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Some graph operations on sum divisor cordial labeling related to H- graph

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Abstract

A sum divisor cordial labeling of a graph G with vertex set V is a bijection $f:V \to \{1,2,...,|V(G)|\}$ such that each edge uv assigned the label 1 if 2 divides $f(u)+f(v)$ and 0 otherwise. Further, the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. A graph with a sum divisor cordial labeling is called a sum divisor cordial graph. In this paper, we prove that graphs H_n (n is even), duplication of all edges of H - graph H_n (n is even), $H_n \odot K_1$, $P(r.H)$, $C(r.H)$ are sum divisor cordial graphs.

Keywords: Divisor cordial labeling, sum divisor cordial labeling, H- graph

Introduction

By a graph, we mean a finite undirected graph without loops or multiple edges. For standard terminology and notations related to graph theory, we refer to Harary [2]. A labeling of graph is a map that carries the graph elements to the set of numbers, usually to the set of non-negative or positive integers. If the domain is the set of edges, then we speak about edge labeling. If the labels are assigned to both vertices and edges, then the labeling is called total labeling. Cordial labeling is extended to divisor cordial labeling, prime cordial labeling, total cordial labeling, Fibonacci cordial labeling etc.

Varatharajan *et al.* [10] introduced the concept of divisor cordial labeling. For dynamic survey of various graph labeling, we refer to Gallian [1]. Lourdusamy and Patrick [4] introduced the concept of sum divisor cordial labeling. Sugumaran and Rajesh [6] proved that Swastik graph Sw_n, path union of finite copies of Swastik graph Sw_n, cycle of k copies of Swastik graph Sw_n (k is odd), Jelly fish $J(n,n)$ and Petersen graph are sum divisor cordial graphs. Sugumaran and Rajesh [7] proved that the Theta graph and some graph operations in Theta graph are sum divisor cordial graphs. Sugumaran and Rajesh [8] proved that the Herschel graph and some graph operations in Herschel graph are sum divisor cordial graphs. Sugumaran and Rajesh [9] proved that the H_n (n is odd), $C_3 @ K_{1,n}$, $<F_n^1 \Delta F_n^2>$, open star of Swastik graph $S(t.Sw_n)$, when t is odd are sum divisor cordial graphs.

In this paper we investigate the sum divisor cordial labeling on the graphs such as graphs H_n where n is even, duplication of all edges of H - graph H_n, where n is odd, $H_n \odot K_1$, $P(r.H)$, $C(r.H)$ are sum divisor cordial graphs.

Definition 1.1: Let $G=(V(G),E(G))$ be a simple graph and let $f:V(G) \to \{1,2,...,|V(G)|\}$ be a bijection. For each edge uv, assign the label 1 if either $f(u) \mid f(v)$ or $f(v) \mid f(v)$ and the label 0 otherwise. The function f is called a *divisor cordial labeling* if $|e_f(0)-e_f(1)| \leq 1$. A graph which admits a divisor cordial labeling is called a *divisor cordial graph*.

Definition 1.2: Let $G=(V(G),E(G))$ be a simple graph and let $f:V(G)\rightarrow\{1,2,...,|V(G)|\}$ be a bijection. For each edge uv, assign the label 1 if either $2|(f(u)+f(v))$ and the label 0 otherwise. The function f is called a *sum divisor cordial labeling* if $|e_f(0)-e_f(1)|\leq 1$. A graph which admits a sum divisor cordial labeling is called a *sum divisor cordial graph*.

Definition 1.4: Let P_n^1, P_n^2 be any two paths with n vertices. Let $V(P_n^1)=\{u_1,u_2,...,u_n\}$ and $V(P_n^2)=\{v_1,v_2,...,v_n\}$. We join the vertices $u_{\frac{(n+1)}{2}}$ and $v_{\frac{(n+1)}{2}}$ by an edge, if n is odd and join the vertices $u_{\frac{n}{2}}$ and $v_{\frac{n}{2}+1}$ if n is even, then the resulting graph is called a H - graph on $2n$ vertices.we denote it by H_n.

Definition 1.5: The graph $H_n \odot K_1$ is obtained by adding a pendant edge to each vertex of an H - graph H_n.

Definition 1.6: The cycle union of a graph G is the graph obtained from a cycle C_n $(n\geq 3)$ by replacing each vertex of the cycle by G and it is denoted by $C(n.G)$.

Definition 1.7: The path union of a graph G is the graph obtained from a path P_n $(n\geq 2)$ by replacing each vertex of the path by G and it is denoted by $P(n.G)$.

Definition 1.8: The duplication of an edge $e=uv$ of a graph G is the graph G' obtained from G by adding a new vertex v' to G such that v' is adjacent to both u and v.

2. Main Results

Theorem 2.1 The graph H_n admits sum divisor cordial labeling, where n is even.

Proof: Let $G=H_n$. Let $V(G)=\{u_i,v_i:1\leq i\leq n\}$ and $E(G)=\{u_iu_{i+1}:1\leq i\leq n-1\}\cup\{v_iv_{i+1}:1\leq i\leq n-1\}\cup\{u_{\frac{n}{2}}v_{\frac{n}{2}+1}\}$ be the set of vertices and edges of $H-$graph respectively. Then G has $2n$ vertices and $2n-1$ edges.

We define the vertex labeling $f:V(G)\rightarrow\{1,2,...,|V(G)|\}$ as follows.

$f(u_i^k)=i$; $i=1,2,...,\frac{n}{2}$.

$f(u_i^k)=2i$; $i=\frac{n}{2}+1,...,n$.

$f(v_i^k)=\frac{n}{2}+i$; $i=1,2,...,\frac{n}{2}+1$.

$f(v_i^k)=2i-1$; $i=\frac{n}{2}+2,...,n$.

From the above labeling pattern, we have $|e_f(0)-e_f(1)|\leq 1$.

Hence G admits sum divisor cordial labeling.

Example 2.2 The sum divisor cordial labeling of H_5 is shown in Figure 1.

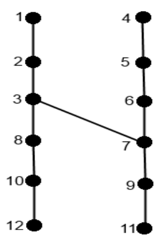


Fig 1: H_5

Theorem 2.3 Duplication of all edges of $H-$graph H_n admits sum divisor cordial labeling, where n is odd.

Proof: Let $V(H_n)=\{u_i, v_i : 1\le i\le n\}$ and $E(H_n)=\{u_iu_{i+1} : 1\le i\le n-1\}\cup\{v_iv_{i+1} : 1\le i\le n-1\}\cup\{u_iv_i : i=\frac{(n+1)}{2}\}$ be the set of vertices and edges of $H-$graph respectively. Let G be the graph obtained by duplicating of all the edges $u_1u_2, u_2u_3, \ldots, u_{n-1}u_n$ and $v_1v_2, v_2v_3, \ldots, v_{n-1}v_n$ by a new vertices $u_1', u_2', \ldots, u_{n-1}'$ and $v_1', v_2', \ldots, v_{n-1}'$ repectively. Let w be a new vertex obtained by duplicating the edge $u_{\frac{n+1}{2}}v_{\frac{n+1}{2}}$.

Then G has $4n-1$ vertices and $6n-3$ edges.
We define the vertex labeling $f : V(G)\to\{1,2,\ldots,|V(G)|\}$ as follows.

Case 1: when $n\equiv 3(\text{mod}\,4)$

$f(u_i)=2i-1,\ i=1,2,\ldots,\frac{(n+1)}{2},$

$f(u_i)=n+i,\ i=\frac{(n+1)}{2}+1,\ldots,n,$

$f(v_i)=2i,\ i=1,2,\ldots,\frac{(n+1)}{2},$

$f(v_i)=\frac{(n+1)}{2}+i,\ i=\frac{(n+1)}{2}+1,\ldots,n.$

$f(u_i')=2n+i,\ i=1,2,\ldots,n-1,$

$f(v_i')=3n-1+i,\ i=1,2,\ldots,n-1,$

$f(w)=4n-1.$

Case 2: when $n\equiv 1(\text{mod}\,4)$

$f(u_i)=i+1,\ i=1,2,\ldots,\frac{(n+1)}{2},$

$f(u_i)=2i,\ i=\frac{(n+1)}{2}+1,\ldots,n,$

$f(v_i)=\frac{(n+1)}{2}+1+i,\ i=1,2,\ldots,\frac{(n+1)}{2},$

$f(v_i)=2i+1,\ i=\frac{(n+1)}{2}+1,\ldots,n.$

$f(u_i')=2n+1+i,\ i=1,2,\ldots,n-1,$

$f(v_i')=3n+i,\ i=1,2,\ldots,n-1,$

$f(w)=1.$

From the above labeling pattern, we have $|e_f(0)-e_f(1)|\le 1$.

Hence G is a sum divisor cordial graph.

Example 2.4 The sum divisor cordial labeling of duplication of all the edges of H_3 and H_5 are shown in Figure 2 and Figure 3 respectively.

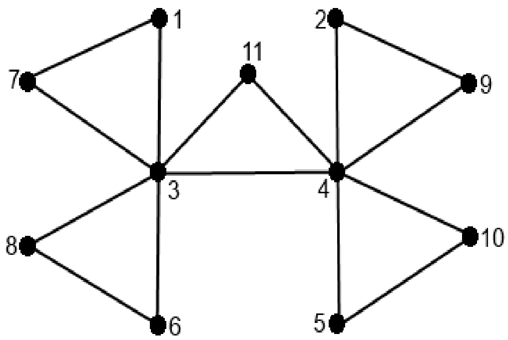


Fig 2: H_3

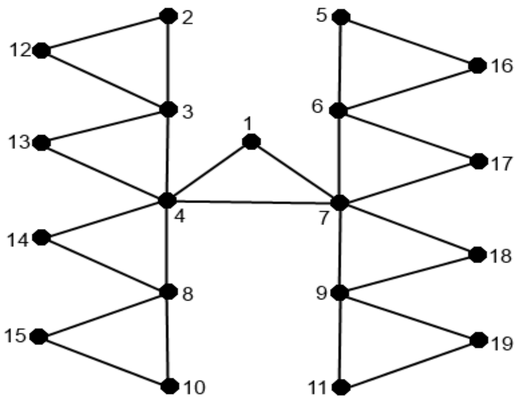


Fig 3: H_5

Theorem 2.5 The graph $H_n \odot K_1$ admits sum divisor cordial labeling.

Proof: let $G = H_n \odot K_1$. Let $V(H_n) = \{u_i, v_i : 1 \le i \le n\}$ and let $u'_1, u'_2, ..., u'_n$ be the pendant vertices connected to $u_1, u_2, ..., u_n$ and $v'_1, v'_2, ..., v'_n$ be the pendant vertices connected to $v_1, v_2, ..., v_n$ respectively in G. Then G has $4n$ vertices $4n-1$ edges. Here, the vertices $u_{\frac{n+1}{2}}$ and $v_{\frac{n+1}{2}}$ are connected by an edge, if n is odd or the vertices $u_{\frac{n}{2}}$ and $v_{\frac{n}{2}+1}$ by an edge, if n is even. We define the vertex labeling $f : V(G) \to \{1, 2, ..., |V(G)|\}$ as follows.

$f(u_i) = 2i - 1,\ i = 1, 2, ..., n.$

$f(v_i) = 2n + 2i - 1,\ i = 1, 2, ..., n.$

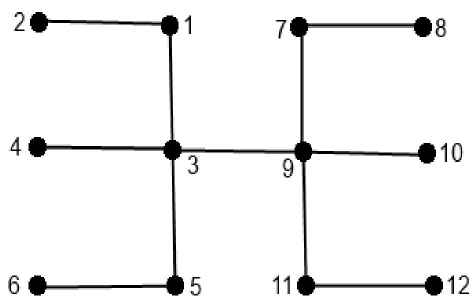
$f(u'_i) = 2i,\ i = 1, 2, ..., n.$

$f(v'_i) = 2n + 2i,\ i = 1, 2, ..., n.$

From the above labeling pattern, we have $|e_f(0) - e_f(1)| \le 1$.

Hence G admits sum divisor cordial labeling.

Example 2.6 The sum divisor cordial labeling of $H_3 \odot K_1$ and $H_4 \odot K_1$ are shown in Figure 4 and Figure 5 respectively.



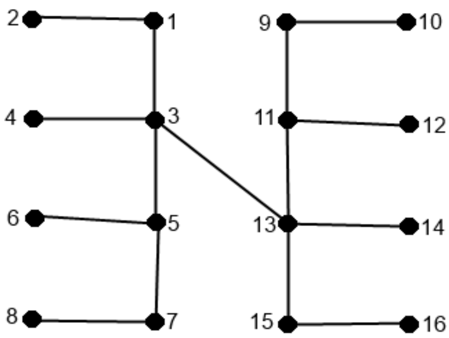


Fig 4: $H_3 \odot K_1$ **Fig 5:** $H_4 \odot K_1$

Theorem 2.7 Path union of r copies of H - graph H_n is a sum divisor cordial cordial graph.

Proof: Let $G = P(r.\ H_n)$ be the path union of r copies of H - graph. In graph G, $|V(G)| = 2nr$ and $|E(G)| = 2nr - 1$. We denote u_i^k and v_i^k are the i^{th} vertices in the k^{th} copy of H - graph respectively, where $i = 1, 2, ..., n$ and $k = 1, 2, ..., r$. Notice that the vertices v_1^k and v_1^{k+1} are connected by an edge in G, where $k = 1, 2, ..., r-1$. We define vertex labeling $f : V(G) \to \{1, 2, ..., |V(G)|\}$ as follows.

Case 1: When n is odd

$f(u_i^k) = i + (k-1)2n$; $i = 1, 2, ..., \frac{(n+1)}{2}$.

$f(u_i^k) = 2i - 1 + (k-1)2n$; $i = \frac{(n+1)}{2} + 1, ..., n$.

$f(v_i^k) = \frac{(n+1)}{2} + i + (k-1)2n$; $i = 1, 2, ..., \frac{(n+1)}{2}$.

$f(v_i^k) = 2i + (k-1)2n$; $i = \frac{(n+1)}{2} + 1, ..., n$.

Case 2: When n is even

$f(u_i^k) = i + (k-1)2n$; $i = 1, 2, ..., \frac{n}{2}$.

$f(u_i^k) = 2i + (k-1)2n$; $i = \frac{n}{2} + 1, ..., n$.

$f(v_i^k) = \frac{n}{2} + i + (k-1)2n$; $i = 1, 2, ..., \frac{n}{2} + 1$.

$f(v_i^k) = 2i - 1 + (k-1)2n$; $i = \frac{n}{2} + 2, ..., n$.

From the above labeling pattern, we have $|e_f(0) - e_f(1)| \leq 1$.

Hence G admits sum divisor cordial labeling.

Example 2.8 The sum divisor cordial labeling of $P(4.\ H_3)$ and $P(4.\ H_4)$ are shown in Figure 6 and Figure 7 respectively.

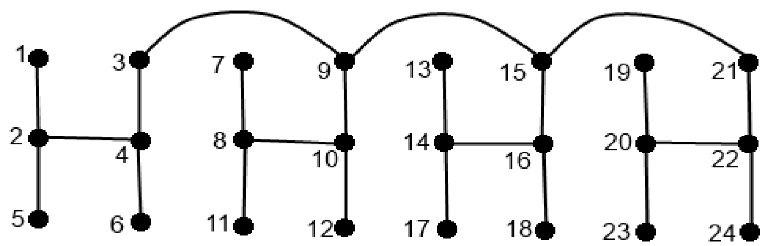


Fig 6: $P(4.\ H_3)$

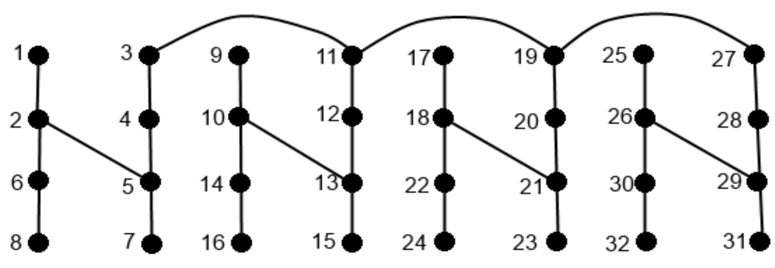


Fig 7: $P(4.\ H_4)$

Theorem 2.9 Cycle union of r copies of H - graph H_n is a sum divisor cordial cordial graph.

Proof: Let $G = C(r.\ H_n)$ be the cycle union of r copies of H - graph. In graph G, $|V(G)| = 2nr$ and $|E(G)| = 2nr$. We denote u_i^k and v_i^k are the i^{th} vertices in the k^{th} copy of H - graph respectively, where $i = 1, 2, ..., n$ and $k = 1, 2, ..., r$.

Notice that the vertices v_1^k and v_1^{k+1} are connected by an edge and the vertices v_1^r and v_1^1 are connected by an edge in G, where $k = 1, 2, ..., r-1$. We define vertex labeling $f : V(G) \rightarrow \{1, 2, ..., |V(G)|\}$ as follows.

Case 1: When n is odd

$f(u_i^k) = i + (k-1)2n$; $i = 1, 2, ..., \frac{(n+1)}{2}$.

$f(u_i^k) = 2i - 1 + (k-1)2n$; $i = \frac{(n+1)}{2} + 1, ..., n$.

$f(v_i^k) = \frac{(n+1)}{2} + i + (k-1)2n$; $i = 1, 2, ..., \frac{(n+1)}{2}$.

$f(v_i^k) = 2i + (k-1)2n$; $i = \frac{(n+1)}{2} + 1, ..., n$.

Case 2: When n is even

$f(u_i^k) = i + (k-1)2n$; $i = 1, 2, ..., \frac{n}{2}$.

$f(u_i^k) = 2i + (k-1)2n$; $i = \frac{n}{2} + 1, ..., n$.

$f(v_i^k) = \frac{n}{2} + i + (k-1)2n$; $i = 1, 2, ..., \frac{n}{2} + 1$.

$f(v_i^k) = 2i - 1 + (k-1)2n$; $i = \frac{n}{2} + 2, ..., n$.

From the above labeling pattern, we have $|e_f(0) - e_f(1)| \leq 1$.

Hence G admits sum divisor cordial labeling.

Example 2.10 The sum divisor cordial labeling of $C(4.\ H_3)$ and $C(4.\ H_4)$ are shown in Figure 8 and Figure 9 respectively.

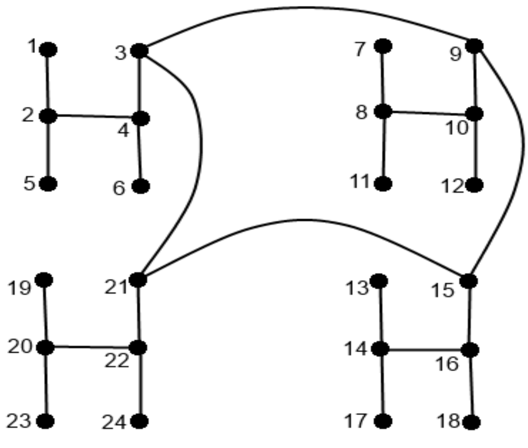


Fig 8: $C(4.\ H_3)$

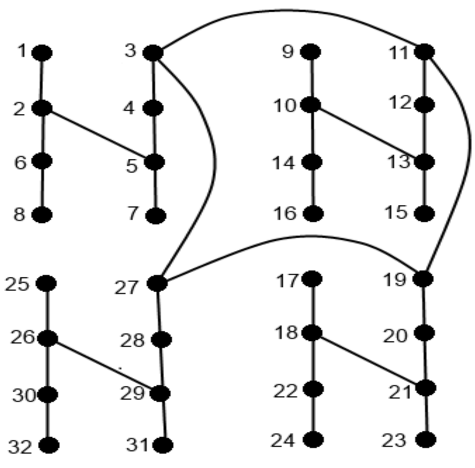


Fig 9: $C(4.\ H_4)$

3. Concluding Remarks

In this paper, we have proved that the graph H_n (n is even), duplication of all edges of H - graph H_n (n is odd), $H_n \odot K_1$, $P(r.\ H_n)$, $C(r.\ H_n)$ are sum divisor cordial graphs.

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