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Path and cycle related families of E-Cordial graph

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Abstract

The two copies of graph $G(p, q)$ are joined by t paths on n -points each. We represent the family by $G(tP_n)$. The paths are attached at the same fixed point on G . We discuss E-cordiality of $C_3(P_n)$, $K_4(P_n)$, $S_4(P_n)$ (shel graph S_4) We show that under certain conditions these graphs are E-cordial.

Keywords: graph, E-cordial, shel graph, C_3 , C_4

Introduction

In 1997 Yilmaz and Cahit introduced weaker version of edge graceful labeling E-cordial labeling. Let G be a (p, q) graph. $f: E \rightarrow \{0, 1\}$ Define f on V by $f(v) = \sum \{f(vu) | (vu) \in E(G)\} \pmod{2}$. The function f is called as E-cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$ where $v_f(i)$ is the number of vertices labeled with $i = 0, 1$. And $e_f(i)$ is the number of edges labeled with $i = 0, 1$. We follow the convention that $v_f(0, 1) = (a, b)$ for $v_f(0) = a$ and $v_f(1) = b$ further $e_f(0, 1) = (x, y)$ for $e_f(0) = x$ and $e_f(1) = y$. A graph that admits E-cordial labeling is called as E-cordial graph. Yilmaz and Cahit prove that trees T_n are E-cordial iff for n not congruent to $2 \pmod{4}$, K_n are E-cordial iff n not congruent to $2 \pmod{4}$, Fans F_n are E-cordial iff for n not congruent to $1 \pmod{4}$.

Yilmaz and Cahit observe that A graph on n vertices can not be E-cordial if n is congruent to $2 \pmod{4}$. One should refer Dynamic survey of graph labeling by Joe Gallian [2] for more results on E-cordial graphs.

The graphs we consider are finite, undirected, simple and connected. For terminology and definitions we refer Harary [3] and Dynamic survey of graph labeling by Joe Gallian [2]. The families we discuss are obtained by taking two copies of graph G and join them by t paths of equal length. The paths are attached at the same fixed point on G . we represent these families by $G(tP_n)$. We take $t = 1$ and $G = C_3, S_4, K_4$.

Theorems proved

$G = C_3(1P_n)$ is E-cordial iff n' is not congruent to $2 \pmod{4}$. where $n' = |V(G)|$

Proof: We define G as $V(G) = \{u_1, u_2, u_3, u_4\} \cup \{v_1, v_2, \dots, v_n\}$ $E(G) = \{e_i = (v_i v_{i+1}), i = 1, 2, n-1, c_1 = (u_1 u_2), c_2 = (u_2 v_1), c_3 = (u_1 v_1), c_4 = (v_n u_3), c_5 = (u_3 u_4), c_6 = (u_4 v_n)\}$ $|V(G)| = n+4$ and $|E(G)| = n+3$.

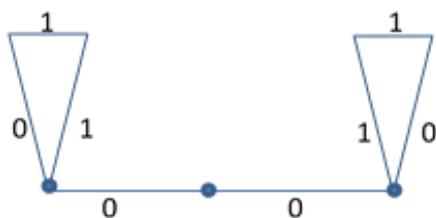


Fig 4.1 : E-cordial labeling of $C_3(P_3)$.
 The numbers are edge labels.

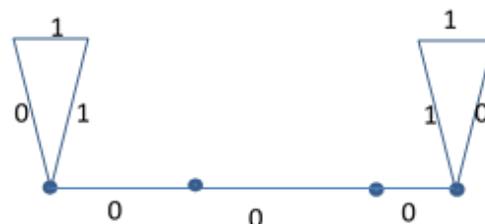


Fig 4.2 : E-cordial labeling of $C_3(P_4)$.
 The numbers are edge labels.

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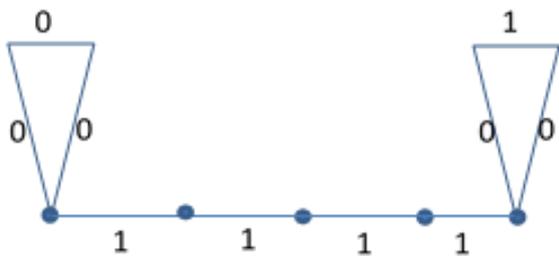


Fig 4.3: E-cordial labeling of $C_3(P_5)$.
The numbers are edge labels.

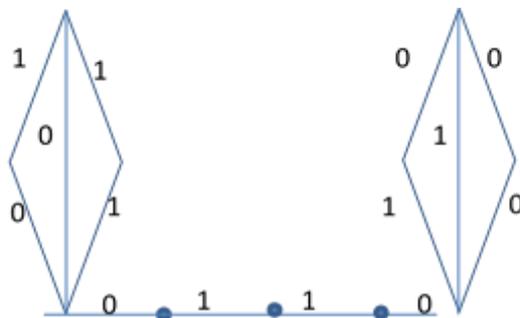


Fig 4.6: $S_4(p_5)$ The edge label numbers are shown

For $n > 5$ define a function $f: V(G) \rightarrow \{0, 1\}$ as $f(u_1) = f(u_2) = 0$
 $f(c_4) = f(c_5) = f(c_6) = 0, f(c_1) = f(c_3) = 0, f(c_2) = 1$
 for $i \leq t$;
 $f(e_i) = 1$ for i is odd number
 $f(e_i) = 0$ i is even number
 Where $t = \min\{\frac{n-i}{4}/i=0,1,2,3\}$

Theorem $S_4(P_n)$ is E-cordial iff n is not congruent to $2 \pmod{4}$

Proof: $S_4(p_n)$ is defined as $V(G) = \{u_1, u_2, u_3, u_4, u_5, u_6\} \cup \{v_1, v_2, v_3, \dots, v_n\}$ and the edge set is given by $E(G) = \{c_1 = (v_1 u_1), (u_1 u_2), c_3 = (u_2 u_3), c_4 = (u_3 u_4), c_5 = (u_4 u_5), c_6 = (u_5 u_6)\} \cup \{e_i = (v_i v_{i+1}), i = 1, 2, \dots, n-1\}$

For $n > 5$ define a function $f: V(G) \rightarrow \{0, 1\}$ as $f(u_1) = f(u_2) = 0$
 $f(c_4) = f(c_5) = f(c_6) = 0, f(c_1) = f(c_3) = 0, f(c_2) = 1$
 for $i \leq t$;
 $f(e_i) = 1$ for i is odd number
 $f(e_i) = 0$ i is even number
 Where $t = \min\{\frac{n-i}{4}/i=0, 1, 2, 3\}$

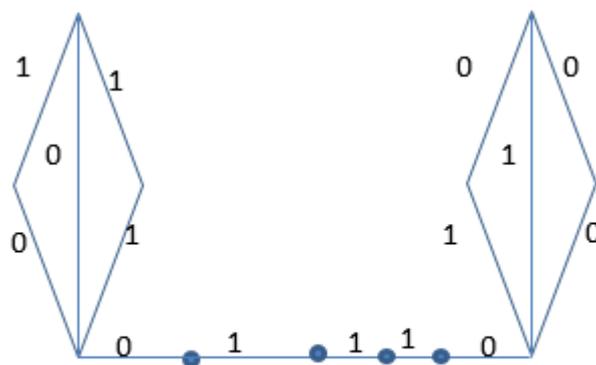


Fig 4.7: $S_4(p_5)$ The edge label numbers are shown

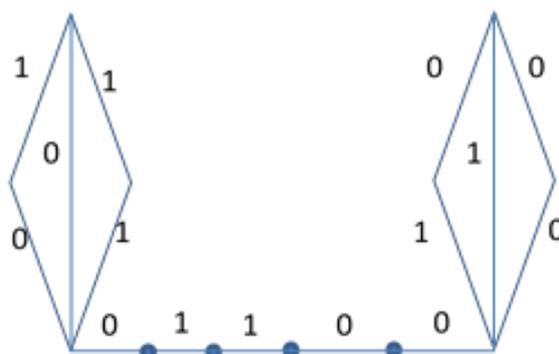


Fig 4.8: $S_4(p_6)$ The edge label numbers are shown

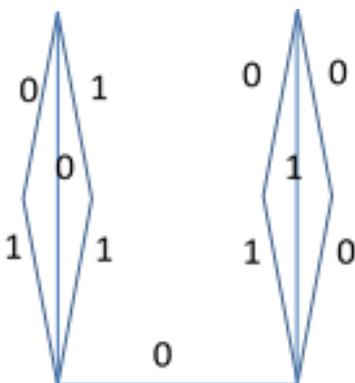


Fig 4.4: $S_4(p_2)$ The edge label numbers are shown

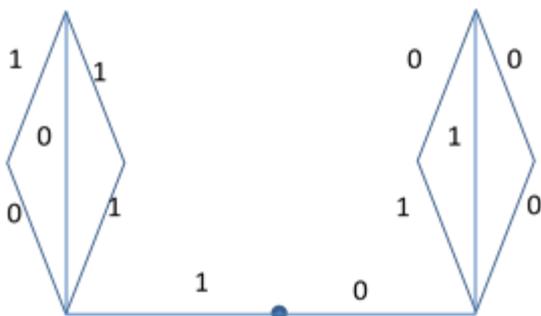


Fig 4.5: $S_4(p)$ The edge label numbers are shown

Define a function $f: V(G) \rightarrow \{0, 1\}$ as $f(u_1) = f(u_2) = 0$
 $f(c_4) = f(c_5) = f(c_6) = 0,$
 $f(c_1) = f(c_3) = 0, f(c_2) = 1$

take $t = \frac{n-j}{4}/j=1, 2, 3.$ (n is divisible by 4 then graph is not E-cordial)

For $i \leq 2t$

$f(e_i) = 0$ for i is odd number $\leq 2t$

$f(e_i) = 1$ for i is even number $< 2t.$

$f(e_i) = 1$ for $i = 2t, 2t+1, 2t+k$ where $k = p-t+1$ where $p = \frac{n-1}{2}$ for $n \equiv 1, 3 \pmod{4}$ and $p = \frac{n-2}{2}$ when $n \equiv 2 \pmod{4}.$

$f(e_i) = 0$ for $2t+k < i \leq n$

Note that The number distribution is

1) for $n \equiv 1 \pmod{4}$ $v_f(0,1) = (\frac{n}{2} + 3, \frac{n}{2} + 4), e_f(0,1) = (\frac{n}{2} + 4, \frac{n}{2} + 4)$

2) for $n \equiv 2 \pmod{4}$ $v_f(0,1) = (\frac{n}{2} + 4, \frac{n}{2} + 4), e_f(0,1) = (\frac{n}{2} + 5, \frac{n}{2} + 4)$

3) for $n \equiv 3 \pmod{4}$ $v_f(0,1) = (\frac{n}{2} + 5, \frac{n}{2} + 4), e_f(0,1) = (\frac{n}{2} + 5, \frac{n}{2} + 5)$

Theorem $K_4(P_n)$ is E-cordial.

Proof: The $K_4(P_n)$ is defined as $V(G) = \{u_1, u_2, u_3, u_4, u_5, u_6\} \cup \{v_1, v_2, \dots, v_n\}$ and $E(G) = \{c_1=(v_1u_1), c_2=(u_1u_2), c_3=(u_2u_3), c_4=(u_3u_4), c_5=(v_1u_2), c_6=(u_1u_3), c_7=(v_nu_4), c_8=(u_4u_5), c_9=(u_5u_6), c_{10}=(u_6v_n), c_{11}=(v_nu_5), c_{12}=(u_5u_6)\} \cup \{e_i=(v_i v_{i+1}) / i = 1, 2, \dots, n-1\}$

We diagrammatically show below the E-cordial labeling of $K_4(p_2)$, $K_4(p_3)$, $K_4(p_5)$.

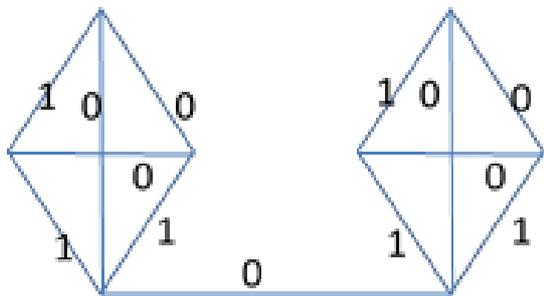


Fig 4.9: $k_4(P_2)$ a labeled copy.

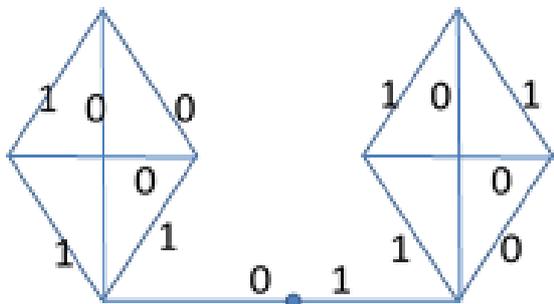


Fig 4.10: $k_4(P_3)$ a labeled copy.

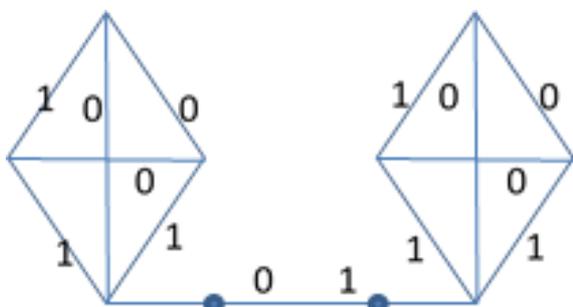


Fig 4.11: $k_4(P_3)$ a labeled copy .

Define a function $f : E(G) \rightarrow \{0,1\}$ as

- $f(c_1)=1,$
- $f(c_2)=1,$
- $f(c_4)=1,$
- $f(c_7)=1,$
- $f(c_8)=1,$

$f(c_{10})=1.$ take $t = \frac{n-j}{4}, j=1,2,3.$ (n is divisible by 4 then graph is not E-cordial)

For $i \leq 2t$

$f(e_i)=0$ for i is odd number $\leq 2t.$

$f(e_i)=1$ for i is even number $< 2t.$

$f(e_i)=1$ for $i= 2t, 2t+1, 2t+k$ where $k = p-t+1$ where $p = \frac{n-1}{2}$ for

$n \equiv 1,3 \pmod{4}$ and $p = \frac{n-2}{2}$ when $n \equiv 2 \pmod{4}.$

$f(e_i) = 0$ for $2t+k < i \leq n$

Note that The number distribution is

- 1) for $n \equiv 1 \pmod{4} v_f(0, 1) = (\frac{n}{2}+3, \frac{n}{2}+4), e_f(0,1) = (\frac{n}{2}+4, \frac{n}{2}+4)$
- 2) for $n \equiv 2 \pmod{4} v_f(0, 1) = (\frac{n}{2}+4, \frac{n}{2}+4), e_f(0,1) = (\frac{n}{2}+5, \frac{n}{2}+4)$
- 3) for $n \equiv 3 \pmod{4} v_f(0, 1) = (\frac{n}{2}+5, \frac{n}{2}+4), e_f(0,1) = (\frac{n}{2}+5, \frac{n}{2}+5)$

Conclusions

We have discussed only for single path between two copies of graphs. It is necessary to evaluate E-cordiality if there are 2 or more paths between twocopies of graphs.

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