_1-h_2\right)\right)+\sqrt{A}\left(h_1-h_2\right)\sinh\left(\sqrt{A}\left(h_1-h_2\right)\right)} \tag{27}$$

First order Solution

On solving equation (20) along with boundary conditions given by Eqn.(21) and Eqn.(22), we get

$$u_1=c_3\cosh\left(\sqrt{A}y\right)+c_4\sinh\left(\sqrt{A}y\right)-\frac{1}{A}\frac{\partial p_1}{\partial x}-\frac{3(n-1)}{2}\left[\frac{\left(c_1^3+3c_1c_2^2\right)A}{32}\cosh\left(3\sqrt{A}y\right)+\frac{\left(c_2^3+3c_2c_1^2\right)A}{32}\sinh\left(3\sqrt{A}y\right)\right]$$

$$-\frac{3(n-1)}{2}\left[-\frac{\left(3c_2c_1^2\right)A}{8}y\cosh\left(\sqrt{A}y\right)+\frac{\left(3c_1c_2^2A^{\frac{1}{2}}-\left(c_1^3+2c_1c_2^2\right)+\left(c_2^3+2c_2c_1^2\right)\right)A}{8}y\sinh\left(\sqrt{A}y\right)\right] \tag{28}$$

Where

$$c_3=\left(\frac{1}{A}\frac{\partial p_1}{\partial x}\right)\left(\frac{\sinh\left(\sqrt{A}h_2\right)-\sinh\left(\sqrt{A}h_1\right)}{\sinh\left(\sqrt{A}\left(h_2-h_1\right)\right)}\right)+\frac{3(n-1)}{2}\left(\frac{X_1\sinh\left(\sqrt{A}h_2\right)-X_2\sinh\left(\sqrt{A}h_1\right)}{\sinh\left(\sqrt{A}\left(h_2-h_1\right)\right)}\right) \tag{29}$$

$$c_4=-\left(\frac{1}{A}\frac{\partial p_1}{\partial x}\right)\left(\frac{\cosh\left(\sqrt{A}h_2\right)-\cosh\left(\sqrt{A}h_1\right)}{\sinh\left(\sqrt{A}\left(h_2-h_1\right)\right)}\right)-\frac{3(n-1)}{2}\left(\frac{X_1\cosh\left(\sqrt{A}h_2\right)-X_2\cosh\left(\sqrt{A}h_1\right)}{\sinh\left(\sqrt{A}\left(h_2-h_1\right)\right)}\right) \tag{30}$$

$$X_1=\frac{\left(c_1^3+3c_1c_2^2\right)A}{32}\cosh\left(3\sqrt{A}h_1\right)+\frac{\left(c_2^3+3c_2c_1^2\right)A}{32}\sinh\left(3\sqrt{A}h_1\right)-\frac{\left(3c_2c_1^2\right)A}{8}h_1\cosh\left(\sqrt{A}h_1\right)$$

$$+\frac{\left(3c_1c_2^2A^{\frac{1}{2}}-\left(c_1^3+2c_1c_2^2\right)+\left(c_2^3+2c_2c_1^2\right)\right)A}{8}h_1\sinh\left(\sqrt{A}h_1\right)$$

$$X_2 = \frac{\left(c_1^3 + 3c_1c_2^2\right)A}{32}\cosh\left(3\sqrt{A}h_2\right) + \frac{\left(c_2^3 + 3c_2c_1^2\right)A}{32}\sinh\left(3\sqrt{A}h_2\right) - \frac{\left(3c_2c_1^2\right)A}{8}h_2\cosh\left(\sqrt{A}h_2\right)$$
$$+\frac{\left(3c_1c_2^2A^{\frac{1}{2}} - \left(c_1^3 + 2c_1c_2^2\right) + \left(c_2^3 + 2c_2c_1^2\right)\right)A}{8}h_2\sinh\left(\sqrt{A}h_2\right)$$

The volume flow rate of first order is given by,

$$q_1 = \int_{h_2}^{h_1}(u_1+1)dy = -\frac{\partial p_1}{\partial x}\frac{1}{A}\left[\frac{\left[\sinh\left(\sqrt{A}h_1\right) - \sinh\left(\sqrt{A}h_2\right)\right]^2}{\sqrt{A}\sinh\left(\sqrt{A}(h_2-h_1)\right)}\right] + \frac{3(n-1)}{2\sqrt{A}}\left[\frac{\left[X_1\sinh\left(\sqrt{A}h_2\right) - X_2\sinh\left(\sqrt{A}h_1\right)\right]}{\sinh\left(\sqrt{A}(h_2-h_1)\right)}\right]$$
$$\left[\sinh\left(\sqrt{A}h_1\right) - \sinh\left(\sqrt{A}h_2\right)\right] + \frac{\partial p_1}{\partial x}\frac{1}{A}\left[\frac{\left[\cosh\left(\sqrt{A}h_1\right) - \cosh\left(\sqrt{A}h_2\right)\right]^2}{\sqrt{A}\sinh\left(\sqrt{A}(h_2-h_1)\right)}\right] - \tag{31}$$
$$\frac{3(n-1)}{2\sqrt{A}}\left[\frac{\left[X_1\cosh\left(\sqrt{A}h_2\right) - X_2\cosh\left(\sqrt{A}h_1\right)\right]}{\sinh\left(\sqrt{A}(h_2-h_1)\right)}\right]\left[\cosh\left(\sqrt{A}h_1\right) - \cosh\left(\sqrt{A}h_2\right)\right] + \frac{\partial p_1}{\partial x}\frac{1}{A}(h_2-h_1) + B$$

$$B = -\frac{3(n-1)}{2}\left[\frac{\left(c_1^3 + 3c_1c_2^2\right)A}{32}\right]\left[\frac{\left[\sinh\left(3\sqrt{A}h_1\right) - \sinh\left(3\sqrt{A}h_2\right)\right]}{3\sqrt{A}}\right] -$$
$$\frac{3(n-1)}{2}\left[\frac{\left(c_2^3 + 3c_2c_1^2\right)A}{32}\right]\left[\frac{\left[\cosh\left(3\sqrt{A}h_1\right) - \cosh\left(3\sqrt{A}h_2\right)\right]}{3\sqrt{A}}\right] -$$
$$\frac{3(n-1)}{2}\left[\frac{\left(3c_1c_2^2A^{\frac{1}{2}} - \left(c_1^3 + 2c_1c_2^2\right) + \left(c_2^3 + 2c_2c_1^2\right)\right)A}{8}\right]$$
$$\left[\frac{\left[h_1\cosh\left(\sqrt{A}h_1\right) - h_2\cosh\left(\sqrt{A}h_2\right)\right]}{\sqrt{A}}\right] +$$
$$\frac{3(n-1)}{2A}\left[\frac{\left(3c_1c_2^2A^{\frac{1}{2}} - \left(c_1^3 + 2c_1c_2^2\right) + \left(c_2^3 + 2c_2c_1^2\right)\right)A}{8}\right]$$

Where $\left[\sinh\left(\sqrt{A}h_1\right) - \sinh\left(\sqrt{A}h_2\right)\right] + \frac{3(n-1)}{2}\left[\frac{\left(3c_2c_1^2\right)A}{8}\right]\left[\frac{\left[h_1\sinh\left(\sqrt{A}h_1\right) - h_2\sinh\left(\sqrt{A}h_2\right)\right]}{\sqrt{A}}\right]$

$$-\frac{3(n-1)}{2A}\left[\frac{\left(3c_2c_1^2\right)A}{8}\right]\left[\cosh\left(\sqrt{A}h_1\right) - \cosh\left(\sqrt{A}h_2\right)\right]$$

From equation (31), we get

$$\frac{\partial p_1}{\partial x} = \frac{A\left[(q_1 - B)\sqrt{A}\sinh\left(\sqrt{A}(h_2-h_1)\right) - X_3 + X_4\right]}{2 - 2\cosh\left(\sqrt{A}(h_1-h_2)\right) + \sqrt{A}(h_1-h_2)\sinh\left(\sqrt{A}(h_1-h_2)\right)} \tag{32}$$

Where

$$X_3 = \frac{3(n-1)}{2}\left[X_1\sinh\left(\sqrt{A}h_2\right) - X_2\sinh\left(\sqrt{A}h_1\right)\right]\left[\sinh\left(\sqrt{A}h_1\right) - \sinh\left(\sqrt{A}h_2\right)\right]$$

$$X_4 = \frac{3(n-1)}{2}\left[X_1 \cosh\left(\sqrt{A}h_2\right) - X_2 \cosh\left(\sqrt{A}h_1\right)\right]\left[\cosh\left(\sqrt{A}h_1\right) - \cosh\left(\sqrt{A}h_2\right)\right]$$

Substituting the equations (27) and (32) in the relation $\frac{\partial p}{\partial x} = \frac{\partial p_0}{\partial x} + we^2 \frac{\partial p_1}{\partial x}$ and neglecting the terms greater than $O\left(we^2\right)$, we get

$$\frac{\partial p}{\partial x} = \frac{A\sqrt{A}\sinh\left(\sqrt{A}(h_2 - h_1)\right)\left(q - we^2 B\right) + we^2 A(X_4 - X_3)}{2 - 2\cosh\left(\sqrt{A}(h_1 - h_2)\right) + \sqrt{A}(h_1 - h_2)\sinh\left(\sqrt{A}(h_1 - h_2)\right)} \tag{33}$$

The dimensionless pressure rise per one wave length in the wave frame is given by

$$\Delta p = \int_{h_2}^{h_1} \frac{\partial p}{\partial x} dx = \int_{h_2}^{h_1} \left[\frac{A\sqrt{A}\sinh\left(\sqrt{A}(h_2 - h_1)\right)\left(q - we^2 B\right) + we^2 A(X_4 - X_3)}{2 - 2\cosh\left(\sqrt{A}(h_1 - h_2)\right) + \sqrt{A}(h_1 - h_2)\sinh\left(\sqrt{A}(h_1 - h_2)\right)}\right] dx \tag{34}$$

The frictional forces, at $y = h_1$ and $y = h_2$ are

$$F_{\lambda_1} = \int_0^1 - h_1^2 \left(\frac{dp}{dx}\right) dx,$$

$$F_{\lambda_2} = \int_0^1 - h_2^2 \left(\frac{dp}{dx}\right) dx, \tag{35}$$

Results and Discussion

We made the calculation of the pressure rise Δp as a function of mean flow $\bar{Q}$ for different values of a, b, d, Da, ϕ and M for the non-Newtonian fluid $(n = 1.4)$. The observation showed gradual fall of the pressure rise in conformity to the increment in flow rate. It is also came to our notice that for a fixed flow rate the pressure rise comes down as the values of a, b, d, Da, ϕ and M goes up. Further more for a given Δp the increment in the values of a, b, d, Da, ϕ and M decrease the flow rate in pumping region $(\Delta p > 0)$.

It can be observed that the Figs. (7)- (11) are drawn to study the effects of the pressure gradient with “x” for diffrent values of a, b, d, M, and We. It can be seen from there that the magnitude of pressure gradient increases with increasing “x” value upto x=0.48 and after that it decreases, that is, the maximum pressure gradient occurs at x = 0.48.

Also observed that the Figs. (12)- (13) are drawn to study the effects of the frictional force with mean flow for different values of a. We observe that the frictional force shows opposite behaviour to that pressure rise for the corresponding variation in a.

Bolus is the volume of fluid trapped within a streamline and this phenomenon is called trapping. It is clear that no trapping occurs at the lower limit of flow rate. This shows that without sufficient volume of the fluid flowing per unit time through the channel, trapping is unlikely to occur.

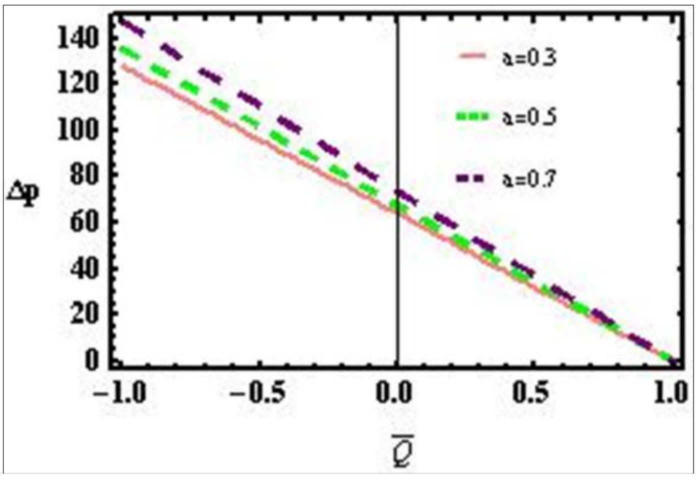

Fig 1: Variation of pressure rise Δp with $\bar{Q}$ for different values $"a"$ of when $n = 1.4$

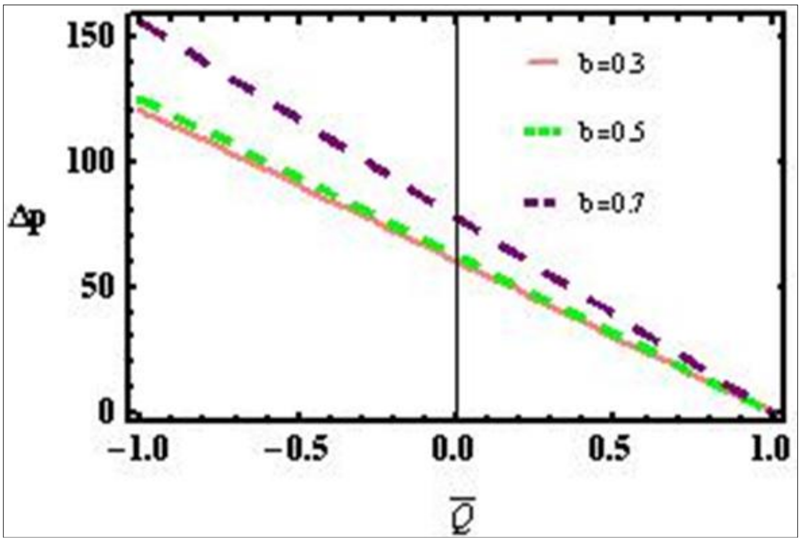

Fig 2: Variation of pressure rise Δp with $\bar{Q}$ for different values "b" of when $n = 1.4$

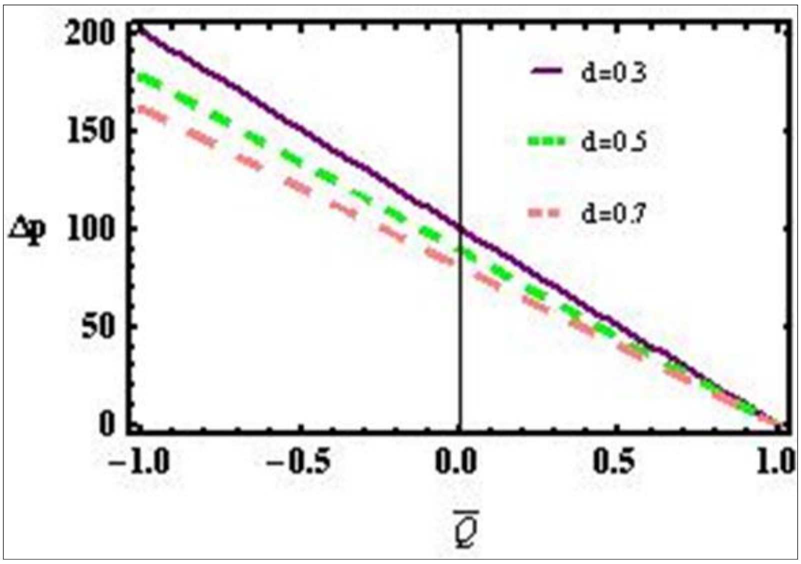

Fig 3: Variation of pressure rise Δp with $\bar{Q}$ for different values of "d" of when $n = 1.4$

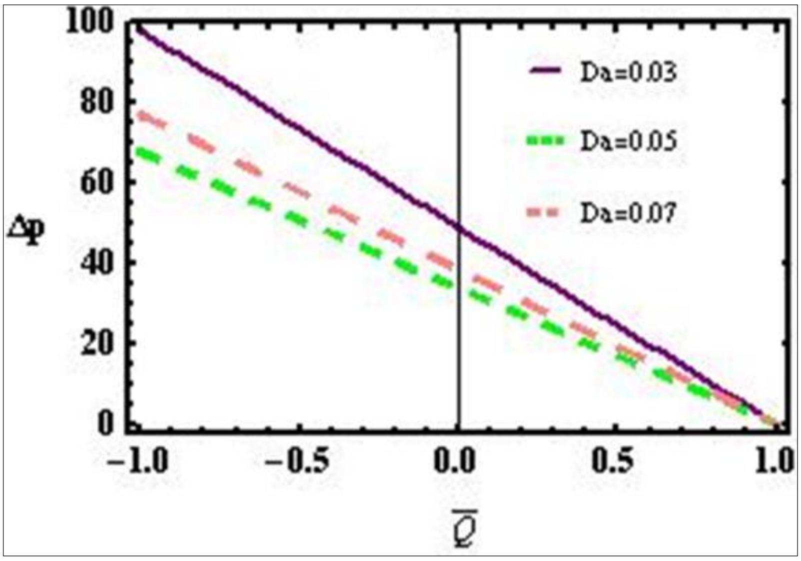

Fig 4: Variation of pressure rise Δp with $\bar{Q}$ for different values of "Da" of when $n = 1.4$

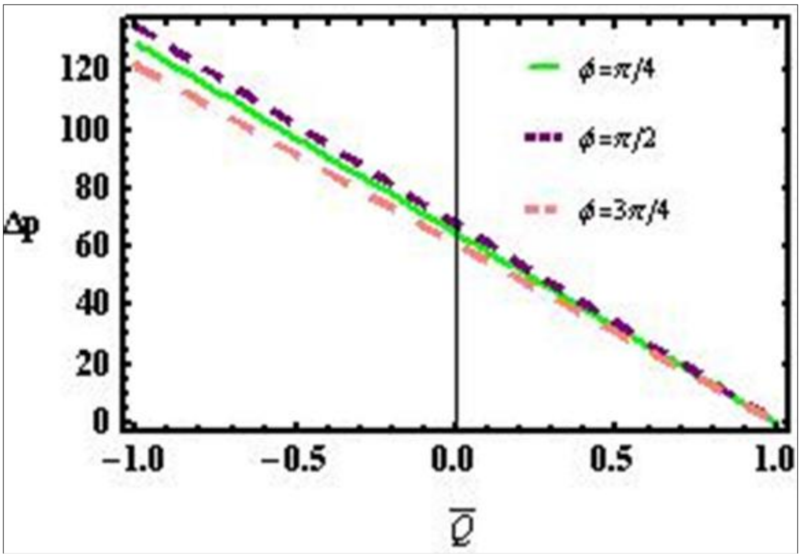

Fig 5: Variation of pressure rise Δp with $\bar{Q}$ for different values of "ϕ" of when $n=1.4$

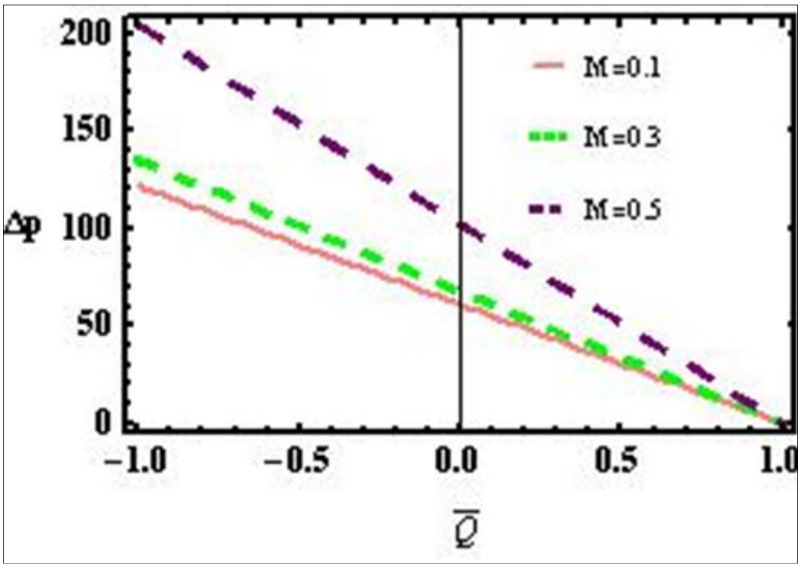

Fig 6: Variation of pressure rise Δp with $\bar{Q}$ for different values of " M " of when $n=1.4$.

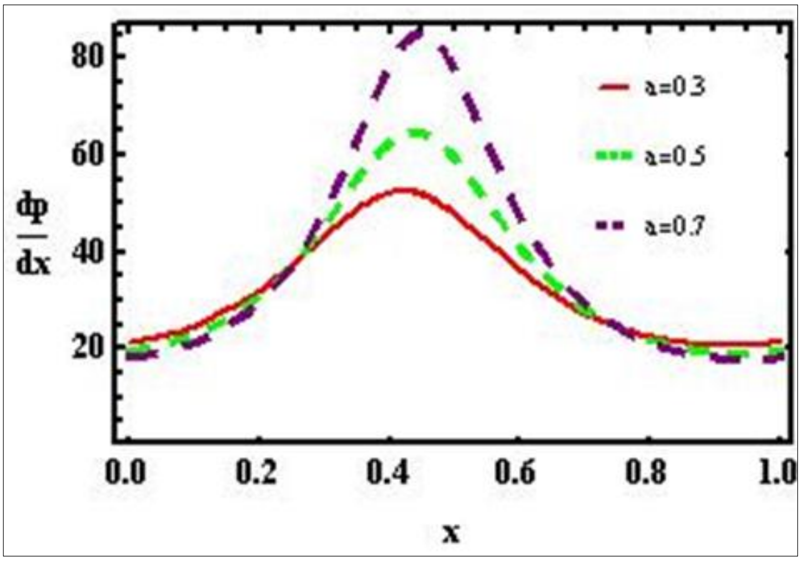

Fig 7: Variation of pressure gradient dp/dx with x different values of "a".

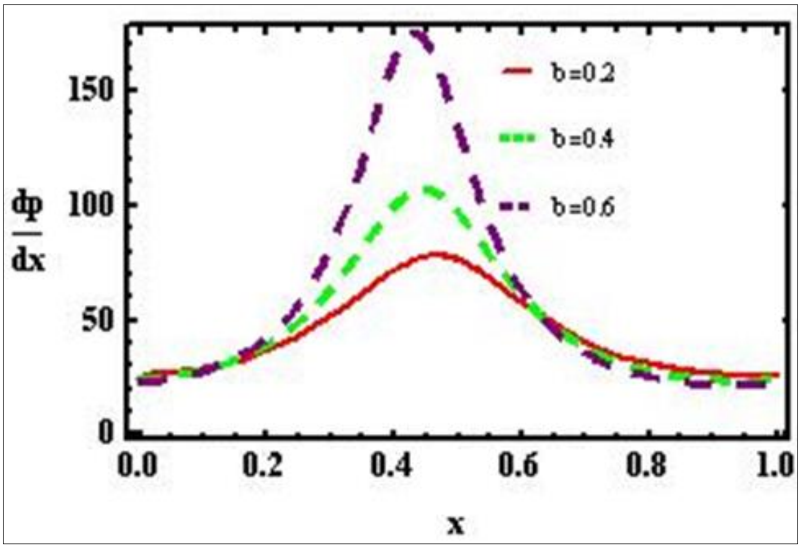

Fig 8: Variation of pressure gradient dp/dx with x different values of "b".

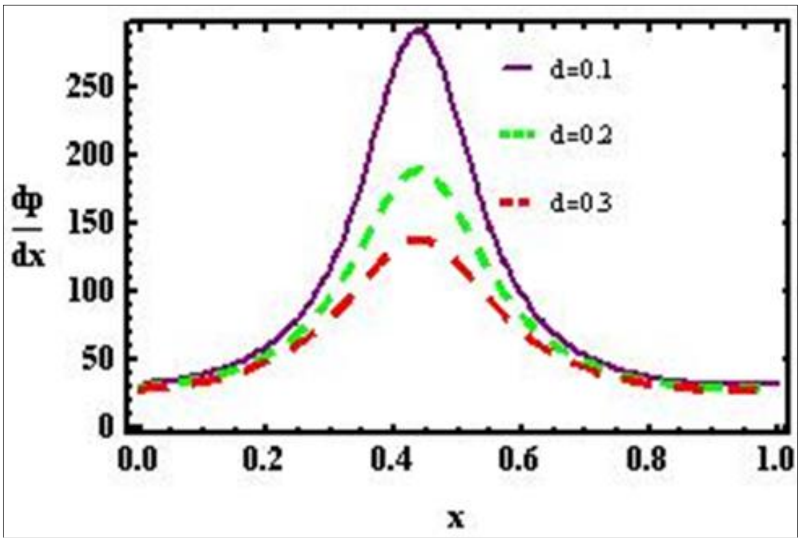

Fig 9: Variation of pressure gradient dp/dx with x different values of "d"

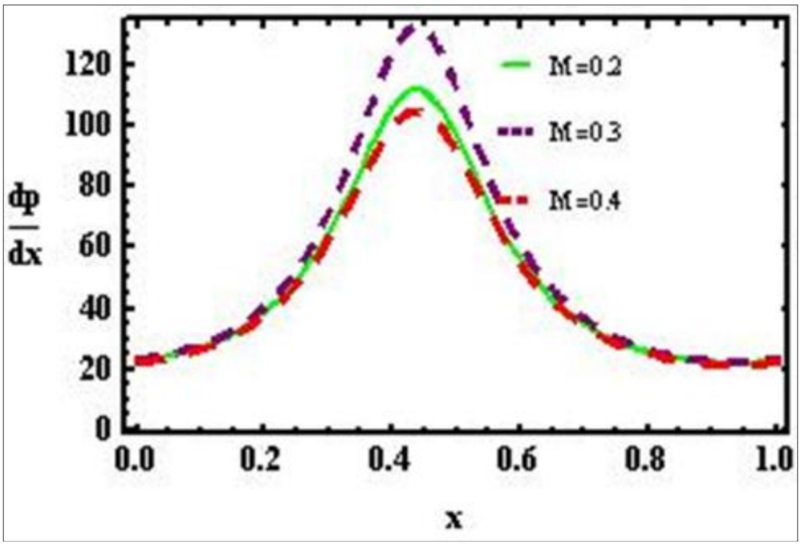

Fig 10: Variation of pressure gradient dp/dx with x different values of "M".

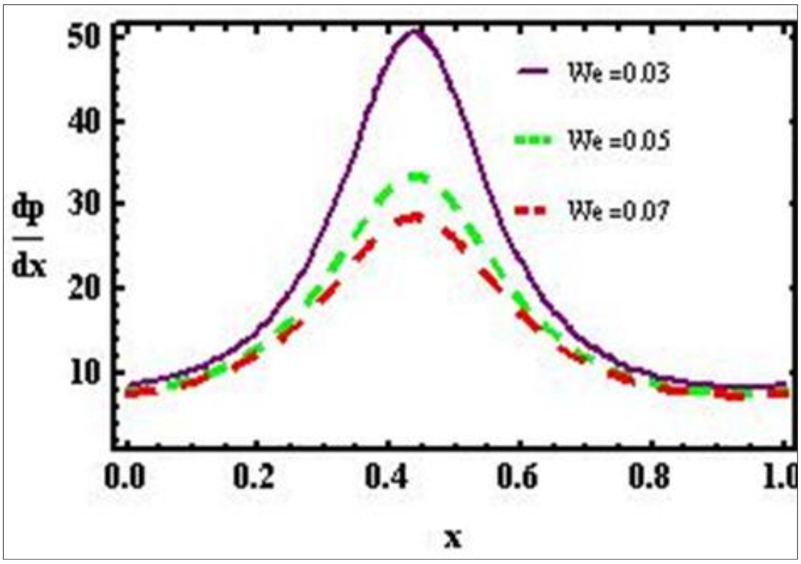

Fig 11: Variation of pressure gradient dp/dx with x different values of "We".

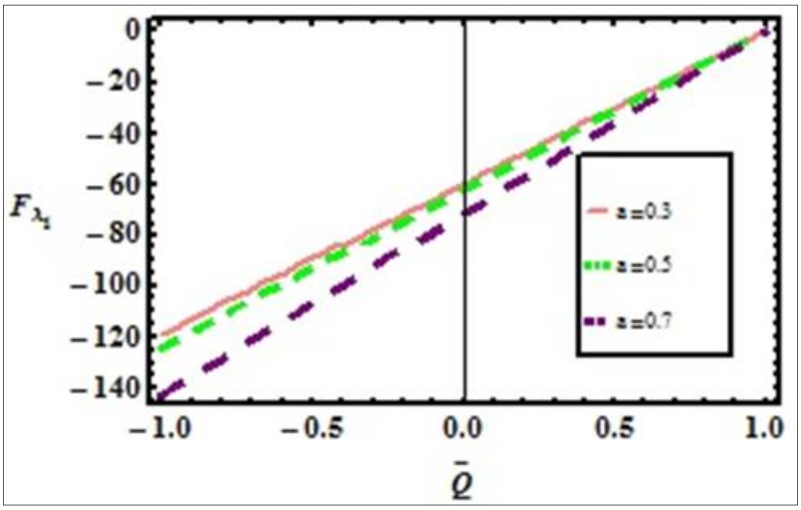

Fig 12: Variation of frictional force F_{λ_1} with $\bar{Q}$ different values of "a".

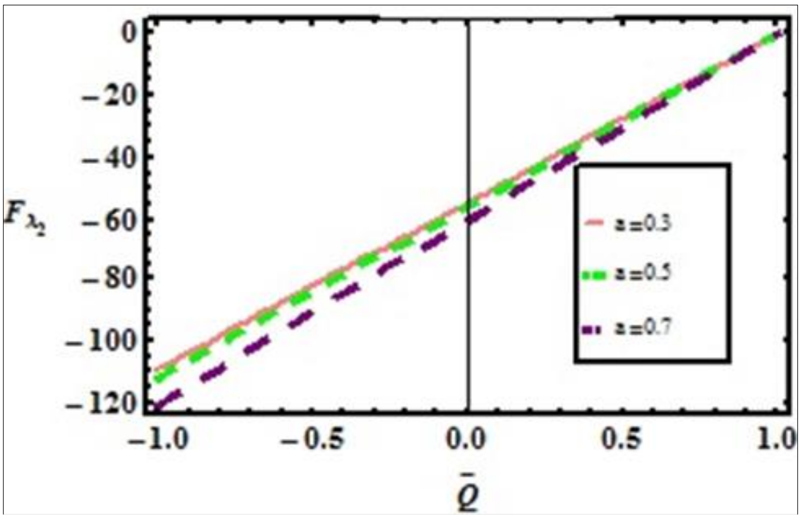

Fig 13: Variation of frictional force F_{λ_2} with $\bar{Q}$ different values of "a".

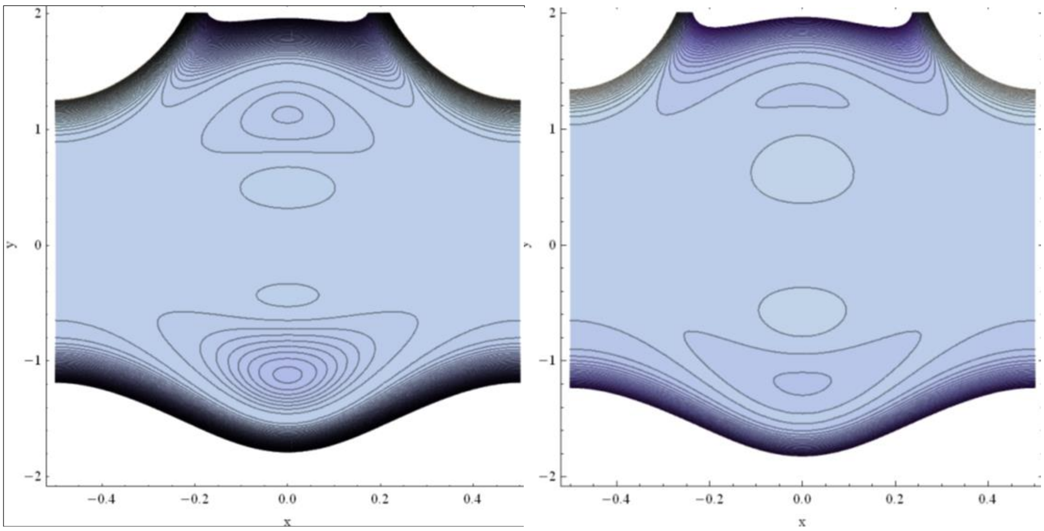

Fig 14: Stream lines for different values of $We = 0.01;\ We = 0.02;$ The other parameters are $a = 0.5;\ b = 0.5;\ d = 0.5$ $\phi = \pi/4;\ Da = 0.01;\ M = 0.6$

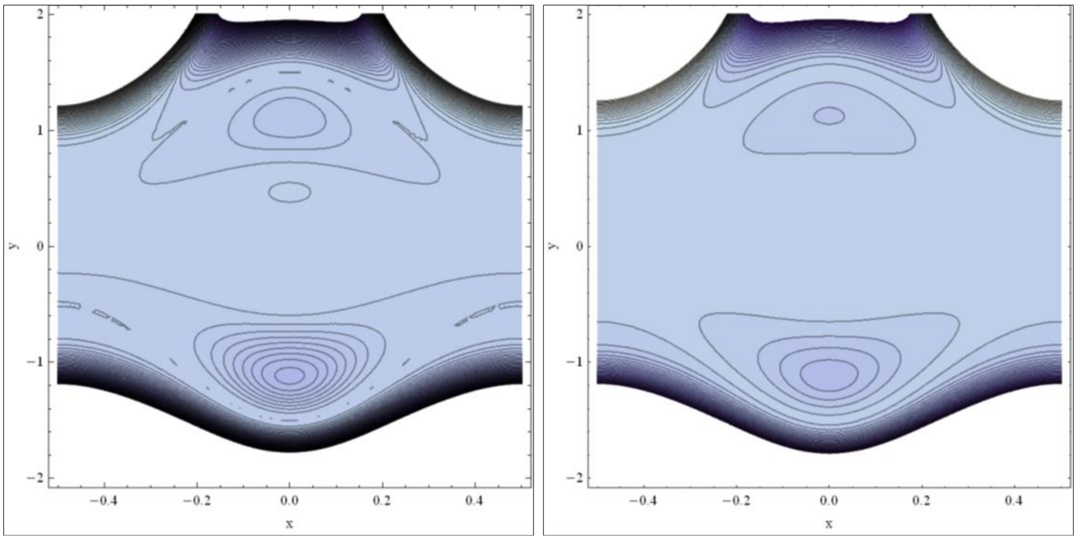

Fig 15: Stream lines for different values of $\phi = 0.4;\ \phi = 0.5;$ The other parameters are $a = 0.5;\ b = 0.5;\ d = 0.5$ $We = 0.03;$ $Da = 0.01;\ M = 0.6$

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