Path and cycle related families of E-Cordial graph

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Abstract
The two copies of graph G(p, q) are joined by t paths on n-points each. We represent the family by G(tPn). The paths are attached at the same fixed point on G. We discuss E-cordiality of C3(Pn), K4(Pn), S4(Pn) (shell graph S4). We show that under certain conditions these graphs are E-cordial.

Keywords: graph, E-cordial, shell graph, C3, C4

Introduction
In 1997 Yilmaz and Cahit introduced weaker version of edge graceful labeling E-cordial labeling. Let G be a (p, q) graph. \( f: E \rightarrow \{0, 1\} \) Define \( f \) on V by \( f(v) = \sum f(\nu u) \text{mod} 2 \). The function \( f \) is called as E-cordial labeling if \( |v(f(0)-f(1))| \leq 1 \) and \( |e(f(0)-e(1))| \leq 1 \) where \( v(f(i)) \) is the number of vertices labeled with \( i = 0, 1 \). And \( e(f(i)) \) is the number of edges labeled with \( i = 0, 1 \). We follow the convention that \( v(f(0)) = a, b \) and \( v(f(1)) = b \). Further \( e(f(0)) = (x, y) \) for \( e(f(1)) = y \). A graph that admits E-cordial labeling is called as E-cordial graph. Yilmaz and Cahit prove that trees \( T_n \) are E-cordial iff \( n \) not congruent to 2 (mod 4), \( K_n \) are E-cordial iff \( n \) not congruent to 2 (mod 4), \( F_n \) are E-cordial iff \( n \) not congruent to 2 (mod 4). One should refer Dynamic survey of graph labeling by Joe Gallian \[2\] for more results on E-cordial graphs.

The graphs we consider are finite, undirected, simple and connected. For terminology and definitions we refer Harary \[3\] and Dynamic survey of graph labeling by Joe Gallian \[2\]. The families we discuss are obtained by taking two copies of graph G and join them by t paths of equal length. The paths are attached at the same fixed point on G. We represent these families by G(tPn). We take \( t = 1 \) and \( G = C_3, S_4, K_4 \).

Theorems proved
\( G = C_3(1P_n) \) is E-cordial iff \( n \) is not congruent to 2 (mod 4). \( \text{where } n' = |V(G)| \)

**Proof:** We define Gas \( V(G) = \{u_1, u_2, u_3, u_4\} \cup \{v_1, v_2, \ldots, v_n\} \) \( E(G) = \{e_i = (v_i v_i+1), i = 1, 2, n-1, c_1 = (u_1 u_2), c_2 = (u_2 v_1), c_3 = (u_1 v_1), c_4 = (v_n u_3), c_5 = (u_3 u_4), c_6 = (u_4 v_n)\} \) \( |V(G)| = n+4 \) and \( |E(G)| = n+3 \).

**Fig 4.1:** E-cordial labeling of \( C_3(P_3) \). Then numbers are edge labels.

**Fig 4.2:** E-cordial labeling of \( C_3(P_4) \). Then numbers are edge labels.
For $n > 5$ define a function $f: V(G) \to \{0, 1\}$ as $f(u_1) = f(u_2) = 0$

$f(c_4) = f(c_5) = f(c_6) = 0, f(c_1) = f(c_3) = 0, f(c_2) = 1$

for $i \leq 2t$;

$f(e_i) = 1$ for $i$ is odd number

$f(e_i) = 0$ for $i$ is even number

Where $t = \min\{\frac{n-4}{4} / i = 0, 1, 2, 3\}$

Theorem $S_4(P_n)$ is $E$-cordial iff $n$ is not congruent to 2 (mod 4)

Proof: $S_4(P_n)$ is defined as $V(G) = \{u_1, u_2, u_3, u_4, u_5, u_6\} \cup \{v_1, v_2, v_3, v_n\}$ and the edge set is given by $E(G) = \{c_1 = (v_1u_1), (u_1u_2), c_3 = (u_2u_3), c_4 = (u_3u_4), c_4 = (u_4v_1), c_5 = (v_1u_2)\} \cup \{e_i = (v_i v_{i+1}), i = 1, 2, ..., n\}$

For $n > 5$ define a function $f: V(G) \to \{0, 1\}$ as $f(u_1) = f(u_2) = 0$

$f(c_4) = f(c_5) = f(c_6) = 0, f(c_1) = f(c_3) = 0, f(c_2) = 1$

for $i \leq 2t$;

$f(e_i) = 1$ for $i$ is odd number

$f(e_i) = 0$ for $i$ is even number

Where $t = \min\{\frac{n-4}{4} / i = 0, 1, 2, 3\}$

Define a function $f: V(G) \to \{0, 1\}$ as $f(u_1) = f(u_2) = 0$

$f(c_4) = f(c_5) = f(c_6) = 0, f(c_1) = f(c_3) = 0, f(c_2) = 1$

take $t = \frac{n-4}{4} / j = 1, 2, 3$. ($n$ is divisible by 4 then graph is not $E$-cordial)

For $i \leq 2t$

$f(e_i) = 0$ for $i$ is odd number $\leq 2t$

$f(e_i) = 1$ for $i$ is even number $< 2t$.

$f(e_i) = 1$ for $i = 2t, 2t+1, 2t+k$ where $k = p-t+1$ where $p = \frac{n-1}{2}$ for $n \equiv 1, 3$ (mod 4) and $p = \frac{n-2}{2}$ when $n \equiv 2$ (mod 4).

$f(e_i) = 0$ for $2t+k<i \leq n$

Note that The number distribution is

1) for $n \equiv 1$ (mod 4) $v_1(0, 1) = (\frac{n-4}{2} + n, \frac{n}{2} + 4)$, $v_1(0, 1) = (\frac{n-4}{2} + n, \frac{n}{2} + 4)$

2) for $n \equiv 2$ (mod 4) $v_1(0, 1) = (\frac{n}{2} + 4, \frac{n}{2} + 4)$, $v_1(0, 1) = (\frac{n}{2} + 4, \frac{n}{2} + 4)$

3) for $n \equiv 3$ (mod 4) $v_1(0, 1) = (\frac{n}{2} + 5, \frac{n}{2} + 5)$, $v_1(0, 1) = (\frac{n}{2} + 5, \frac{n}{2} + 5)$
Theorem $K_4(P_n)$ is E-cordial. 

**Proof:** The $K_4(P_n)$ is defined as $V(G)=\{u_1, u_2, u_3, u_4, u_5, u_6\} \cup \{v_1, v_2, \ldots, v_n\}$ and $E(G)=\{c_1=(v_1u_1), c_2=(u_1u_2), c_3=(u_2u_3), c_4=(u_3u_4), c_5=(v_1u_2), c_6=(u_1u_3), c_7=(u_5v_n), c_8=(u_4u_5), c_9=(u_5u_6), c_{10}=(u_6v_n), c_{11}=(v_nv_4), c_{12}=(v_nv_5)\} \cup \{e_i=(v_iv_{i+1})/1 \leq i \leq n\}$

We diagrammatically show below the E-cordial labeling of $K_4(p_2), K_4(p_3), K_4(P_5)$.

Note that The number distribution is
1) for $n\equiv1(\text{mod } 4)v_f(0,1)=\binom{n}{2}+3, \binom{n}{2}+4, e_f(0,1)=\binom{n}{2}+4, \binom{n}{2}+4$
2) for $n\equiv2(\text{mod } 4)v_f(0,1)=\binom{n}{2}+4, \binom{n}{2}+4, e_f(0,1)=\binom{n}{2}+5, \binom{n}{2}+4$
3) for $n\equiv3(\text{mod } 4)v_f(0,1)=\binom{n}{2}+4, \binom{n}{2}+5, \binom{n}{2}+4, e_f(0,1)=\binom{n}{2}+5, \binom{n}{2}+5$

**Conclusions**

We have discussed only for single path between two copies of graphs. It is necessary to evaluate E-cordiality if there are 2 or more paths between two copies of graphs.

**References**