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Fuzzy mathematical approach to game theory

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Abstract
The notion of fuzzy set tries to show to what degree an element is a member of a set, the degree to which an element belongs to a set is an element of the interval [0,1] rather than the Boolean value {0,1}. The purpose of this paper is to study some components in a game such as set of players, set of strategies, set of payoffs, cores, stable sets etc and their fuzzification with the help of fuzzy sets. These notions lead to introduce fuzzy games.

Keywords: Fuzzy set, Strategies, Payoff, Core, Stable set, Fuzzy Game

1. Introduction
A game is determined by informations, decisions and goals. But human ideas and decision are fuzzy. For a man with immense entropy function may err or set right and understanding a little may increase his understanding in the pursuit of some knowledge. Therefore in a game, perfect informations, decision and goals may not be feasible. We are therefore led to the fuzzy mathematical approach to game theory.

2. Fuzzy sets
In an ordinary set the characteristic function assigns a value of either 0 or 1 to each individual in the universal set. But this function is generalized is the case of fuzzy concept in such a way that the values assigned to the elements of universal set fall within a specified range in [0, 1], a closed interval of real numbers.

2.1. Definition
Let X be the universal set and [0, 1] be the closed interval of real numbers. A mapping A: $X \to [0, 1]$ is called a fuzzy set A or fuzzy subset A of X for which $A(x) = \alpha$, where x is a member of X and α is a real number belonging to [0, 1]. Every member of X is a member of fuzzy set A with its grade α which is denoted by $x \in_\alpha A$ or $(x,\alpha) \in A$. A is called 'the membership function and A(x) is called the membership grade of $x \in X$. A fuzzy set A is called universal set if A(x) = 1 for all $x \in X$.

3. Fuzzy mathematical approach
In classical game theory it is assumed that all data of a game are known exactly by players. However, in real games, the players are often not able to evaluate exactly the game due to lack of information and precision in the available informations of the situation. Some mathematicians treated imprecision in games with a probabilistic method to solve such problem of imprecision in games but could not succeed in general. However, in reality imprecision is of different type and can be modeled by fuzzy sets. The notion of fuzzy sets tries to show that to what degree an element is a member of a set It helps to fuzzify each component in a game such as: set of players, set of strategies, set of payoffs, cores, stable sets etc. These notions lead to the proposition of fuzzy games.

3.1 Study of some component in a game
Let N = {1, 2, 3} Be the set of players / participants. They may co-operate each other or may not. Here we consider the possibilities of co-operation. A non – empty subset of N is called a coalition.

Let v: $Z^N \to R$ be a characteristic function. For any coalition S, v(s) is the total payoff that the coalition S earns collectively. Sometimes v(s) is called value or worth of the coalition S. N and v(s) uniquely determine a game (N, v) called a co-operative game of n players. It is important to know what coalition should be formed and how the payoff should be distributed among players of each of this coalition when a game is played. An answer to this question is called a 'solution concept' for the game (N, v).

In a game (N, v), S(⊆N) is called crisp coalition, its characteristic vector being e^s such that

$(e^s)_i = 1$ if i∈s

= 0 if i∈ N-S

Let the game be denoted by G^N. In a crisp game each player may have only two variants of participation (full participation or non involvement at all) is a crisp coalition. However, more freedom may be given to players by considering full co-operation and non-cooperation (i.e. fuzzy co-operation).

A fuzzy coalition of player set N is a vector S in $[0, 1]^N$ where the i^{th} co-ordinate Si is referred to as the participation level of player i. The empty coalition in a fuzzy setting is

$e^{ø}$ = (0, 0, 0…….0,0).

The grand coalition is thus e^v = {1, 1….1} The set of Fuzzy coalition is denoted by F^N. Hence, a cooperative fuzzy game with player set N is a function w: $F^N \to R$ with $w(e^{ø}) = 0$ assigning to each fuzzy coalition S the value achieved by cooperation. We denote the set of fuzzy games with player set N by fG^N.

3.2. Cores and stable sets of crisp games

Let v ∈ G^N. Also let xi be the payoff for player i ∈N, then x = (x_1,x_2, …. x_n) is called a pay off vector. x is called an imputation, if v (N) = $\sum_{i \in N} x_i$ and $x_i v(\{i\})$. The imputation set of v is denoted by I(v) and is defined by I(v) = {x∈R^N: x is an imputation for each i ∈N}, where R^N is the n-dimensional Euclidean space.

The core of crisp game v is denoted by C(v) and is defined by the subset of imputations which are stable against any possible deviation by a coalition i.e.

C(v) = {x ∈ R^N:v(N) = $\sum_{i \in N} x_i$, v(S) = $\sum_{i \in S} x_i$, for each S⊆N}. Let there be two imputations x and y and S⊆N. Imputation x is said to dominate y, and denoted by x dom y if the following hold:

(a) $x_i > y_i$ for each i ∈ S i.e. x is better than y for all i.

(b) $\sum_{i \in S} x_i \le$ v(S) i.e. the payoff $\sum_{i \in S} x_i$ is reachable by S The negation of x dom y is denoted by ¬ x dom y

(c) The dominance core of a crisp game v is denoted by DC(v) and is defined by the set of imputations which are not dominated by any other imputation i.e.

DC(v) = { x∈I(v): ¬y dom x for all y ∈ I(v)

A stable set of a crisp game v is a nonempty set K of imputations if the following hold:

(a) ¬ x dom y for all x,y ∈ K

(b) For all z ∈ I(v) -K there exists an imputation x ∈ K such that x dom z.

3.3 Cores and stable sets of Fuzzy games

Let W ∈ FG^N and I(w) be the imputation set such that

I(w) ={ x ∈ R^N :$\sum_{i \in N} x_i$ =w(e^N), $x_i \ge$ w(e^i), for all i ∈ N}

The core of a fuzzy game W is denoted by c(w), and which is the subset of these

Imputations which are stable against any possible deviation by fuzzy coalition

i.e. c(w) ={x∈ R^N:w(e^N) = $\sum_{i \in N} x_i$, $\sum_{i \in N} S_i x_i \ge$ W(s) for each s∈FG^N

Let S ∈ F^N and Car(S) = {i ∈ N: S_i > 0} If car (s) ≠ N, then S is called "a proper fuzzy coalition". The set of proper fuzzy coalition is denoted by PF^N.

The proper core of a fuzzy game W, denoted by CP(w), is defined as C^P(w) = {x ∈ R^N: $\sum_{i \in N} x_i$ =W(e^N), $\sum_{i \in N} S_i\ x_i \ge v$, for each S ∈ F^N}

We consider only crisp like coalition e^S in the stability condition then the crisp core of the fuzzy game w is defined by

C^{cr} (w) = {x ∈ R^N: $\sum x_i$ = w(e^N), $\sum x_i \ge$ w(e^S) over i ∈ Car (e^S) and for all s ∈ Z^N }

Let x,y ∈ I(w) and S ∈ F^N. Imputation x is said to dominate imputation y via S, denoted by x dom y if the following holds:

(a) $x_i > y_i$ for all i∈ Car (S)

(b) $\sum_{i \in N} S_i\ x_i \le$ w(s)

The negation of x dom y is ¬ x dom y.

The dominance core of a fuzzy game w is denoted by DC(w) is the set of all imputation which are not dominated by any other imputation i.e.

DC(w) = { x ∈ I (w) : ¬ y dom x for all y ∈ I (w)}

A stable set of a fuzzy game w is denoted by nonempty set K of imputations if the following hold:

(a)¬ x dom y for all x,y ∈ K

(b) There exists an imputation x ∈ K such that x dom z for all z∈ I (w) - K

4. Conclusion
In a similar fashion many other components of a game may be forefied with the help of fuzzy set theory which are left open for further study.

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6. References

1. Abreu D. On the Theory of In. nitely Repeated Games with Discount-ing, Econometrica, 1988; 56(2):383-39.
2. Atanassov K. Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 1986; 20:87-96.
3. Atanassov K. New operations defined over Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 1994; 61:137-142.
4. Atanassov K. More on Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 1989; 33:37-46.
5. Atanassov K. Remarks on the Intuitionistic fuzzy sets-III, Fuzzy Sets and Systems, 1995; 75:401-402.
6. Athey S, Bagwell K. Optimal Collusion with Private Information. RAND Journal of Economics. 2001; 32:428-65.
7. Baldwin JF. Fuzzy logic and fuzzy reasoning, in Fuzzy Reasoning and Its Applications, E.H. Mamdani and B.R. Gaines (eds.), London: Academic Press, 1981.
8. Bellman RE, Zadeh LA. local and fuzzy logics, Memorandum No. ERL-M584, Electronics Res. lab Berkeley, 1978.
9. Bouchon-Meunier B, Yager RR. Zadeh (Eds.) LA. Uncertainty in Intelligent and Information Systems Advances in Fuzzy, 2000.
10. Brown JG. A note on fuzzy sets, information and control, 1971; 18:201-204.
11. Chen G. (Guanrong). Introduction to fuzzy sets, fuzzy logic, and fuzzy control systems / Guanrong Chen, 2000.
12. Korner S. Laws of thought, Encyclopedia of Philosophy, MacMillan, NY, 1967; 4:414-417.
13. Reigber A. My life with Kostas, unpublished report, Neverending Story Press, 1999.
14. Zadeh LA. Fuzzy sets, Information and control, 1965, 338-353.
15. Zadeh LA. Fuzzy algorithms, Info. & Ctl. 1968; 12:94-102.
16. Zadeh LA. Making computers think like people, IEEE. Spectrum, 1984; 8:26-32.